Gravitational-Radiation Damping of Compact Binary Systems to Second Post-Newtonian Order

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(Received 20 January 1995)

The rate of gravitational-wave energy loss from inspiralling binary systems of compact objects of arbitrary mass is derived through second post-Newtonian (2PN) order $O((Gm/rc^2)^2)$ beyond the quadrupole approximation. The result has been derived by two independent calculations of the (source) multipole moments. The 2PN terms, and, in particular, the finite mass contribution therein (which cannot be obtained in perturbation calculations of black hole spacetimes), are shown to make a significant contribution to the accumulated phase of theoretical templates to be used in matched filtering of the data from future gravitational-wave detectors.

PACS numbers: 04.30.Db, 04.25.Nx, 97.60.Jd, 97.80.Fk

One of the most promising astrophysical sources of gravitational radiation for detection by large-scale laser-interferometer systems such as the U.S. LIGO or the French-Italian VIRGO projects [1] is the inspiralling compact binary. This is a binary system of neutron stars or black holes whose orbit is decaying toward a final coalescence under the dissipative effect of gravitational radiation reaction. For much of the evolution of such systems, the gravitational wave-form signal is an accurately calculable "chirp" signal that sweeps in frequency through the dectectors' sensitive bandwidth, typically between 10 and 1000 Hz. Estimates of the rate of such inspiral events range from 3 to 100 per year, for signals detectable out to hundreds of Mpc by the advanced version of LIGO [2].

In addition to outright detection of the waves, it will be possible to determine parameters of the inspiralling systems, such as the masses and spins of the bodies [3–5], to measure cosmological distances [6], to probe the nonlinear regime of general relativity [7], and to test alternative gravitational theories [8]. This is made possible by the technique of matched filtering of theoretical waveform templates, which depend on the source parameters, against the broadband detector output [9].

Roughly speaking, any effect that causes the template to differ from the actual signal by one cycle over the 500 to 16 000 accumulated cycles in the sensitive bandwidth will result in a reduction in the signal-to-noise ratio. This necessitates knowing the prediction of general relativity for gravitational radiation damping, and its effect on the orbital phase, to substantially higher accuracy than that provided by the lowest-order quadrupole or Newtonian approximation. If post-Newtonian corrections to the quadrupole formula scale as powers of $v^2 \approx m/r$ ($G = c = 1$), then, say, for a double neutron-star inspiral in the LIGO/VIRGO bandwidth, with m/r typically around 10^{-2}, corrections *at least* of order $(m/r)^2 \sim 10^{-4}$ will be needed in order to be accurate to one cycle out of the 16 000 cycles accumulated for this process. This corresponds to corrections of *second* post-Newtonian (2PN) order.

Although numerous corrections to the quadrupole energy-loss formula have previously been calculated (for a summary, see [10]), the 2PN contributions to the energy loss rate for arbitrary masses have not been derived. This paper presents these contributions for the first time, discusses their significance for gravitational-wave data analysis, and outlines the derivation.

The central result is as follows: Through 2PN order, the rate of energy loss, dE/dt, from a binary system of two compact bodies of mass m_1 and m_2, orbital separation r, and spins $\mathbf{S_1}$, and $\mathbf{S_2}$ in a nearly circular orbit (apart from the adiabatic inspiral) is given by

$$-\frac{dE}{dt} = \frac{32}{5}\eta^2\left(\frac{m}{r}\right)^5\left[1 - \frac{m}{r}\left(\frac{2927}{336} + \frac{5}{4}\eta\right) + \left(\frac{m}{r}\right)^{3/2}\left\{4\pi - \frac{1}{12}\sum\nolimits_i\left(73\frac{m_i^2}{m^2} + 75\eta\right)\hat{\mathbf{L}}\cdot\boldsymbol{\chi}_i\right\} + \left(\frac{m}{r}\right)^2\left\{\frac{293\,383}{9072} + \frac{380}{9}\eta - \frac{\eta}{48}\left(223\boldsymbol{\chi}_1\cdot\boldsymbol{\chi}_2 - 649\hat{\mathbf{L}}\cdot\boldsymbol{\chi}_1\hat{\mathbf{L}}\cdot\boldsymbol{\chi}_2\right)\right\}\right], \quad (1)$$

where $\hat{\mathbf{L}}$ is a unit vector directed along the orbital angular momentum, $m = m_1 + m_2$, $\eta = m_1 m_2/m^2$, $\boldsymbol{\chi}_1 = \mathbf{S}_1/m_1^2$, $\boldsymbol{\chi}_2 = \mathbf{S}_2/m_2^2$, and $\sum_i$ denotes the sum over $i = 1, 2$. The terms in square brackets in Eq. (1) are, respectively, at lowest order, Newtonian (quadrupole); at order m/r, 1PN [11]; at order $(m/r)^{3/2}$, the nonlinear effect of "tails" (4π term) [12,13] and spin-orbit effects [14,15]; and at order $(m/r)^2$, the 2PN terms (new with this paper), and spin-spin effects [14–16].

