Tunneling conductance of Luttinger liquids: Resonances

V. Ashvin
Jawaharlal Nehru Centre for Advanced Scientific Research, Bangalore 560064, India

G. V. Vijayagovindan
School of Physics, Mahatma Gandhi University, Kottayam 686631, India

N. Kumar
Raman Research Institute, Bangalore 560080, India
and Jawaharlal Nehru Centre for Advanced Scientific Research, Bangalore 560064, India
(Received 19 September 1995; revised manuscript received 12 February 1996)

We have calculated the two-probe Landauer conductance of a one-channel quantum wire containing a Luttinger liquid and connected to two noninteracting leads through tunnel barriers. The tunneling conductance shows broad resonances as a function of the bias voltage, this being a manifestation of the spin-charge separation. In the limit of zero bias and zero barrier, the tunneling conductance reduces to the ballistic contact value e^2/h per channel. [S0163-1829(96)51020-9]

A quasi-one-dimensional micrometer-length quantum wire containing repulsively interacting electrons and connected to wide electron reservoirs at the two ends is a nontrivial mesoscopic transport system of considerable current interest, made realizable experimentally by the recent advances in nanoheterostructure technology.[1] A minimum electronic model for such a system is a homogeneous one-dimensional Luttinger liquid (1DLL) for the quantum wire, terminating into two 1D leads containing noninteracting electrons. The latter is to simulate the higher-dimensional reservoirs that act as perfect absorbers and emitters of electrons. An exact well-known result for the ballistic case of noninteracting electrons is the quantization of the two-probe Landauer conductance in units of $e^2/2\pi\hbar$ per spin orientation per transverse channel.[2,3] Very recently, however, Maslov and Stone[4] have shown theoretically that the same result holds equally exactly also in the absence of a repulsive interaction among electrons in the quantum wire, modeled as a 1DLL. That there is no renormalization of the quantized conductance by a repulsive interaction is also consistent with the recent experimental results[5] on very long, high mobility GaAs wires, but disagrees with some earlier calculations,[6–8] where the interaction parameter (i.e., the *correlation exponent* K) for the wire appears explicitly in the conductance $=Ke^2/2\pi\hbar$ per channel. Here $K=1$ for the noninteracting electrons and $K<1$ for repulsive interaction.

This absence of renormalization of the conductance by interaction, while presumably exact, does conceal the essential feature of the Luttinger liquid, namely, the spin-charge separation.[9–11] In this work, we have calculated a related quantity of interest, namely, the tunneling (differential) conductance dI/dV as function of the bias voltage (V) for a 1DLL quantum wire of length L connected to the two ideal (noninteracting electrons) leads through tunnel barriers. [The latter make it possible to inject electrons at a tunable energy $E=(eV)$ above the Fermi level for the wire.] The derived expression for the conductance shows broad, roughly equispaced resonances as a function of the bias voltage, or wire length, that clearly reveal the spin-charge separation inherent to 1DLL, in that the oscillations may be interpreted as the *beat* phenomenon resulting from the different speeds of the spin and the charge carriers. Our calculations are for the zero-temperature case.

For an infinitely long spin-charge-separated 1D Luttinger liquid, the retarded one-electron Green function $G(x,E)$ in the mixed space $(x)-$energy (E) representation is (in the obvious notation)[9–11]

$$\begin{aligned} G(x,E) &= \frac{i}{v_g}\Theta(x)e^{ik_Fx+ixE/\hbar v_H}J_0(\Delta v xE/2\hbar v_g^2) \\ &\quad + \frac{i}{v_g}\Theta(-x)e^{-ik_Fx-ixE/\hbar v_H}J_0(\Delta v xE/2\hbar v_g^2) \\ &\equiv G_R+G_L \end{aligned}$$

with

$$v_g=(v_\rho v_\sigma)^{1/2}; \quad v_H^{-1}=\tfrac{1}{2}(v_\rho^{-1}+v_\sigma^{-1}), \qquad \Delta v=v_\rho-v_\sigma. \tag{1}$$

Here we have assumed the simplest case of absence of the large-momentum transfer backscattering. Also, the forward scattering momentum cutoff (Λ^{-1}) has been set to zero.[9] The essential feature of $G(x,E)$ is its decomposition into the right-only- and the left-only-moving parts. It is this feature of 1DLL that enables us, despite interactions, to treat the two-probe tunneling conductance problem—*a la* Landauer—as one of multiple scattering and transmission of an electron through the two tunnel barriers separating the finite-length quantum wire and the terminal leads as follows.

