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RAPIDLY ROTATING STRANGE STARS FOR A NEW EQUATION OF STATE OF STRANGE QUARK MATTER

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ABSTRACT

For a new equation of state of strange quark matter, we construct equilibrium sequences of rapidly rotating strange stars in general relativity. The sequences are the normal and supramassive evolutionary sequences of constant rest mass. We also calculate equilibrium sequences for a constant value of Ω corresponding to the most rapidly rotating pulsar PSR 1937+21. In addition, we calculate the radius of the marginally stable orbit and its dependence on Ω, relevant for modeling of kilohertz quasi-periodic oscillations in X-ray binaries.

Subject headings: equation of state — relativity — stars: neutron

1. INTRODUCTION

One of the most fascinating aspects of modern astrophysics is the possible existence of a new family of collapsed stars consisting completely of a deconfined mixture of *up* (*u*), *down* (*d*), and *strange* (*s*) quarks together with an appropriate number of electrons to guarantee electrical neutrality. Such compact stars have been referred to in the literature as strange stars (SSs) and their constituent matter as strange quark matter (SQM). The possible existence of SSs is a direct consequence of the so-called *strange matter hypothesis*, formulated by Witten (1984; see also Bodmer 1971; Terazawa 1979). According to this hypothesis, SQM, in equilibrium with respect to the weak interactions, could be the absolute ground state of strongly interacting matter, rather than ^{56}Fe. Ever since the formulation of this hypothesis, there have been several reports in the literature on the structure of nonrotating SSs (e.g., Haensel, Zdunik, & Schaefer 1986; Alcock, Farhi, & Olinto 1986). However, these have remained largely theoretical and speculative in nature. Observational data from the *Rossi X-Ray Timing Explorer* (*RXTE*) of certain low-mass X-ray binaries (LMXBs) have recently given rise to a resurgence in the astrophysical interest associated with SSs. It has been suggested that at least a few LMXBs could be harboring SSs. For example, recent studies have shown that the compact objects associated with the X-ray burster 4U 1820−30 (Bombaci 1997), the bursting X-ray pulsar GRO J1744−28 (Cheng et al. 1998), and the X-ray pulsar Her X-1 (Dey et al. 1998) are likely SS candidates. The most promising and convincing SS candidates are the compact objects in the bursting millisecond X-ray pulsar SAX J1808.4−3658 (Li et al. 1999a) and in the atoll source 4U 1728−34 (Li et al. 1999b).

The compact nature of these sources make general relativity important in describing these systems. Furthermore, their existence in binary systems implies that these may possess rapid rotation rates (Bhattacharya & van den Heuvel 1991 and references therein). Particularly, the two SS candidates in SAX J1808.4−3658 and 4U 1728−34 are millisecond pulsars having spin periods $P = 2.49$ and 2.75 ms, respectively. These two properties make the incorporation of general relativistic effects of rotation imperative for satisfactory treatment of the problem.

Most of the calculations on the rotational properties of SSs reported so far have relied on the slow-rotation approximation (Colpi & Miller 1992; Glendenning & Weber 1992). This approximation loses its validity as the star's spin frequency approaches the mass-shedding limit. Rapidly rotating SS sequences have been recently computed by Gourgoulhon et al. (1999), Stergioulas, Kluźniak, & Bulik (1999), and Zdunik et al. (2000). However, all the calculations mentioned above make use of a very schematic model (Freedman & McLerran 1978; Farhi & Jaffe 1984), related to the MIT bag model (Chodos et al. 1974) for hadrons, for the equation of state (EOS) of SQM. Within the MIT bag model EOS, the SS radii calculated are seen to be incompatible with the mass-radius (*M*-*R*) relation (Li et al. 1999a) for SAX J1808.4−3658 and only marginally compatible (see Fig. 2) with that for 4U 1728−34 (Li et al. 1999b).

In this Letter, we present calculations of equilibrium sequences of rapidly rotating SSs in general relativity using a new model for the EOS of SQM derived by Dey et al. (1998). This model is based on a "dynamical" density-dependent approach to confinement. In contrast, in the simple EOS based on the MIT bag model, the medium effects on the quark degrees of freedom and on the quark-quark interaction are not considered. For illustrative purposes, we compare our results, obtained using this new EOS, with those for the MIT bag model EOS.