For the special case of a test mass orbiting a massive black hole, perturbation theory has been used to derive an analogous analytic formula (apart from spin-spin effects) [12,17], and for nonrotating holes, to extend the expansion through the equivalent of 4PN order [18]. The test-body $\eta = 0$ limit of Eq. (1) agrees completely with these results to the corresponding order.

The equations of motion for circular orbits, correct to 2PN order including spin effects, yield for the orbital angular velocity $\omega \equiv v/r$ and the orbital energy [10,14,19]

$$\omega^2 = \frac{m}{r^3}\left[1 - \frac{m}{r}(3-\eta) - \left(\frac{m}{r}\right)^{3/2}\sum_i\left(2\frac{m_i^2}{m^2} + 3\eta\right)\hat{\mathbf{L}}\cdot\boldsymbol{\chi}_i + \left(\frac{m}{r}\right)^2\left\{6 + \frac{41}{4}\eta + \eta^2 - \frac{3\eta}{2}(\boldsymbol{\chi}_1\cdot\boldsymbol{\chi}_2 - 3\hat{\mathbf{L}}\cdot\boldsymbol{\chi}_1\hat{\mathbf{L}}\cdot\boldsymbol{\chi}_2)\right\}\right], \tag{2a}$$

$$-E = \eta\frac{m^2}{2r}\left[1 - \frac{1}{4}\frac{m}{r}(7-\eta) + \left(\frac{m}{r}\right)^{3/2}\sum_i\left(2\frac{m_i^2}{m^2} + \eta\right)\hat{\mathbf{L}}\cdot\boldsymbol{\chi}_i + \left(\frac{m}{r}\right)^2\left\{\frac{1}{8}(-7 + 49\eta + \eta^2) + \frac{\eta}{2}(\boldsymbol{\chi}_1\cdot\boldsymbol{\chi}_2 - 3\hat{\mathbf{L}}\cdot\boldsymbol{\chi}_1\hat{\mathbf{L}}\cdot\boldsymbol{\chi}_2)\right\}\right]. \tag{2b}$$

Combining Eqs. (1) and (2), one can express the rate of change $\dot{\omega}$ of the angular velocity as a function of ω and get

$$\dot{\omega} = \frac{96}{5}\eta m^{5/3}\omega^{11/3}\left[1 - \left(\frac{743}{336} + \frac{11}{4}\eta\right)(m\omega)^{2/3} + (4\pi - \beta)(m\omega) + \left(\frac{34\,103}{18\,144} + \frac{13\,661}{2016}\eta + \frac{59}{18}\eta^2 + \sigma\right)(m\omega)^{4/3}\right], \tag{3}$$

where the spin-orbit (β) and spin-spin (σ) parameters are given by $\beta = \frac{1}{12}\sum_i(113m_i^2/m^2 + 75\eta)\hat{\mathbf{L}}\cdot\boldsymbol{\chi}_i$ and $\sigma = (\eta/48)(-247\boldsymbol{\chi}_1\cdot\boldsymbol{\chi}_2 + 721\hat{\mathbf{L}}\cdot\boldsymbol{\chi}_1\hat{\mathbf{L}}\cdot\boldsymbol{\chi}_2)$. From that one calculates the accumulated number of gravitational-wave cycles $\mathcal{N} = \int(f/\dot{f})df$, where $f = \omega/\pi$ is the frequency of the quadrupolar waves, in terms of the frequencies at which the signal enters and leaves the detectors' bandwidth. In order to avoid complications caused by spin-induced precessions of the orbital plane [15,20], we assume that the spins are aligned parallel to the orbital angular momentum (in particular, β and σ remain constant). Using 10 Hz as the entering frequency of LIGO/VIRGO-type detectors, set by seismic noise, and, as the exit frequency, the smaller of either 1000 Hz (set by photon-shot noise) or the frequency corresponding to the innermost stable circular orbit and the onset of plunge (for small mass ratio, $f_{\max} = 1/6^{3/2}\pi m$ [21]), we find contributions to the total number of observed wave cycles from the various post-Newtonian terms listed in Table I.

