# Comparison of search templates for gravitational waves from binary inspiral: 3.5PN update

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Phasing formulas in a recent paper of ours' [Phys. Rev. D **63**, 044023 (2001)] are updated taking into account the recent 3.5PN results. Some misprints in our recent paper are also corrected.

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## I. INTRODUCTION

In this paper we update Tables I and II in Ref. [1] [henceforth referred to as Damour-Iyer-Sathyaprakash 3 (DIS3)] in view of the recent theoretical progress made in the dynamics of, and radiation from, binary systems to 3.5 post-Newtonian (3.5PN) order [2–5]. Before giving a comprehensive list of the corresponding updates, we take this opportunity to correct some misprints in Ref. [1]. The results in [1] are not changed since they used the correct formulas free of the misprints below.

The expansion coefficients in Table I and Table II of DIS3 are all Newton-normalized coefficients. In the notation of the paper, there should be carets on all these coefficients, except $e_k$ (that is, $\hat{E}_k$, $\hat{\mathcal{F}}_k$, $\hat{t}^v_k$, $\hat{\phi}^v_k$, $\hat{\phi}^t_k$, $\hat{F}^t_k$, and $\hat{\tau}_k$). The coefficients $\hat{t}^v_5$, $\hat{F}^t_5$, and $\hat{\tau}_2$ in Table II (which were correct in the eprint version) contain typographical errors in the published version of DIS3. Their correct expressions are

$$\hat{t}^v_5=-\left(\frac{7729}{252}+\eta\right)\pi, \tag{1.1}$$

$$\hat{F}^t_5=-\left(\frac{7729}{21504}+\frac{3}{256}\eta\right)\pi, \tag{1.2}$$

$$\hat{\tau}_2=\frac{5}{9}\left(\frac{743}{84}+11\eta\right). \tag{1.3}$$

The second of the equations in Eq. (4.5) of DIS3 should read

$$p^0_\varphi=\left[\frac{r_0^2-3\eta}{r_0^3-3r_0^2+5\eta}\right]^{1/2}r_0, \tag{1.4}$$

the factor $r_0$ outside the square brackets on the right-hand side is missing in the previous versions of DIS3.

## II. UPDATES

The energy [2–4] and flux [5] functions have now been computed up to order $v^7$ in post-Newtonian theory. The corresponding expansion coefficients are as follows. The 3PN coefficients in the expansion of the various energy functions are

$$\hat{E}_3=-\frac{675}{64}+\left[\frac{34445}{576}-\frac{205\pi^2}{96}+\frac{10\omega_s}{3}\right]\eta-\frac{155}{96}\eta^2-\frac{35}{5184}\eta^3, \tag{2.1}$$

$$e_3=-9+\left(\frac{4309}{72}-\frac{205}{96}\pi^2+\frac{10}{3}\omega_s\right)\eta-\frac{103}{36}\eta^2+\frac{1}{81}\eta^3, \tag{2.2}$$

with the Padé approximant $e_{P_6}$ determined using Eq. (2.17) of DIS3, wherein $c_1$ and $c_2$ are as in DIS3 and $c_3$ is given by

$$c_3=\frac{e_1e_3-e_2^2}{e_1(e_1^2-e_2)}. \tag{2.3}$$

The dimensionless parameter $\omega_s$ (used in [2]) is related to the parameter $\lambda$ (used in [3]) by $\omega_s=-1987/840-11\lambda/3$, so that we alternatively have

$$\hat{E}_3=-\frac{675}{64}+\left[\frac{209323}{4032}-\frac{205\pi^2}{96}-\frac{110\lambda}{9}\right]\eta-\frac{155}{96}\eta^2-\frac{35}{5184}\eta^3, \tag{2.4}$$

$$e_3=-9+\left(\frac{26189}{504}-\frac{205}{96}\pi^2-\frac{110}{9}\lambda\right)\eta-\frac{103}{36}\eta^2+\frac{1}{81}\eta^3. \tag{2.5}$$

The numerical value of $\omega_s$ has been recently determined by dimensional regularization [4] to be simply equal to $\omega_s=0$, which corresponds to $\lambda=-1987/3080$. [Note that there is a sign misprint in the second term on the right-hand side of Eq. (4.7) in the last reference in [2]; it should read $\lambda=-3\omega_s/11-1987/3080$.] Concerning the 3PN update of the effective one-body Hamiltonian, it is explicitly given in Sec. IV D of the second reference in [2].