Consider an electron wave of unit amplitude incident on the barrier at $x=0$ from the left lead at an energy E above the Fermi energy. Let the amplitude transmission coefficient of the tunneling barrier be $t(E)$. Then the amplitude for the electron injected at $x=0$ will be $t(E)$. Now this injected amplitude at $x=0$ will be propagated adiabatically towards the right barrier by the right-moving propagator $G_R(x,E)$ properly normalized to unity at $x=0$. Thus the propagated amplitude at $x=L$ will be $t(e)g_R(L,E)$ with $g_R(x,E)\equiv G_R(x,E)/G_R(0,E)$. The barriers at $x=L$ transmits $t(E)g_R(L,E)t(E)$ and reflects $t(E)g_R(L,E)r(E)$, where $r(E)$ is the amplitude reflection coefficient for the barrier, with $|t(E)|^2+|r(E)|^2=1$. This reflected amplitude is now propagated to the left barrier at $x=0$ with an amplitude $t(E)g_R(L,E)r(E)g_L(-L,E)$, and so on for the multiple scatterings. The total transmitted amplitude $T(E)$ is then given by the series

$$\begin{aligned} T(E) &= t(E)g_R(L,E)t(E)+t(E)g_R(L,E)r(E) \\ &\quad \times g_L(-L,E)r_L(E)g_R(L,E)t(E)+\cdots \\ &= \frac{t^2(E)g_R(L,E)}{1-r^2(E)g_R(L,E)g_L(-L,E)}. \end{aligned} \tag{2}$$

Here we have assumed the two barriers to be identical and symmetrical. The tunneling conductance $G(V)$ with E set equal to eV is now given by

$$\begin{aligned} G(V) &\equiv \left(\frac{dI}{dV}\right) = e^2\left(\frac{\partial n}{\partial E}\right)|T(E)|^2 \\ &\equiv \frac{(e^2/\pi\hbar)|t(E)|^4 J_0^2(e\Delta v LV/2\hbar v_g^2)}{|1-\exp(iQ)r^2(E)J_0^2(e\Delta v LV/2\hbar v_g^2)|^2}, \end{aligned} \tag{3}$$

where $Q=2L[k_F+E/\hbar v_H]$. This is our main result.

It is readily seen that in the limit $V\to 0$ and $t(E)\to 1$, $r(E)\to 0$, i.e., in the zero-bias zero-barrier limit, the conductance reduces to the well-known expression $e^2/\pi\hbar$ (as for the spin case). As function of the bias voltage V, the differential tunneling conductance $G(V)$ shows oscillations, with equispaced peaks separated by $\delta V\sim(\hbar v_g/eL)(v_g/\Delta v)$. For $v_g\sim 10^5$ ms^{-1}, $\Delta v/v_g\sim 0.1$, and $L\sim 10$ μm, we get $\delta V\sim 0.5$ mV. Of course, the bias voltage must be kept low enough to keep the number of active transverse channels fixed ($=1$ in the present case). These oscillations, of course, disappear with the velocity difference $\Delta v\to 0$, suggesting their origin in the spin-charge separation.

Above, we have taken the leads to contain noninteracting electrons. If the leads are modeled by yet another 1D Luttinger liquid, all we have to do is to recalculate the prefactor $e^2(\partial n/\partial E)v_E$ in our Eq. (3) for a 1DLL. This can be done very generally through an identity stating that for a system in a stationary state the expectation value of the time derivative of the current operator $j(x)$ must vanish.[12] Applying this to a 1DLL in the presence of a local potential $\delta U(x)$, and using the well-known expression[11] for $j(x)$, we get for the change in the local electron density $\delta n(x)$ due to $\delta U(x)$ as $-\delta U(x)(K/2\pi h v_F')$ giving for $(\partial_n/\partial E)v_F'=K/2\pi\hbar$, where K is the standard interaction parameter as defined in Ref. 11. This gives $G(V)$ (with interacting leads) $=KG(V)$ (with noninteracting leads).

The following comment on our Eq. (2) for the transmission amplitude seems in order. As mentioned earlier, this equation is exact in the case of a strictly one-electron problem, or, equivalently, in the absence of mutual interaction in the wire, where this can be obtained directly by wave-function matching. In this case we can propagate the injected wave amplitude $\psi_{\rm in}(x=0,E)$ at the incident energy E through the composition rule[13]

$$\psi(x,E)=G_0(x-x',E)\left(\frac{i\hbar}{2m}\frac{\overleftrightarrow{\partial}}{\partial x'}\right)\psi_{\rm in}(x',E), \tag{4}$$

where $(i\hbar/2m)\overleftrightarrow{\partial}/\partial x'$ gives the mean of the velocities on the two sides of $x'=0$, and G_0 is the Green function for a one-electron problem for $x>0$. This ensures that the $\psi(x,E)$ for $x>0$ is matched to the incident amplitude $\psi_{\rm in}(0,E)$ at $x=0$. In the present case we have assumed such a matching for the case of our 1DLL, made plausible in the absence of backscattering.

