Modified photon equation of motion as a test for the principle of equivalence

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We have considered a modified equation of motion based on the principle of covariance. Some astronomical observations are used to place limits on the presence of the extra terms in the modified equation.

The equivalence principle has been put to several tests, including a recent one based on the analysis of the differential time delay between the arrival of left- and right-handed circularly polarized (LCP and RCP) signals from the millisecond pulsar PSR 1937+214 [1,2]. The analysis carried out so far has been based on phenomenological arguments invoking some symmetry-breaking terms which can be added to the ordinary Newtonian potential. The equivalence principle dictates that the equation of motion of a freely falling body be given by the geodesic equation. On the other hand, the principle of general covariance is not so restrictive. For example, the geodesic equation could be modified [3] for a freely falling particle of spin S_δ to

$$\frac{d^2x^a}{dp^2}+\Gamma^\alpha_{\beta\gamma}\frac{dx^\beta}{dp}\frac{dx^\gamma}{dp}+fR^{\ \alpha}_{\beta\gamma\delta}\frac{dx^\beta}{dp}\frac{dx^\gamma}{dp}S^\delta=0\ , \quad (1)$$

where p is a parameter specifying the position of a particle on its trajectory. It has been argued that, since the arbitrary constant f has dimension of length, the ratio of the third term to the second term would be roughly of order d/D, where d and D are the characteristic linear dimension of the particle and the characteristic spacetime dimension, respectively. The contribution of the last term would, therefore, be negligible. It is of interest to see whether any observational limit on such a term, however small, can be obtained. In this note, we will examine the pulsar data on the basis of Eq. (1), thereby arriving at an estimate of the coupling constant f, if such a term exists.

We consider the motion of a light ray in the Schwarzschild metric, based on Eq. (1). We note that the last term in the equation is parity violating. This leads to different travel times for LCP and RCP light rays. We write

$$ds^2=-B(r)dt^2+A(r)dr^2+r^2(d\theta^2+\sin^2\theta d\phi^2)\ , \quad (2)$$

where $B(r)=A^{-1}(r)=1-2GM/(c^2r)=1-2m/r$. In Eq. (1), we can always choose $S^t=0$. We also find that if $S^\theta=0$, then $\theta=\pi/2$ is a solution of the second-order differential equation in θ. So, in this case, the motion can be confined to the $\theta=\pi/2$ invariant plane. The other three equations of motion can be written as

$$0=\frac{d^2t}{dp^2}+\frac{B'}{B}\frac{dr}{dp}\frac{dt}{dp}+f\frac{B''}{2B}\frac{dr}{dp}\frac{dt}{dp}S^r+f\frac{B'r}{2}\frac{d\phi}{dp}\frac{dt}{dp}S^\phi\ , \quad (3a)$$

$$0=\frac{d^2r}{dp^2}-\frac{B'}{2B}\left[\frac{dr}{dp}\right]^2-rB\left[\frac{d\phi}{dp}\right]^2+\frac{1}{2}B'B\left[\frac{dt}{dp}\right]^2+f\left[\frac{B''B}{2}\left[\frac{dt}{dp}\right]^2-\frac{B'r}{2}\left[\frac{d\phi}{dp}\right]^2\right]S^r+f\frac{B'r}{2}\frac{d\phi}{dp}\frac{dr}{dp}S^\phi\ , \quad (3b)$$

$$0=\frac{d^2\phi}{dp^2}+\frac{2}{r}\frac{dr}{dp}\frac{d\phi}{dp}+f\left[\frac{B'B}{2r}\left[\frac{dt}{dp}\right]^2-\frac{B'}{2Br}\left[\frac{dr}{dp}\right]^2\right]S^\phi+f\frac{B'}{2Br}\frac{dr}{dp}\frac{d\phi}{dp}S^r\ . \quad (3c)$$

We will now solve these equations, keeping only terms up to order $m=GM/c^2$. Since the last term in Eq. (1) contains $R^{\ \alpha}_{\beta\gamma\delta}$ which is of order m, we can substitute values for the other quantities in this term, namely dx^β/dp and S^δ, by their zeroth-order ones. That is, these values are obtained by treating the light ray as traveling in a straight line, as shown in Fig. 1.

The zeroth-order values are

$$\frac{dr}{dp}=\sin\phi,\quad \frac{dt}{dp}=1,\quad \frac{d\phi}{dp}=\frac{r_0}{r^2}=\frac{\cos^2\phi}{r_0}\ ,$$

where r_0 is the radial distance of closest approach of the light ray. Thus, for the RCP beam, we substitute $S^r=\sin\phi$ and $S^\phi=\cos\phi/r$ in the terms containing spin in Eqs. (3a)–(3c). Then the Eq. (3a) gives

$$\frac{d^2t}{dp^2}+\frac{B'}{B}\frac{dr}{dp}\frac{dt}{dp}+fm\left[-\frac{2}{r^3}\sin\phi\frac{dr}{dp}+\frac{\cos^3\phi}{r_0r^2}\right]=0 \quad (4)$$

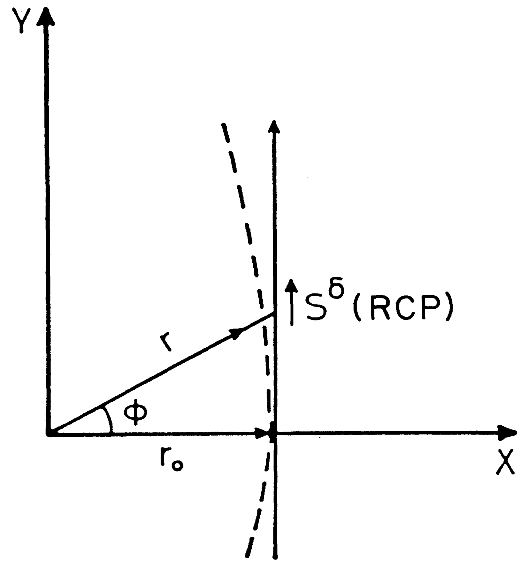

FIG. 1. The geometry of the light ray and the helicity vector.