TABLE I. Contributions to the accumulated number $\mathcal{N}$ of gravitational-wave cycles in a LIGO/VIRGO-type detector. Frequency entering the bandwidth is 10 Hz (seismic limit); frequency leaving the detector is 1000 Hz for two neutron stars (photon shot noise), and ~360 and ~190 Hz for the two cases involving black holes (innermost stable orbit). Spin parameters β and σ are defined in the text. Numbers in parentheses indicate contribution of finite-mass (η) effects.

	$2\times1.4M_\odot$	$10M_\odot + 1.4M_\odot$	$2\times10M_\odot$
Newtonian	16 050	3580	600
First PN	439(104)	212(26)	59(14)
Tail	−208	−180	−51
Spin-orbit	17β	14β	4β
Second PN	9(3)	10(2)	4(1)
Spin-spin	-2σ	-3σ	$-\sigma$

Because $\chi \leq 1$ for black holes, and $\lesssim$0.63–0.74 for neutron stars (depending on the equation of state, see [22]), β and σ are always less than ~9.4 and ~2.5, respectively. However, if we consider models for the past and future evolution of observed binary pulsar systems such as PSR 1534 + 12 and PSR 1913 + 16, we find (using a conservative upper limit for moments of inertia) that $\chi_1^{1534+12} < 5.2\times10^{-3}$, $\chi_1^{1913+16} < 6.5\times10^{-3}$, and we expect $\chi_2 \lesssim 7\times10^{-4}$. If such values are typical, both the spin-orbit and (*a fortiori*) the spin-spin terms will make negligible contributions to the accumulated phase.

Table I demonstrates that the 2PN terms, and notably the finite-mass (η-dependent) contributions therein (which cannot be obtained by test-body approaches), make a significant contribution to the accumulated phase, and thus must be included in theoretical templates to be used in matched filtering. The additional question of how the presence of these terms will affect the accuracy of estimation of parameters in the templates can only be answered reliably using a full matched filter analysis [4,5]. This is currently in progress [23].

The remainder of this paper outlines the derivations leading to this result. Two entirely independent calculations were carried out, using different approaches, one by (BDI), using their previously developed generation formalism [24,25], the other by Will and Wiseman (WW), using a formal slow-motion expansion originated by Epstein and Wagoner [26]. Details of these calculations will be published elsewhere [27].

Both approaches begin with Einstein's equations written in harmonic coordinates (see [25] for definitions and notation). We define the field $h^{\alpha\beta}$, measuring the deviation of the "gothic" metric from the Minkowski metric $\eta^{\alpha\beta} = \mathrm{diag}(-1,1,1,1)$: $h^{\alpha\beta} = \sqrt{-g}g^{\alpha\beta} - \eta^{\alpha\beta}$. (Greek indices range from 0 to 3, while latin indices range from

1 to 3.) Imposing the harmonic coordinate condition $\partial_\beta h^{\alpha\beta} = 0$ then leads to the field equations

$$\Box h^{\alpha\beta} = 16\pi(-g)T^{\alpha\beta} + \Lambda^{\alpha\beta}(h) \equiv 16\pi\tau^{\alpha\beta}, \quad (4)$$

where $\Box$ denotes the flat spacetime d'Alembertian operator, $T^{\alpha\beta}$ is the matter stress-energy tensor, and $\Lambda^{\alpha\beta}$ is an effective gravitational source containing the nonlinearities of Einstein's equations. It is a series in powers of $h^{\alpha\beta}$ and its derivatives; both quadratic and cubic nonlinearities in $\Lambda^{\alpha\beta}$ play an essential role in our calculations.