We use the methodology described in detail in Datta, Thampan, & Bombaci (1998) to calculate the structure of rapidly rotating SSs. For completeness, we briefly describe the method here. For a general axisymmetric and stationary spacetime, assuming a perfect fluid configuration, the Einstein field equations reduce to ordinary integrals (using Green's function approach). These integrals may be self-consistently (numerically and iteratively) solved to yield the value of metric coefficients in all space. Using these metric coefficients, one may then compute the structure parameters, angular momentum, and moment of inertia corresponding to initially assumed central density and polar-to-equatorial radius ratio. These may then be used (as described in Thampan & Datta 1998) to calculate parameters connected with stable circular orbits (like the innermost stable orbit and the Keplerian angular velocities) around the configuration in question.

The sequences that we calculate are constant rest mass sequences, constant angular velocity sequences, constant central density sequences, and constant angular momentum sequences. We also calculate the radius $r_{\rm orb}$ of the marginally stable orbit and its dependence on the spin rate of the SS, which will be relevant for modeling X-ray burst sources involving SSs.

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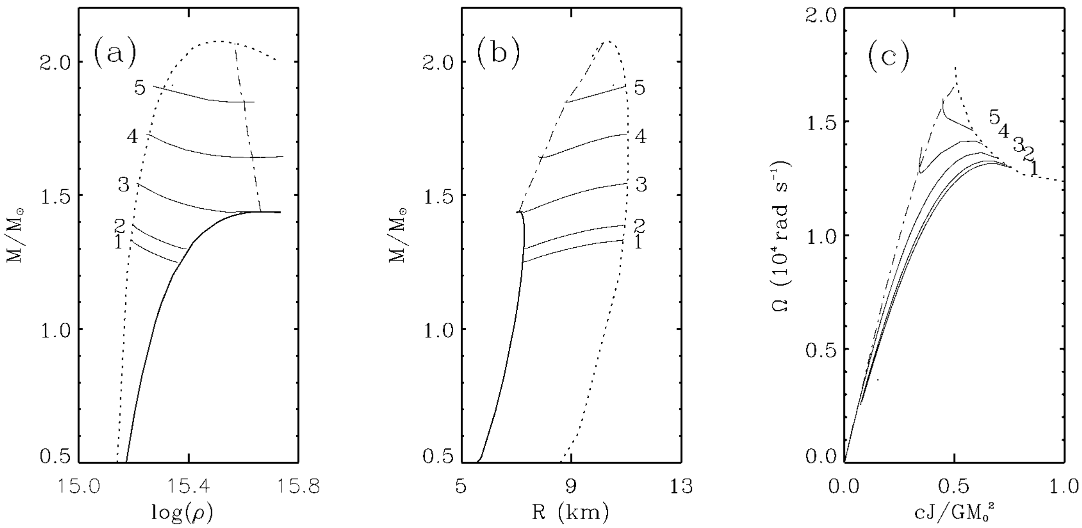

FIG. 1.—Structure parameters for rotating SSs corresponding to EOS SS1. The thick solid line represents the nonrotating limit, the thick dotted line represents the mass-shed limit, and the almost vertical thin dot-dashed line tilted to the left is the instability limit to quasi-radial mode perturbations. The thin solid lines (labeled 1, 2, …) represent constant rest mass sequences: 1: 1.59 $M_\odot$, 2: 1.66 $M_\odot$, 3: 1.88 $M_\odot$, 4: 2.14 $M_\odot$, and 5: 2.41 $M_\odot$.

2. THE EQUATION OF STATE FOR SQM

As mentioned earlier, the schematic EOS for SQM based on the MIT bag model has become the "standard" EOS model for SS studies. However, this EOS model becomes progressively less trustworthy as one goes from a very high density region (asymptotic freedom regime) to one of low density, where confinement (hadron formation) takes place. Recently, Dey et al. (1998) derived a new EOS for SQM using a "dynamical" density-dependent approach to confinement. This EOS has asymptotic freedom built in, shows confinement at zero baryon density, and shows deconfinement at high density. In this model, the quark interaction is described by a color–Debye-screened interquark vector potential originating from gluon exchange and by a density-dependent scalar potential which restores chiral symmetry at high density (in the limit of massless quarks). This density-dependent scalar potential arises from the density dependence of the in-medium effective quark masses M_q, which are taken to depend on the baryon number density n_B according to (see Dey et al. 1998)

