Mon. Not. R. Astron. Soc. 327, L5–L9 (2001)

Heating of the intergalactic medium as a result of structure formation

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Accepted 2001 July 18. Received 2001 July 9; in original form 2001 February 28

ABSTRACT

We estimate the heating of the intergalactic medium as a result of shocks arising from structure formation. The heating of the gas outside the collapsed regions, with small overdensities $[(n_b/\bar{n}_b) \leqslant 200]$ is considered here, with the aid of a Zel'dovich approximation. We estimate the equation of state of this gas, relating the density to its temperature and its evolution in time, considering the shock heating caused by 1σ density peaks as being the most dominant. We also estimate the mass fraction of gas above a given temperature as a function of redshift. We find that the baryon fraction above 10^6 K at $z = 0$ is $\sim$10 per cent. We estimate the integrated Sunyaev–Zel'dovich distortion from this gas at the present epoch to be of the order of 10^{-6}.

Key words: galaxies: formation – intergalactic medium – cosmology: theory – large-scale structure of Universe.

1 INTRODUCTION

It has become evident from recent numerical simulations that a significant fraction of the baryons in the universe reside in the warm-hot phase of the intergalactic medium (WHIM), with temperatures of the order of 10^5–10^7 K (Cen & Ostriker 1999, hereafter CO99). The gas in this phase is raised to a high temperature by shock heating as a result of the formation of structure. Recent simulations by Davé et al. (2001) and Croft et al. (2001) have also calculated that the equation of state of this phase is approximately $\rho \propto T$.

There have also been a few analytical attempts to understand the heating process in the intergalactic medium through analytic means. Pen (1999) pointed out that there is a need for non-gravitational heating in the intra-cluster and intergalactic medium (IGM) to avoid the constraints from the soft X-ray background. Extra heating decreases the amount of clustering of the gas and therefore reduces the flux of soft X-ray radiation. Wu, Fabian & Nulsen (1999) have also addressed the question with detailed calculations and came to the same conclusions.

These are, however, relevant for the heating of the gas which is already within collapsed objects. For example, the work of Wu et al. (1999) refers to the heating of the gas which is already within a collapsed halo, with overdensities larger than ~ 200. The numerical simulations of CO99 and Davé et al. (2001), on the other hand, point out the heating in the gas which have overdensities much smaller than this.

It is interesting to note that Zel'dovich and his colleagues had reached similar conclusions to that of the recent numerical simulations in the context of their study of the formation of pancakes. As a by-product of their study of the formation of large pancakes, they had worked out the magnitude of gravitational heating of the intergalactic gas. Although much of the earlier motivation has been lost now, a substantial part of their work sounds prescient. To quote from Sunyaev & Zeldovich (1972) – '*It is possible that a significant fraction of the intergalactic gas* (10–50 *per cent) was not subjected to compression in the 'pancakes' and was heated only by the damped shock waves moving away from them*.' This is exactly what the numerical simulations have unearthed, namely, the heating of the gas which with overdensities smaller than ~ 200, outside the collapsed region but worked upon by shock waves caused by gravitational collapse.

In this Letter we attempt to understand the heating of this phase of IGM with the help of the Zel'dovich approximation. As the gas in warm–hot IGM is only mildly non-linear, this approximation can shed light on the gravitational heating process, if used within its limitations. Below, we attempt to estimate the amount of the gravitational heating, and the state of the gas, by including other heating and cooling effects. We also attempt to estimate the mass fraction of baryons which are affected by this heating as a function of redshift.

We assume a cosmological model with $\Omega_\Lambda = 0.7$, $\Omega_m = 0.3$ and $h = 0.65$, with $\Omega_B h^2 = 0.015$, the big bang nucleosynthesis value.

2 SHOCK HEATING IN THE VICINITY OF COLLAPSED OBJECTS

Consider the gas surrounding a high-density peak. As the gas flows inwards, it is compressed, and depending on its adiabatic exponent, it stops at a place away from the centre of accretion, and a shock

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wave travels outward. We will concentrate on this shock wave as it compresses and heats the very outer parts of the collapsed region. Sunyaev & Zel'dovich (1972, hereafter SZ72) tried to model this shock wave in the context of one-dimensional collapse of gas onto pancakes. In their idealized picture, as the gas flows towards the inner region, a singularity appears and a shock travels outwards through the gas. This shock velocity can be easily determined for the perturbation of a given length-scale $\lambda(=2\pi/k)$, assuming a single sinusoidal perturbation. Suppose the singularity appears at a redshift z_c. They defined a parameter, μ which corresponds to a given Lagrangian coordinate, and is given by $(\sin\pi\mu)/(\pi\mu) = (1+z)/(1+z_c)$. The parameter μ, therefore, is equivalent to a time parameter. In the case of a sinusoidal perturbation, it also gives the fraction of matter that has passed through the shock wave up to a given moment.