Post-Minkowski matching approach (BDI).—This approach proceeds through several steps. The first consists of constructing an iterative solution of Eq. (4) in an inner domain (or near zone) that includes the material source but whose radius is much less than a gravitational wavelength. Defining source densities $\sigma = T^{00} + T^{kk}$, $\sigma_i = T^{0i}$, $\sigma_{ij} = T^{ij}$, and the retarded potentials $V = -4\pi\Box_R^{-1}\sigma$, $V_i = -4\pi\Box_R^{-1}\sigma_i$, and $W_{ij} = -4\pi\Box_R^{-1}[\sigma_{ij} + (4\pi)^{-1}(\partial_i V\partial_j V - \frac{1}{2}\delta_{ij}\partial_k V\partial_k V)]$, where $\Box_R^{-1}$ denotes the usual flat spacetime retarded integral, one obtains the inner metric $h_{\rm in}^{\alpha\beta}$ to some intermediate accuracy $O(6,5,6)$: $h_{\rm in}^{00} = -4V + 4(W_{ii} - 2V^2) + O(6)$, $h_{\rm in}^{0i} = -4V_i + O(5)$, $h_{\rm in}^{ij} = -4W_{ij} + O(6)$, where $O(n)$ means a term of order ε^n in the post-Newtonian parameter $\varepsilon \sim v/c$. From this, one constructs the inner metric with the higher accuracy $O(8,7,8)$ needed for subsequent matching as $h_{\rm in}^{\alpha\beta} = \Box_R^{-1}[16\pi\overline{\tau}^{\alpha\beta}(V,W)] + O(8,7,8)$, where $\overline{\tau}^{\alpha\beta}(V,W)$ denotes the right-hand side of Eq. (4) when retaining all the quadratic and cubic nonlinearities to the required post-Newtonian order in the near zone, and given as explicit combinations of derivatives of V, V_i, and W_{ij}. The second step consists of constructing a generic solution of the *vacuum* Einstein equations [Eq. (4) with $T^{\alpha\beta} = 0$] in the form of a multipolar-post-Minkowskian expansion that is valid in an external domain which overlaps with the near zone and extends into the far wave zone. The construction of $h^{\alpha\beta}$ in the external domain is done algorithmically as a functional of a set of parameters, called the "canonical" multipole moments $M_{i_1,\ldots i_l}(t)$, $S_{i_1,\ldots i_l}(t)$ which are symmetric and trace-free (STF) Cartesian tensors. Schematically, $h_{\rm ext}^{\alpha\beta} = \mathcal{F}^{\alpha\beta}[M_L, S_L]$, where $L \equiv i_1, \ldots i_l$ and where the functional dependence includes a nonlocal time dependence on the past "history" of $M_L(t)$ and $S_L(t)$. The third, "matching" step consists of requiring that the inner and external metrics be equivalent (modulo a coordinate transformation) in the overlap between the inner and the external domains. This requirement determines the relation between the canonical moments and the inner metric (itself expressed in terms of the source variables). Performing the matching through 2PN order [25] thus determines $M_L(t) = I_L[\overline{\tau}^{\alpha\beta}] + O(5)$, $S_L(t) = J_L[\overline{\tau}^{\alpha\beta}] + O(4)$, where the "source" moments I_L and J_L are given by some mathematically well-defined (analytically continued) integrals of the quantity $\overline{\tau}^{\alpha\beta}(V,W)$ which appeared as source of $h_{\rm in}^{\alpha\beta}$. When computing the source moments we neglect all finite size effects, such as spin (which to 2PN accuracy can be added separately) and internal quadrupole effects. The final result for the 2PN quadrupole moment reads, in the case of a *circular* binary,

$$I_{ij} = {\rm STF}_{ij}\eta m[Ar^i r^j + Br^2 v^i v^j], \quad (5)$$

with $A = 1 - \frac{1}{42}(m/r)(1 + 39\eta) - \frac{1}{1512}(m/r)^2(461 + 18\,395\eta + 241\eta^2)$ and $B = \frac{11}{21}(1 - 3\eta) + \frac{1}{378}(m/r) \times (1607 - 1681\eta + 229\eta^2)$. The final step consist of computing from the external metric $h_{\rm ext}^{\alpha\beta}$ the gravitational radiation emitted at infinity. This entails introducing a (nonharmonic) "radiative" coordinate system $X^\mu = (T, X^i)$ adapted to the falloff of the metric at future null infinity. The transverse-traceless (TT) asymptotic wave form $h_{ij}^{\rm TT}$ can be uniquely decomposed into two sets of STF radiative multipole moments U_L, V_L which are then computed as some nonlinear functionals of the canonical moments, and therefore of the source multipole moments. For instance, up to $O(5)$,

$$U_{ij}(T) = I_{ij}^{(2)}(T) + 2m\int_{-\infty}^{T} dT' \ln\left(\frac{T - T'}{2b_1}\right) I_{ij}^{(4)}(T'), \quad (6)$$