The expansion coefficients in Table II of DIS3 at 3PN and 3.5PN are as follows. The coefficients in the expansion of the flux are

 

$$\hat{\mathcal{F}}_6^v = \frac{6643739519}{69854400} + \frac{16\pi^2}{3} - \frac{1712}{105}\gamma + \left( -\frac{11497453}{272160} + \frac{41\pi^2}{48} + \frac{176}{9}\lambda - \frac{88}{3}\Theta \right)\eta - \frac{94403}{3024}\eta^2 - \frac{775}{324}\eta^3, \tag{2.6}$$

$$\hat{\mathcal{F}}_7^v = \left( -\frac{16285}{504} + \frac{176419}{1512}\eta + \frac{19897}{378}\eta^2 \right)\pi, \tag{2.7}$$

and $\hat{\mathcal{F}}_{l6}^v = -\frac{1712}{105}$. Here $\gamma$ is the Euler constant, $\gamma = 0.577\ldots$, and $\Theta$ and $\lambda$ are two undetermined parameters in Ref. [5]. We use the letter $\Theta$ to denote what is denoted by $\theta$ in [5] (this should not be confused with the related undetermined 3PN quantity $\hat{\theta} = \theta - 7\lambda/3$ which is also used in some formulas of [5]). The $\lambda$ appearing in the flux function is the same quantity as in the energy function, arising, as it does, from the time derivatives of the mass quadrupole moment involved in computing the far-zone flux. The quantity $\hat{F}_{l6}^v$ is the coefficient of the logarithmic term that arises, for the first time, at the 3PN order; to the usual *Newton-normalized* Taylor expansion [6] one must add $\hat{\mathcal{F}}_{l6}^v \log(4v) v^6$ to complete the PN expansion. Beware that if expressions are rewritten in terms of $\omega_s$ rather than $\lambda$, the rational numerical coefficient in the $\eta$ term will change. For ready reckoning, in the above and subsequent formulas, we indicate this explicitly as follows: $[-11497453/272160 + (176\lambda)/9 \rightarrow -14930989/272160 - (16\omega_s)/3]$. This means the flux formula may be alternatively written in terms of $\omega_s$ by the indicated replacement.

Coefficients in the expansion of time as a function of the invariant velocity parameter $v = (\pi m f)^{1/3}$, where $f$ is the gravitational-wave frequency, are

$$\hat{t}_6^v = -\frac{10052469856691}{23471078400} + \frac{128}{3}\pi^2 + \left( \frac{15335597827}{15240960} - \frac{451}{12}\pi^2 + \frac{352}{3}\Theta - \frac{2464}{9}\lambda \right)\eta + \frac{6848}{105}\gamma - \frac{15211}{1728}\eta^2 + \frac{25565}{1296}\eta^3, \tag{2.8}$$

$$\hat{t}_7^v = \left( -\frac{15419335}{127008} - \frac{202823}{1512}\eta + \frac{7373}{189}\eta^2 \right)\pi, \tag{2.9}$$

and $\hat{t}_{l6}^v = \frac{6848}{105}$, where $\hat{t}_{l6}^v$ is the coefficient of the logarithmic term; to the usual Newton-normalized Taylor expansion one must add $\hat{t}_{l6}^v \log(4v) v^6$ to complete the PN expansion $[15335597827/15240960 - (2464\lambda)/9 \rightarrow 18027490051/15240960 + (224\omega_s)/3]$.

Coefficients in the expansion of the gravitational wave phase as a function of the invariant velocity $v$ are

$$\hat{\phi}_6^v = \frac{12348611926451}{18776862720} - \frac{160}{3}\pi^2 - \frac{1712}{21}\gamma + \left( -\frac{15335597827}{12192768} + \frac{2255}{48}\pi^2 + \frac{3080}{9}\lambda - \frac{440}{3}\Theta \right)\eta + \frac{76055}{6912}\eta^2 - \frac{127825}{5184}\eta^2, \tag{2.10}$$

$$\hat{\phi}_7^v = \left( \frac{77096675}{2032128} + \frac{1014115}{24192}\eta - \frac{36865}{3024}\eta^2 \right)\pi, \tag{2.11}$$

and $\hat{\phi}_{l6}^v = -\frac{1712}{21}$, where, as in the previous cases, $\hat{\phi}_{l6}^v$ is the coefficient of the logarithmic term; to the usual Newton-normalized Taylor expansion one must add $\hat{\phi}_{l6}^v \log(4v) v^6$ to complete the PN expansion $[-15335597827/12192768 + (3080\lambda)/9 \rightarrow -18027490051/12192768 - (280\omega_s)/3]$.