The velocity of matter falling onto the shock, V_s, as derived by SZ72, can be generalized for any cosmological model as,

$$V_s \sim \frac{dz}{dt}\frac{\lambda}{2\pi}\frac{1}{(1+z_c)^2}(\mu\pi)^{1/2}\sin^{1/2}(\mu\pi)$$

$$\sim \frac{dz}{dt}\frac{\lambda}{2\pi}\frac{1}{(1+z_c)^2}(\mu\pi), \qquad (1)$$

where λ is the comoving length-scale of perturbation. The temperature behind the shock wave is given by $T_s \sim V_s^2 m_p/(6k_B)$, where m_p is the mass of a proton and k_B is the Boltzmann's constant (SZ72, equation 2).

Note that this temperature reaches a maximum at $\mu \sim 0.5$, when approximately half of the matter has passed through the shock. This happens when $1+z \sim (2/\pi)(1+z_c)$. The maximum temperature is given by, [noting that $(dz/dt) = H(z)(1+z)$]

$$T_{\max} \sim \frac{m_p}{6k_B}H(z)^2\left(\frac{\lambda}{2\pi}\right)^2(\mu\pi)^2\left(\frac{1+z}{1+z_c}\right)^2\frac{1}{(1+z_c)^2}$$

$$\sim \frac{m_p}{6k_B}H(z)^2\frac{L_{\rm ln}^2}{(1+z_c)^2}, \qquad (2)$$

where we have written $L_{\rm ln} = 1/k$ for the comoving length-scale of the perturbation, in the notation of CO99. This is the typical length of perturbations that becomes non-linear at $1+z_c$. It is interesting to compare equation (4) of CO99 with this equation. They derived a value of $K = 0.3$ from their simulation where the maximum temperature or, equivalently, the maximum sound velocity was given by $C_s^2 = KH^2[L_{\rm ln}/(1+z_c)]^2$. Comparing this with the above expression, we obtain $K \sim 5/18$ for a monoatomic gas.

There is, however, a crucial difference. The parameter $L_{\rm ln}$ in CO99 is defined as the perturbation that becomes non-linear at a given redshift z. This provided the value of the maximum temperature reached by the gas at a given z. In the above formulation, however, there are three important epochs: $z_{\rm ln}$ is the epoch when the perturbation has an overdensity larger than unity and it becomes non-linear, z_c is the epoch when the singularity appears and z_m is the epoch when $\mu \sim 0.5$, when the maximum gas temperature is achieved. Naturally $z_{\rm ln} > z_c > z_m$. Here we have assumed that $z_{\rm ln} \sim z_c$ but that it is larger than z_m. This difference becomes non-negligible especially at high redshifts. This difference is shown in Fig. 1, where the maximum temperature reached at a given redshift is plotted for the Λ cosmological model (as in CO99). The solid line is the prediction from the above formulation (with $z_{\rm ln} \sim z_c > z_m$ and the dotted line is from CO99. The only difference is that we have treated $L_{\rm ln}$ as the length-scale of the perturbation that becomes non-linear at a redshift slightly larger than z_m to give the maximum temperature at z_m. Naturally, there is a difference in these two epochs, of the order of a Hubble time.