which can be integrated to obtain

$$\frac{dt}{dp}B+\frac{fm}{r_0^2}\sin\phi\cos^2\phi=1\ , \tag{5}$$

where the parameter p has been appropriately chosen to make the constant of motion 1. Then

$$\frac{dt}{dp}=\frac{1}{B}-\frac{fm}{r^3}(r^2-r_0^2)^{1/2}\ . \tag{6}$$

From Eq. (3c), we obtain

$$0=\frac{d^2\phi}{dp^2}+\frac{2}{r}\frac{dr}{dp}\frac{d\phi}{dp}+\frac{fm}{r_0r^3}\cos^2\phi\sin\phi\frac{dr}{dp}+\frac{fm}{r^4}\cos^3\phi \tag{7}$$

which can be integrated to give

$$\frac{d\phi}{dp}=\frac{k}{r^2}-\frac{fm}{r_0r^3}(r^2-r_0^2)^{1/2}\ , \tag{8}$$

where k is a constant of motion. As $m\to 0$, $(d\phi/dp)\to r_0/r^2$. So, $k=r_0+O(m)$. Now, Eq. (3b) can be written as

$$\frac{d^2r}{dp^2}-\frac{B'}{2B}\left[\frac{dr}{dp}\right]^2-rB\left[\frac{d\phi}{dp}\right]^2+\frac{m}{r^2}\left[\frac{dt}{dp}\right]^2-\frac{2fm}{r^3}\sin\phi=0\ . \tag{9}$$

Using Eqs. (6) and (8) and upon integration, we find that all the f terms cancel, giving

$$\frac{1}{B}\left[\frac{dr}{dp}\right]^2+\frac{k^2}{r^2}-\frac{1}{B}=0\ . \tag{10}$$

From Eqs. (6) and (10) we arrive at a dr/dt equation, which is integrated to obtain the expression for the time required by the light ray to traverse the path between r_1 to r_2 to be

$$T(r_2,r_1)=t(r_2,r_1)+\frac{fm}{c}\left[\frac{1}{r_2}-\frac{1}{r_1}\right]\ , \tag{11}$$

where $t(r_2,r_1)$ is the corresponding time required by the light beam, if $f=0$. Therefore, the difference between the arrival times of RCP and LCP beams is

$$\Delta t=\frac{2fm}{c}\left[\frac{1}{r_2}-\frac{1}{r_1}\right]\ . \tag{12}$$

Similarly, using Eqs. (8) and (10), we obtain the $d\phi/dr$ equation, which, upon integration, gives an extra term $(=fm/r_0^2)$ in the expression for the bending of the light ray. Therefore, the difference in the deviation between RCP and LCP rays is

$$\Delta\phi=\frac{2fm}{r_0^2}\ . \tag{13}$$

First, let us consider experiments based on the bending of light. The experiments confirm [4] the prediction of the bending of light through an angle $4m/r_0$ to an accuracy of 2%. Had the bending of LCP and RCP rays been different, then there would have been a spread of light beam and an upper limit can be obtained from

$$\left|\frac{\Delta\phi}{4m/r_0}\right|\le 0.02\ ,$$

leading to

$$|f|\le 1.86\times 10^{-4}\ \text{AU}\ . \tag{14}$$

A recent analysis using pulsar measurements to place limits on gravitational symmetry violations can, in principle, be taken over to estimate limits on f. Recently, Klein and Thorsett [2] have reported a measurement of delay between RCP and LCP signals coming from PSR 1937+214 to 0.37 ± 0.67 μsec. The pulsar is located at a distance of 2.5 kpc from Earth and 7.45 kpc from the galactic center. The distance from Earth to the galactic centre is 8.5 kpc. If we wish to use this data to put a limit on f, we shall have to make the approximation of treating the Galaxy as a mass point of mass 6×10^{11} $M_\odot$. Since the pulsar is inside the Galaxy, this would be a rather crude approximation. Then setting $\Delta t\le 10^{-6}$ sec, we obtain, from Eq. (12),

$$f\le 2.10\times 10^{-3}\ \text{AU}\ . \tag{15}$$

However, the sharpest constraint on f can be obtained, if we consider the effect of the pulsar's field, rather than that of the galaxy, on these photons. For neutron stars, $m_p/r_p>0.1$ is typical, where m_p and r_p are mass and radius of a pulsar respectively. Therefore, taking into account the observed delay of 10^{-6} sec, Eq. (12) then implies

$$f<10^{-7}\ \text{AU}\ . \tag{16}$$

The strength of this constraint, however, depends on where on the neutron star's surface the pulse radiation originates.

The supernova SN 1987A data could also be used for obtaining some information on f. It has been concluded [5,6] that the difference between the arrival time of light and neutrinos could, at most, be six hours. One expects a time difference of the order of

$$\Delta t \sim |\Delta t_\nu - \Delta t_\gamma| \sim 2m \left[f_\gamma - \frac{1}{2} f_\nu \right] \left[\frac{1}{r_2} - \frac{1}{r_1} \right] . \quad (17)$$

The supernova-to-Earth distance is about 50 kpc. Then assuming $f_\gamma \sim f_\nu \sim f$ and $\Delta t \leq 6$ h, we get an upper limit on f as

$$f \leq 10^7 \text{ AU} , \quad (18)$$

which is much weaker than the other upper limits. On the other hand, the rather large time delay involved here may have to be attributed to causes totally different from modifications to the geodesic equation.

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