The expansion coefficients of the phase as a function of the time parameter[1] $\theta = [\eta(t_{\rm ref} - t)/(5m)]^{-1/8}$, where $t_{\rm ref}$ is a reference time at which the PN-expanded $GW$ frequency formally goes to infinity, are given by

$$\hat{\phi}_6^t = \frac{831032450749357}{57682522275840} - \frac{53}{40}\pi^2 - \frac{107}{56}\gamma + \left( -\frac{123292747421}{4161798144} + \frac{2255}{2048}\pi^2 + \frac{385}{48}\lambda - \frac{55}{16}\Theta \right)\eta + \frac{154565}{1835008}\eta^2 - \frac{1179625}{1769472}\eta^3, \tag{2.12}$$

$$\hat{\phi}_7^t = \left( \frac{188516689}{173408256} + \frac{140495}{114688}\eta - \frac{122659}{516096}\eta^2 \right)\pi, \tag{2.13}$$

and $\hat{\phi}_{l6}^t = -\frac{107}{56}$, where, as before, $\hat{\phi}_{l6}^t$ is the coefficient of the logarithmic term; to the usual Newton-normalized Taylor expansion one must add $\hat{\phi}_{l6}^t \log(2\theta)\theta^6$ to complete the PN expansion $[-123292747421/4161798144 + (385\lambda)/48 \rightarrow -144827885213/4161798144 - (35\omega_s)/16]$.

Finally, coefficients in the expansion of the gravitational-wave frequency in terms of the time parameter $\theta$ are given by

$$\hat{F}_6^t = -\frac{720817631400877}{288412611379200} + \frac{53}{200}\pi^2 + \frac{107}{280}\gamma + \left( \frac{123292747421}{20808990720} - \frac{451}{2048}\pi^2 - \frac{77}{48}\lambda + \frac{11}{16}\Theta \right)\eta - \frac{30913}{1835008}\eta^2 + \frac{235925}{1769472}\eta^3, \tag{2.14}$$

[1]The dimensionless time variable $\theta$ here is related to $\tau$ in [3] by $\theta = \tau^{-1/8}$ leading to minor differences in the coefficients here and in [3].

$$\hat{F}^t_7 = \left( -\frac{188516689}{433520640} - \frac{28099}{57344}\eta + \frac{122659}{1290240}\eta^2 \right)\pi \tag{2.15}$$

and $\hat{F}^t_{l6} = \frac{107}{280}$, where $\hat{F}^t_{l6}$ is the coefficient of the logarithmic term; to the usual Newton-normalized Taylor expansion one must add $\hat{F}^t_{l6}\log(2\theta)\theta^6$ to complete the PN expansion [$123292747421/20808990720 - (77\lambda)/48 \rightarrow 144827885213/20808990720 + (7\omega_s)/16$].

In computing Padé coefficients of the *new* flux function [6] one needs the first seven continued fraction coefficients. The first six of these are as in Appendix A of Ref. [6], except that the last term in the first line of $c_6$ should be $c_3^2(c_2 - c_1)$. The coefficient $c_7$ is too long to be quoted in this Brief Report; an electronic version can be obtained from the authors.

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# Erratum: Comparison of search templates for gravitational waves from binary inspiral: 3.5PN update [Phys. Rev. D 66, 027502 (2002)]

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Recently [1,2], two errata have appeared correcting some coefficients in the computation of tails in the flux of gravitational waves $\mathcal{L}$ from compact binaries in Refs. [3,4]. This leads to some incorrect post-Newtonian coefficients in Refs. [5,6] at orders 2.5PN and 3.5PN, which have also been corrected in [7,8]. The paper [9] is based on Refs. [5,6] and as a consequence, some post-Newtonian coefficients there are also modified. The correction affects only the $\eta$-dependent coefficients at order 2.5PN and 3.5PN there and hence in the present paper. The other coefficients are not modified. These modify Eqs. (1.1), (1.2), (2.7), (2.9), (2.11), (2.13) and (2.15) which should now read:

$$\hat{t}_5^v = -\left(\frac{7729}{252} - \frac{13}{3}\eta\right)\pi, \tag{1.1}$$

$$\hat{F}_5^t = -\left(\frac{7729}{21504} - \frac{13}{256}\eta\right)\pi, \tag{1.2}$$

$$\hat{\mathcal{F}}_7^v = \left(-\frac{16285}{504} + \frac{214745}{1728}\eta + \frac{193385}{3024}\eta^2\right)\pi \tag{2.7}$$

$$\hat{t}_7^v = \left(-\frac{15419335}{127008} - \frac{75703}{756}\eta + \frac{14809}{378}\eta^2\right)\pi \tag{2.9}$$

$$\hat{\phi}_7^v = \left(\frac{77096675}{2032128} + \frac{378515}{12096}\eta - \frac{74045}{6048}\eta^2\right)\pi \tag{2.11}$$

$$\hat{\phi}_7^t = \left(\frac{188516689}{173408256} + \frac{488825}{516096}\eta - \frac{141769}{516096}\eta^2\right)\pi \tag{2.13}$$

$$\hat{F}_7^t = \left(-\frac{188516689}{433520640} - \frac{97765}{258048}\eta + \frac{141769}{1290240}\eta^2\right)\pi. \tag{2.15}$$

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