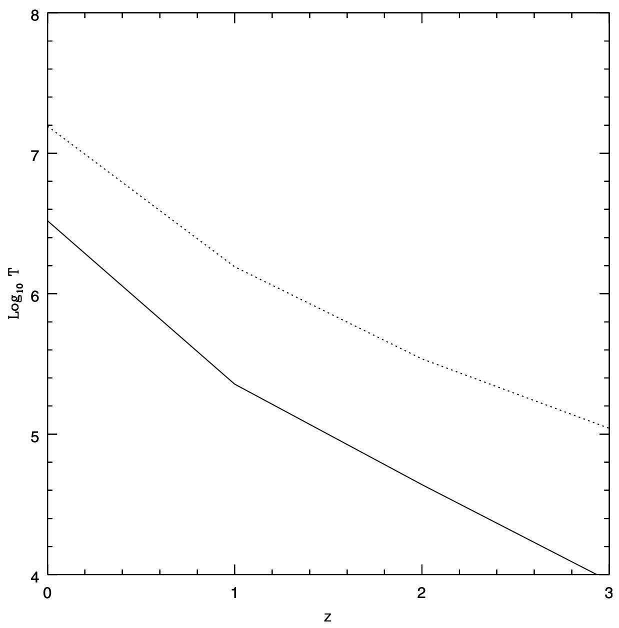

Figure 1. The maximum temperature from shocks as a result of structure formation is shown as a function of redshift (solid line). The dotted line is that from CO99.

3 EVOLUTION OF TEMPERATURE

In order to calculate the evolution of temperature of this gas, we will need to take into account all the sources of heating and cooling and their rates. First, let us consider the rate of heating caused by these shocks. We should note here that the gas with different initial density (or overdensity) will go through the shock wave at different point of time. The gas with larger initial density will be closer to the singularity and will pass through the shock wave sooner. In the ideal case, the gas density profile is smooth and shock travels through it equally affecting all parts of it. In reality, however, we expect the gas closer to the collapsed region will be shocked and heated more effectively than the gas farther away.

We note that the average density profile of the gas away from the collapsed region is expected to be of the type $\rho \propto r^{-2/3}$, where r is the perpendicular distance from the pancake or filament (Zel'dovich 1970). To treat the differential effectiveness of shock heating analytically, we shall define a time-scale for the passing of the shock wave through the gas as

$$t_s \sim \frac{\lambda}{1+z_c}\frac{1}{U}\left(\frac{\rho}{\bar\rho}\right)^{-3/2} \qquad (3)$$

where $\bar\rho$ is the average ambient density, and where $U = V_s/3$ is the velocity of the shock front relative to the plane of symmetry (whereas V_s is the velocity of matter impinging on the shock, see for example Jones, Palmer & Wyse 1981). The extra factor $(\rho/\bar\rho)^{-3/2}$ then accounts for the fact that gas with larger overdensity is heated more effectively than that with lower overdensity. This prescription is valid only for collapsing gas with different overdensities in a given perturbation. In this work, we attempt to calculate the equation of state of gas in a given 1σ perturbation. We then estimate the equation of state of gas in the IGM in general using the relevant filling factor of such perturbations.

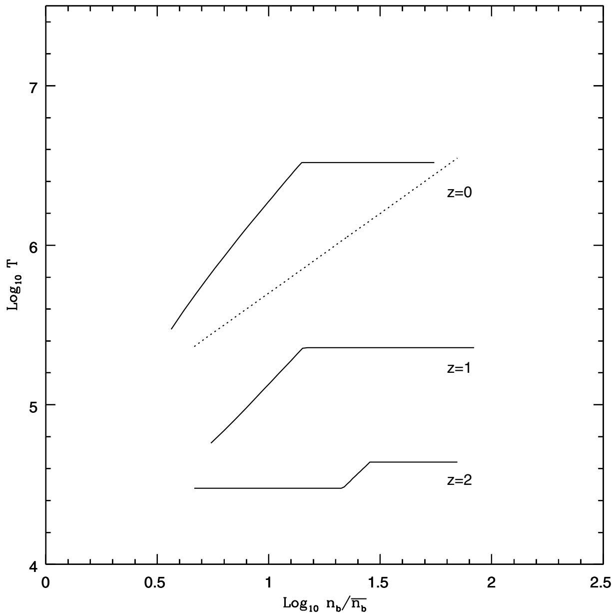

Figure 2. Temperature of the gas is plotted against its overdensity $n_b/\bar{n_b}$. The solid curves are for gas at $z = 0, 1, 2$ from top to bottom. The dotted line shows the equation of state found by Davé et al. (2000, see their fig. 6).

We then write the shock heating rate as simply $T_{\rm shock}/t_{\rm shock}$. For a universe with $\Omega_\Lambda + \Omega_0 = 1$, one has

$$\frac{{\rm d}T_{\rm shock}}{{\rm d}z} \sim -0.4\times10^6\left(\frac{\lambda/2\pi}{1\,{\rm Mpc}}\right)^2(n_{\rm b}/\bar{n}_{\rm b})^{3/2} \times \mu^3\frac{h^2(1+z)^2}{(1+z_{\rm c})^5}[\Omega_\Lambda+(1+z)^3\Omega_0]. \quad (4)$$

We should, however, remember the maximum temperature that the gas can be raised to by the shocks, as discussed in Section 2. We therefore put an upper limit on the temperature, as given by equation (2). This will reflect the physical fact that although lower density gas is not shocked as effectively as the higher density gas closer to the filament or pancake, the higher density gas is not heated to indefinitely higher temperatures this way.