A number of remarks are in order now to clarify the main approximation involved in the idealized Luttinger-liquid model adopted here, and the extent of its robustness relevant to our treatment of tunneling. First, we have considered here a minimum model of spin Luttinger liquid that retains the spin-charge separation, but without the additional complication of the anomalous power-law correlation function. The latter involves scattering between the left- and the right-going branches caused by the unscreened short-ranged two-body repulsion. Our expression [Eq. (1)] for the electron propagator is valid only in the corresponding limit of the coupling constant g-ology.[10,11] Second, the question naturally arises as to whether our tunneling results are robust against this neglect of interbranch backscattering. This question becomes particularly relevant in view of the recent published work[14] where the short-ranged repulsive interaction is shown to lead to vanishing linear conductance even for an arbitrarily weak one-body scatterer—a subtle manifestation of the (soft) Coulomb gap. Inasmuch as in our case the tunnel barrier does cause backscattering, one could conclude that there would be a vanishing tunneling conductance—*at vanishing bias voltage*. The latter, of course, is the whole point. We have considered here the differential tunneling conductance as a function of the bias voltage. The latter involves carrier injection at energies *away* from the Fermi level. The vanishing of the tunneling density of states (and hence of the linear conductance) due to the repulsive interaction and the one-body scattering mentioned above refer strictly to the zero bias-voltage limit, i.e., the condition at the Fermi level. Thus, our tunneling conductance and its resonant qualitative features should persist at nonzero bias voltage. It is only in the limit of zero bias voltage that they will be washed out by the soft Coulomb gap due to the relevant repulsive interaction. Of course, we do expect quantitative modifications at all bias voltages due to the soft Coulomb gap.

Finally, we would like to point out that the earlier study of spin-charge separation[15] involved the effect of an Aharonov-Bohm (AB) magnetic flux through a Luttinger-liquid loop on the transmission through the loop. This is qualitatively quite different from the present study in that our tunneling at finite bias voltage involves carrier injection away from Fermi level while their AB-flux effects are related to the conditions at Fermi level. The latter could, therefore, be relatively more subject to the blockade effects discussed above.

In conclusion, we have shown that the tunneling conductance of a Luttinger-liquid quantum wire connected to the leads through tunnel barriers shows resonances as function of the bias voltage. This is due to the spin-charge separation and should be observable. Physical arguments based on forward-only scattering for the 1DLL are given to justify our Landauer-type one-electron-scattering approach. Our result agrees with the known result in the appropriate limit of zero bias and no barrier.

One of us (V.A.) would like to thank Professor C. N. R. Rao, President of the Jawaharlal Nehru Centre for Advanced Scientific Research, for financial support during the course of this work.

---

[1] For a review, see *Mesoscopic Phenomena in Solids*, edited by B.L. Altshuler, P.A. Lee, and R.A. Webb (North-Holland, New York, 1991).

[2] B.J. van Wees *et al.*, Phys. Rev. Lett. **60**, 848 (1988).

[3] D.A. Wharam *et al.*, J. Phys. C **21**, L209 (1988).

[4] D.L. Maslov and M. Stone, Phys. Rev. B **52**, 5539 (1995).

[5] S. Tarucha, T. Honda, and T. Saku, Solid State Commun. **94**, 413 (1995).

[6] W. Apel and T.M. Rice, Phys. Rev. B **26**, 7063 (1982).

[7] C.L. Kane and M.P.A. Fisher, Phys. Rev. B **46**, 15 233 (1992).

[8] M. Ogata and H. Fukuyama, Phys. Rev. Lett. **73**, 468 (1994).

[9] F.D.M. Haldane, J. Phys. C **14**, 2585 (1981).

[10] J. Voit, J. Phys. Condens. Matter **5**, 8305 (1993).

[11] For a review, see V.J. Emery, in *Highly Conducting One-Dimensional Conductors*, edited by J.T. Devreese (Plenum, New York, 1979).

[12] This closed form for $\delta n/\delta U$ obtains only for the special case of the 1DLL model considered here. For a general case of interacting electrons, there is no such closed form. Surprisingly, the same identity also gives a closed form density-response to $\delta U(x)$ for a quantum Hall liquid consistent with its known incompressibility.

[13] A. Auerbach, S. Kivelson, and Denis Nicole, Phys. Rev. Lett. **53**, 411 (1984).

[14] C.L. Kane and M.P.A. Fisher, Phys. Rev. Lett. **68**, 1220 (1992).

[15] E.A. Jagla and C.A. Balseiro, Phys. Rev. Lett. **70**, 639 (1993).