Comment on "Simple One-Dimensional Model of Heat Conduction which Obeys Fourier's Law"

In a recent Letter, Garrido *et al.* [1] consider heat conduction in a 1D model of N hard point particles of alternating masses. Based on numerical results, the authors claim that this momentum conserving model exhibits Fourier's law. We comment on the contradiction with an earlier result of Prosen and Campbell [2] (PC). We then point out certain inconsistencies in their results.

The authors first measure the system size dependence of the mean current $\langle J \rangle = \langle N^{-1} \sum_l m_l u_l^3/2 \rangle$, where m_l, x_l, and u_l denote the mass, position, and velocity of the lth particle. As they correctly point out, one cannot make definite conclusions from this simulation data, since the asymptotic regime may not have been reached. Next, the authors compute the correlation $C(t) = N\langle J(t)J(0)\rangle$ and find a decay $C(t) \sim t^{-1.3}$, which is sufficiently fast to give a finite Kubo conductivity κ. This contradicts the exact result of PC on infinite κ in momentum conserving systems. Their proof applies to this model. However, Garrido *et al.* work in the zero-momentum ensemble where PC makes no predictions. As pointed out in [3], the correct Kubo formula involves the connected part of $C(t)$ [4], or, one may fix the momentum to be zero as [1] have done. Thus, PC *does not* prove divergence of κ.

However, some aspects of the paper are unsatisfactory. First, the linear temperature profiles obtained are inconsistent with the finding of finite κ. The temperature (T) dependence of κ can be scaled out from the Kubo formula giving $\kappa \sim T^{1/2}$. This follows since the correlation $C_T(t)$ has the scaling form $C_T(t) = T^3 C_1(T^{1/2}t)$. Kinetic theory arguments also give $\kappa \sim T^{1/2}$. This implies nonlinear temperature profiles. In our study [5], we see the expected nonlinear profiles. The difference could be because [1] uses deterministic heat baths while we use stochastic baths. It is not clear how well such deterministic baths simulate true thermal baths. Another source of error is that [1] defines a local temperature from the mean energy and position of each particle. In 1D, position fluctuations are large ($\sim\sqrt{N}$) and the correct method is the one we use: Define local number and energy densities as $n(x) = \langle \sum_l \delta(x - x_l)\rangle$, $\epsilon(x) = \langle \sum_l (m_l u_l^2/2)\delta(x - x_l)\rangle$, and then define $T(x) = 2\epsilon(x)/n(x)$.

Second, our simulations do not verify the results of [1]. The authors computed $C(t)$ and also $c(t) = \langle j_i(t) j_i(0)\rangle$. They find, for $m_1 \neq m_2$, $C(t)$ and $c(t)$ have the same decay $\sim 1/t^{1.3}$, while for $m_1 = m_2$, $c(t) \sim 1/t^3$. Our results are summarized in Fig. 1. The main differences with [1] are: (i) we do not find any evidence for the decay $C(t) \sim 1/t^{1.3}$. The behavior we find is consistent with the decay $J \sim 1/N^{0.83}$ found in [5]. (ii) $c(t)$ behaves differently from $C(t)$ contrary to the claim in [1]. The authors comment that $c(t)$ has better averaging properties, and, because it shows roughly the same decay, this confirms the behavior seen for $C(t)$. But is there any reason to expect $C(t)$ and $c(t)$ decay similarly? In fact, for $m_1 = m_2$, $C(t)$ is a constant while $c(t)$ is not. (iii) The equal mass case is nonergodic since there are a macroscopic number of conservation laws. Thus, time averages depend on initial conditions. Our simulations verify this. We also find that making the masses slightly unequal restores ergodicity. Thus, it is hard to understand the decay $c(t) \sim 1/t^3$ obtained by [1]. We note Jepsen [6] (quoted by [1]) gives only $\langle v_i(t)v_i(0)\rangle \sim 1/t^3$ while $c(t)$ is more like $\langle v_i^3(t)v_i^3(0)\rangle$. Also, Jepsen does not treat zero-momentum ensemble. In our simulations, averages were taken over 10^9–10^{10} collisions. As checks, we found that $C(0)$ and $c(0)$ agree with exact results and that $C(t)$ and $c(t)$ satisfy the scaling forms given above.

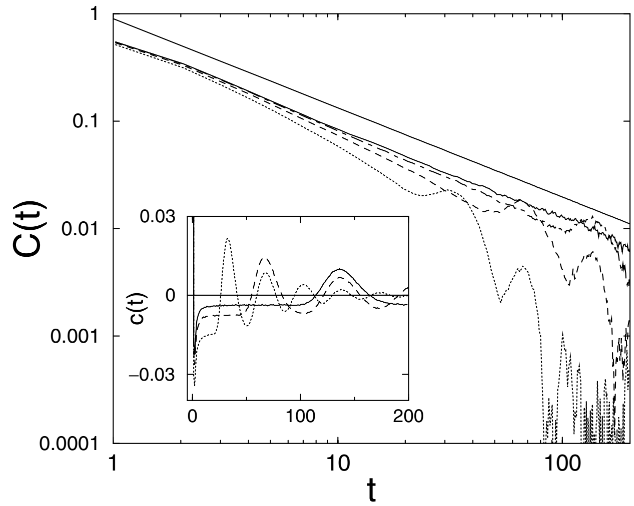

FIG. 1. Plot of $C(t)$ for $N = 100$ (dotted line), 200, 400, and 800 (solid line) ($T = 1$, $m_2/m_1 = 2$). The straight line has a slope -0.83. The inset shows $c(t)$ for $N = 100$ (dotted line), 200, and 400 (solid line).

Thus, there is no evidence for validity of Fourier's law in this model. I thank Onuttom Narayan for discussions.

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