As was pointed out in SZ72, a useful approximation for μ is $\mu \sim (1/\pi)\{6[1-(1+z/1+z_{\rm c})]\}^{1/2}$. We have used this approximation in our calculations below.

Before discussing other sources of heating, we should note here that this formulation is adequate only for a limited duration. Although in principle the gas infall continues until $\mu = 1$, in reality the approximations used to calculate μ break down for large values of μ. We therefore consider the evolution of the temperature only until $\mu = 0.5$.

The second heating source, which is the adiabatic compression of the gas, is easily described as, (for $n_{\rm b}/\bar{n}_{\rm b} \gg 1$)

$$\frac{{\rm d}T_{\rm ad}}{{\rm d}z} = \frac{2}{3}\frac{T}{(n_{\rm b}/\bar{n}_{\rm b})}\frac{{\rm d}(n_{\rm b}/\bar{n}_{\rm b})}{{\rm d}z}. \quad (5)$$

We characterize the growth of the overdensity by the following equation:

$$\frac{{\rm d}(n_{\rm b}/\bar{n}_{\rm b})}{{\rm d}z} = -\eta\frac{(n_{\rm b}/\bar{n}_{\rm b})}{1+z}, \quad (6)$$

where η equals unity for the linear regime in a $\Omega = 1$ universe. In the quasi-linear regime, η could be large. For example, the overdensity evolves as $\delta \propto (1+z)^{-2.15}$ for the range of the scales where the power spectrum has $n = -2$ (Peacock 1999). We adopt a value of $\eta = 2$. The final result is found not to depend on its value strongly.

One cooling process is caused by the expansion of the universe, and is given by

$$\frac{{\rm d}T_{\rm ex}}{{\rm d}z} = \frac{2T}{1+z}. \quad (7)$$

Cooling caused by free–free radiation is given by

$$\frac{{\rm d}T_{\rm ff}}{{\rm d}z} = 0.22\Omega_{\rm b}h\frac{(n_{\rm b}/\bar{n}_{\rm b})T^{1/2}(1+z_{\rm c})^2}{[\Lambda_0+(1+z)^3\Omega_0]^{1/2}}. \quad (8)$$

Although cooling as a result of inverse Compton scattering becomes important at high redshift, at $z = 4$ the cooling time for Compton cooling $[\sim 9\times10^{12}(1+z)^{-4}\,{\rm yr} = 1.5\times10^{10}\,{\rm yr}]$ is larger than that for free–free cooling for a gas at 10^6 K and with an overdensity of ~ 100 ($\sim10^9$ yr). It is shown below that at high redshifts ($z_{\rm c} \gtrsim 3.5$), shock heating contributes to the equation of state only for gas with large overdensities, of order ~ 100. As Compton cooling is not as efficient as Compton cooling for this gas, we neglect it in our calculation.

Combining all the heating and cooling processes, one has for the evolution of the gas temperature,

$$\frac{{\rm d}T}{{\rm d}z} = \frac{{\rm d}T_{\rm s}}{{\rm d}z} + \frac{{\rm d}T_{\rm ad}}{{\rm d}z} + \frac{{\rm d}T_{\rm ex}}{{\rm d}z} + \frac{{\rm d}T_{\rm ff}}{{\rm d}z} \quad (9)$$

We present the numerical solution of this equation below.

4 RESULTS

The process of heating as a result of structure formation is essentially statistical in nature. To track it analytically, however, we focus on the 1σ fluctuations. These are the perturbations that dominate the heating at a given redshift, as found in CO99. Fluctuations with higher degree of non-linearity at a given epoch would involve gas with very large overdensities, and does not concern us here. As has been emphasized earlier, here we are concerned with gas which is rather on the outskirts of highly non-linear perturbations.