$$M_q = m_q + 310 \operatorname{sech}\left(\nu \frac{n_B}{n_0}\right) \text{ (MeV)}, \qquad (1)$$

where $n_0 = 0.16$ fm^{-3} is the normal nuclear matter density, q $(= u, d, s)$ is the flavor index, and ν is a parameter. The effective quark mass M_q (n_B) goes from its constituent mass at $n_B = 0$ to its current mass m_q as $n_B \to \infty$. Here we consider two different parameterizations of this EOS, which correspond to a different choice for the parameter ν. The equation of state SS1 (SS2) corresponds to $\nu = 0.333$ $(\nu = 0.286)$. These two models for the EOS give absolutely stable SQM according to the strange matter hypothesis. In order to compare our results with those of previous studies, we also use the MIT bag model EOS for massless noninteracting quarks and $B = 90$ MeV fm^{-3} (hereafter the B90_0 EOS).

TABLE 1
STRUCTURE PARAMETERS FOR THE NONROTATING MAXIMUM MASS CONFIGURATIONS

EOS	ρ_c[a] ($\times 10^{15}$ g c^{-3})	M[b]	R[c] (km)	M_0[d]	r_{orb}[e] (km)	I[f] ($\times 10^{45}$ g cm^2)
SS1 ………	4.65	1.438	7.093	1.880	12.740	0.733
SS2 ………	5.60	1.324	6.533	1.658	11.730	0.576
B90_0 ……	3.09	1.603	8.745	1.937	14.202	1.146

[a] Central density.
[b] Gravitational mass in solar units.
[c] Equatorial radius.
[d] Baryonic mass in solar units.
[e] Radius of the marginally stable orbit.
[f] Moment of inertia.

3. RESULTS AND DISCUSSION

The equilibrium sequences of rotating SSs depend on two parameters: the central density (ρ_c) and the rotation rate (Ω). For purpose of illustration, we choose three limits in this parameter space. These are (1) the static or nonrotating limit,

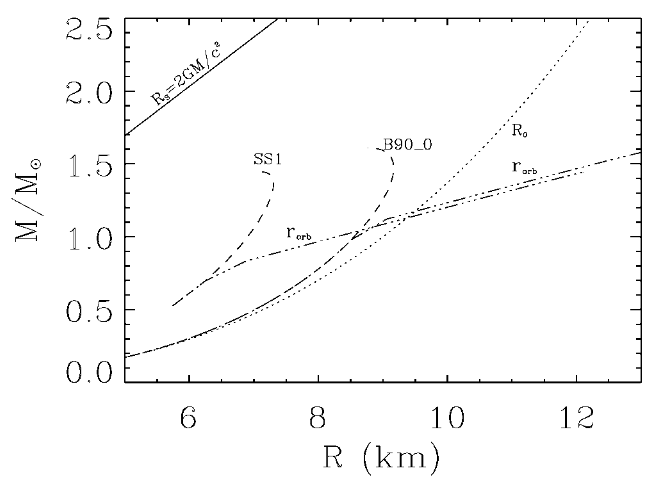

FIG. 2.—Mass-radius relations and r_{orb} for a rotation rate of $\Omega = 2.287 \times 10^3$ rad s^{-1} (i.e., 364 Hz) corresponding to the inferred rotation rate of the central accretor in 4U 1728−34 (Médez & van der Klis 1999). Also plotted are the bounds on the M-R from general considerations and the fit for these values obtained by Titarchuk & Osherovich (1999). See text for further details.

TABLE 2

Structure Parameters for the Maximally Rotating ($\Omega = \Omega_{MS}$) Maximum Mass Configuration

EOS	ρ_c[a] ($\times 10^{15}$ g c^{-3})	Ω[b] ($\times 10^4$ rad s^{-1})	I[c] ($\times 10^{45}$ g cm^2)	M[d]	T/W[e]	R[f] (km)	$\tilde{j}$[g]	r_{orb}[h] (km)	M_0[i]
SS1 ………	3.10	1.613	2.072	2.077	0.219	10.404	0.524	11.656	2.694
SS2 ………	3.60	1.738	1.613	1.904	0.218	9.612	0.570	10.758	2.366
*B*90_0 ……	1.90	1.190	3.369	2.272	0.232	13.213	0.633	14.612	2.683

[a] Central density.
[b] Rotation rate.
[c] Moment of inertia.
[d] Gravitational mass in solar units.
[e] Ratio of the rotational to the total gravitational energy.
[f] Equatorial radius.
[g] Specific angular momentum ($\tilde{j} = cJ/GM_0^2$).
[h] Radius of the marginally stable orbit.
[i] Baryonic mass in solar units.