which contains a nonlocal tail integral [in which $b_1 = be^{-11/12}$, where b is a freely specifiable parameter entering the coordinate transformation $x^\mu \to X^\mu$: $T - R = t - |\mathbf{x}| - 2m\ln(|\mathbf{x}|/b)$]. The superscript (n) denotes n time derivatives. The energy loss is given by integrating the square of $\partial h_{ij}^{\rm TT}/\partial T$ over the sphere at infinity. At 2PN order this leads to

$$-\frac{dE}{dT} = \frac{1}{5}U_{ij}^{(1)}U_{ij}^{(1)} + \frac{1}{189}U_{ijk}^{(1)}U_{ijk}^{(1)} + \frac{16}{45}V_{ij}^{(1)}V_{ij}^{(1)} + \frac{1}{9072}U_{ijkm}^{(1)}U_{ijkm}^{(1)} + \frac{1}{84}V_{ijk}^{(1)}V_{ijk}^{(1)}. \quad (7)$$

Inserting the 2PN expression (5) of the source quadrupole into the radiative quadrupole (6), and using the previously derived 1PN expressions for the other multipole moments, we end up with the energy loss (1).

Epstein-Wagoner approach (WW).—This approach starts by considering $h^{\alpha\beta} = \Box_R^{-1}(16\pi\tau^{\alpha\beta})$ as a formal solution of Eq. (4) everywhere. One then expands the retarded integral $\Box_R^{-1}$ to leading order in $1/R$ in the far zone, while the retardation is expanded in a slow-motion approximation. Using identities, such as $\tau^{ij} = \frac{1}{2}\partial^2(\tau^{00}x^i x^j)/\partial t^2 +$ spatial divergences, which result from $\partial_\beta\tau^{\alpha\beta} = 0$ (a consequence of the harmonic gauge condition), we express the radiative field as a sequence of Epstein-Wagoner multipole moments,

$$h_{\rm TT}^{ij} = \frac{2}{R}\frac{d^2}{dt^2}\sum_{m=0}^{\infty} n_{k_1}\ldots n_{k_m} I_{\rm EW}^{ijk_1\ldots k_m}(t - R)_{\rm TT}, \quad (8)$$

where $I_{\rm EW}^{ijk_1\ldots k_m}$ are integrals over space of moments of the source $\tau^{\alpha\beta}$ (e.g., $I_{\rm EW}^{ij} = \int\tau^{00}x^i x^j d^3x$; see [26,28] for formulas). To sufficient accuracy for the radiative field, the two-index moment must be calculated to 2PN order, the three- and four-index moments to PN order, and the five- and six-index moments to Newtonian order. The moments of the compact-support source distribution $(-g)T^{\alpha\beta}$ are straightforward. Contrary to what happens in the BDI calculation, where the matching leads to mathematically well-defined formulas for the source multipole moments, the EW moments of the noncompact $\Lambda^{\alpha\beta}$ source are given by formally divergent integrals. To deal with this difficulty we define a sphere of radius $\mathcal{R} \gg \lambda \sim r/\varepsilon$ centered on the center of mass of the system, and integrate the noncompact moments within the sphere. Many integrations by parts are carried out to simplify the calculations, and the resulting surface terms are evaluated at $\mathcal{R}$ and kept. The divergent terms are proportional to $\mathcal{R}$, and signal the failure of the slow-motion expansion procedure extended into the far zone. We discard the divergent terms. In order to compare directly with BDI, we transform the EW moments into STF moments using the projection integrals given by Thorne [29]. For the quadrupole moment, for instance, that transformation is given by $I^{ij} = \mathrm{STF}_{ij}[I^{ij}_{\mathrm{EW}} + \frac{1}{21}(11I^{ijkk}_{\mathrm{EW}} - 12I^{kijk}_{EW} + 4I^{kkij}_{\mathrm{EW}}) + \frac{1}{63}(23I^{ijkkll}_{\mathrm{EW}} - 32I^{kijll}_{\mathrm{EW}} + 10I^{kkijll}_{\mathrm{EW}} + 2I^{klklij}_{\mathrm{EW}})]$. We find that those STF moments agree exactly with the source moments of BDI, e.g., Eq. (5). Note that the formal EW approach misses the tail effects in the wave form [see (6)]. They must be added separately.

This work is supported in part by CNRS, the NSF under Grants No. PHY 92-22902 (Washington University), No. PHY 92-13508 (Caltech); and NASA under Grants No. NAGW 3874 (Washington University), No. NAGW 2936 (Northwestern University), and No. NAGW 2897 (Caltech). BRI acknowledges the hospitality of IHES.

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