(2) the limit at which instability to quasi-radial mode sets in, and (3) the centrifugal mass-shed limit. The last limit corresponds to the maximum rotation rate (Ω_{MS}) for which centrifugal forces are able to balance the inward gravitational force.

The result of our calculations for EOS SS1 is displayed in Figure 1. In Figure 1*a* we show the functional dependence of the gravitational mass (M) with ρ_c. In this set of figures, the thick solid curve represents the nonrotating or static limit, and the thick-dashed curve the centrifugal mass-shed limit. The thin solid curves are the constant rest mass (M_0) evolutionary sequences. The evolutionary sequences above the maximum stable nonrotating mass configuration are the supramassive evolutionary sequences, and those that lie below this limit are the normal evolutionary sequences. The maximum mass sequence for this EOS corresponds to $M_0 = 2.2\, M_\odot$. The thin dot-dashed line (slanted leftward) represents instability to quasi-radial perturbations. In Figure 1*b*, we give a plot of M as a function of the equatorial radius R.

In Figure 1*c*, we display the plot of Ω as a function of the specific angular momentum $\tilde{j} = cJ/GM_0^2$ (where J is the angular momentum of the configuration). Unlike for neutron stars (e.g., Cook, Shapiro, & Teukolsky 1994; Datta et al. 1998), the Ω-$\tilde{j}$ curve does not show a turnover to lower $\tilde{j}$-values for SSs. This is due to the effect of the long-range (nonperturbative) interaction in quantum chromodynamics, which is responsible for quark confinement in hadrons, and makes low-mass SSs self-bound objects. The value of Ω for the mass-shed limit appears to asymptotically tend to nonzero for rapidly rotating low-mass stars. A further ramification of this result is that the ratio of the rotational energy to the total gravitational energy (T/W) becomes greater than 0.21 (as also reported by Gourgoulhon et al. 1999), thus probably making the configurations susceptible to triaxial instabilities.

In Table 1 we display the values of the structure parameters for the maximum mass nonrotating SS models. The larger value of the maximum mass for the SS1 model, with respect to the SS2 model, can be traced back to role of the parameter ν in equation (1) for the effective quark mass M_q. In fact, a larger value of ν (SS1 model) gives a faster decrease of M_q with density, producing a stiffer EOS.

Tables 2 and 3 display the maximum mass rotating and maximum angular momentum models for the EOS models under consideration. While for EOS SS1, the maximum mass rotating model and the maximum angular momentum models are the same; for EOS SS2, the two models are slightly different, with the maximum angular momentum model coming earlier (with respect to ρ_c) than the maximum mass rotating configuration.

In Table 4 we list the values of the various parameters for the constant Ω sequences for EOS SS1. The first entry in this table corresponds to ρ_c for which $r_{orb} = R$. For higher values of ρ_c, $r_{orb} > R$; for large values of ρ_c, the boundary layer (separation between the surface of the SS and its innermost stable orbit) can be substantial (~5 km for the maximum value of the listed ρ_c).

In Figure 2, we plot our theoretically calculated *M*-*R* curves (*dashed curves*) for EOS SS1 and *B*90_0 for a rotational frequency of 364 Hz corresponding to the inferred rotational frequency of the compact star in the source 4U 1728−34 (Méndez & van der Klis 1999). In the same figure, we plot the radius R_0 of the inner edge of the accretion disk (*dotted curve*) for 4U 1728−34 as deduced by Titarchuk & Osherovich (1999; see also Li et al. 1999b) from a fit of kilohertz quasi-periodic oscillation data in this source. Since R_0 must be larger than both

TABLE 3

Structure Parameters for the Maximum Angular Momentum Configuration

EOS	ρ_c[a] ($\times 10^{15}$ g c^{-3})	Ω[b] ($\times 10^4$ rad s^{-1})	I[c] ($\times 10^{45}$ g cm^2)	M[d]	T/W[e]	R[f] (km)	$\tilde{j}$[g]	r_{orb}[h] (km)	M_0[i]
SS1 ………	3.10	1.613	2.072	2.077	0.219	10.404	0.524	11.656	2.694
SS2 ………	3.40	1.719	1.633	1.899	0.220	9.693	0.575	10.837	2.355
*B*90_0 ……	1.70	1.161	3.456	2.254	0.239	13.447	0.650	14.864	2.650

[a] Central density.
[b] Rotation rate.
[c] Moment of inertia.
[d] Gravitational mass in solar units.
[e] Ratio of the rotational to the total gravitational energy.
[f] Equatorial radius.
[g] Specific angular momentum ($\tilde{j} = cJ/GM_0^2$).
[h] Radius of the marginally stable orbit.
[i] Baryonic mass in solar units.

TABLE 4
STRUCTURE PARAMETERS FOR THE CONSTANT ANGULAR VELOCITY SEQUENCE FOR EOS SS1

ρ_c[a] ($\times 10^{15}$ g c^{-3})	I[b] ($\times 10^{45}$ g cm^2)	M[c]	T/W[d]	R[e] (km)	$\tilde{j}$[f]	r_{orb}[g] (km)	M_0[h]
1.70	0.353	0.852	0.013	6.663	0.153	6.663	1.027
1.80	0.433	0.963	0.012	6.884	0.143	7.664	1.178
1.90	0.502	1.052	0.019	7.036	0.136	8.378	1.301
2.40	0.701	1.297	0.010	7.326	0.116	10.355	1.660
2.60	0.737	1.346	0.010	7.344	0.112	10.741	1.735
4.60	0.769	1.458	0.008	7.139	0.096	11.732	1.914
5.65	0.734	1.449	0.007	7.007	0.093	11.703	1.899

NOTE.—This sequence corresponds to the rotation rate of the pulsar PSR 1937+21 (Backer et al. 1982), having $\Omega = 4.03 \times 10^3$ rad s^{-1} or period $P = 1.556$ ms.
[a] Central density.
[b] Moment of inertia.
[c] Gravitational mass in solar units.
[d] Ratio of the rotational to the total gravitational energy.
[e] Equatorial radius.
[f] Specific angular momentum ($\tilde{j} = cJ/GM_0^2$).
[g] Radius of the marginally stable orbit.
[h] Baryonic mass in solar units.

R and r_{orb}, it is possible to deduce (Li et al. 1999b) an upper bound for the radius and mass of the central accretor. Using these constraints on M and R, Li et al. (1999b) concluded that 4U 1728−34 is possibly an SS rather than a neutron star. However, in their calculation, Li et al. (1999b) used an approximate EOS-independent expression to account for the effects of rotation on the moment of inertia of the star and hence on r_{orb} (see Li et al. 1999b for further details). In contrast, in this Letter we make an "exact" calculation (as a result of our accurate calculation of angular momentum and hence the moment of inertia) of r_{orb} (*triple-dot–dashed curves*) for the two EOSs SS1 and B90_0. Notice that the upper bound for the mass of 4U 1728−34 (the intersection point between the curves R_0 and r_{orb}) is about 1.15 $M_\odot$, in agreement with the results of Li et al. (1999b). The two EOS models considered in Figure 2 are both consistent with 4U 1728−34. Also, having in mind the scaling with $B^{-1/2}$ of the M-R relation of SSs within the bag model EOS (Witten 1984) and the constraints on the allowed values of the constant B to fulfil the strange matter hypothesis (Farhi & Jaffe 1984), we see that the simple EOS based on the MIT bag model is only marginally compatible with the M-R relation for 4U 1728−34.

To summarize, in this Letter we present calculations of equilibrium sequences of rapidly rotating SSs in general relativity for a set of new EOSs. We compare the results so obtained with those for the generally used MIT bag model EOS. In addition to this, we illustrate the results of our computations for a specific rotation rate inferred for the central accretor in the source 4U 1728−34. We find that if the compact star in this source were indeed an SS, then its mass and radius would be bounded above by a value of $M = 1.15\ M_\odot$ and $R = 9.3$ km. It is expected that future observations may shed more light on this issue.

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