To calculate the equation of state of a gas at a given epoch, we therefore find out the relevant length-scale, given the power spectrum which is *COBE* normalized. For the physical state of the gas at a redshift z, we determine the perturbation which becomes non-linear at $z_{\rm c}$, so that after evolving by a time period equivalent to $\mu = 0.5$, we reach the epoch z. In other words, to find the state of the gas at $z = 0(1,2)$, we adopt $z_{\rm c} = (1+z)(\pi/2) - 1 \sim 0.5(2.0, 3.5)$. We find the scale of the perturbations which are 1σ at this epoch, according to the relevant power spectrum. The value of $L_{\rm ln}$ at $z_{\rm c} \sim 0.5(2., 3.5)$ is $\sim 6(1.6, 0.65)h^{-1}$ Mpc. The initial values of $\rho_{\rm b}/\bar{\rho}$ is taken to be in the range of 1–50. The initial temperature has been fixed at 3×10^4 K, which is the temperature reached by the IGM gas through photoionization heating. The results for the state of gas at $z = 0, 1, 2$ are shown in Fig. 2. The dotted line shows the equation of state as found by Davé et al. (2001).

We can estimate the fraction of the mass that has a given temperature in the following way. First, we note that, in the formulation of SZ72, if the density of gas which has just entered the shock front is $\rho_{\rm i}$, then the fraction of mass (f) that has already gone through the shock wave is given as $(\rho_{\rm i}/\bar{\rho}) \sim 3/(\pi^2 f^2)$. If T is the temperature of the gas corresponding to the initial density $\rho_{\rm i}$,

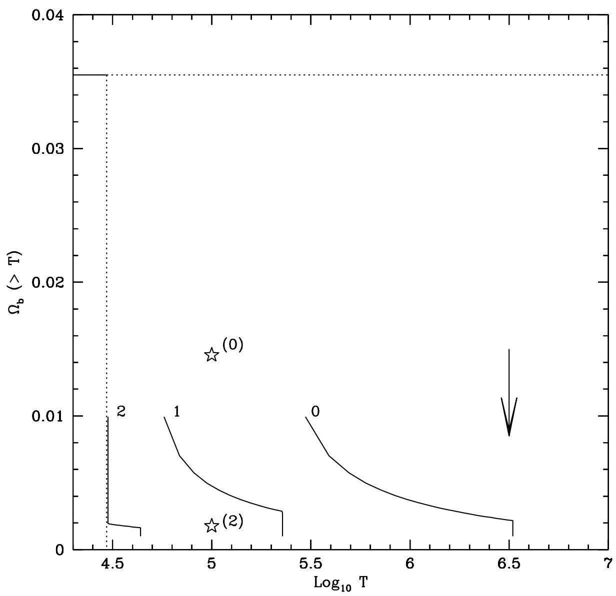

Figure 3. The fraction of shock-heated diffuse gas above a given temperature $\Omega_B(>T)$ is plotted against logarithm of T. The three solid curves from right to left are for gas at $z=0$, 1 and 2 respectively. The stars are the mass fraction at $z=0$, and $z=2$ found by Croft et al. (2000, their section 2). The horizontal dotted line refers to the total baryonic gas in the universe and the vertical dotted line shows the lower limit of the temperature due to photoionization. The arrow is from the limit of soft X-ray emission for gas with $n_b/\bar{n}_b \sim 100$ at $z=0$.

then f is the fraction of mass with temperature larger than T. To be precise, this approximation is valid for the case of instantaneous cooling, which is a reasonable assumption for $\mu \lesssim 0.1$ ($z_c \gtrsim 0.5$, as can be seen from equations 4 and 9). In other words, this approximation is valid only for small values of f ($\ll 1$).

We multiply this fraction with the (comoving) number density of the 1σ peak, as given in Bardeen et al. (1986), to derive the fraction of mass which has temperature larger than a given value. The results for the fractions at $z=0,1,2$ are shown in Fig. 3. Unfortunately, the limitations of the single sinusoidal wave approximation do not allow us to draw a full curve, as one has to stop at $f \sim 1$ (in reality, the above approximation is valid only for small values of f). The curves, however, can serve as pointers to what 1σ peaks can do to the diffuse IGM.

We also show (by stars) the mass fraction derived by Croft et al. (2001) at $z=0,2$, and the fractions are 41 and 5 per cent respectively. To compare these numbers with the curves in Fig. 3, we should remember that the curves show the result of heating by 1σ density peaks only. In reality, there will be contribution from higher sigma peaks, taking the gas to higher temperatures at rarer places. Although we do not have the fraction for gas above 10^5 K at $z=0$, a naive extrapolation of the existing curve above 10^6 K is consistent with this value. We note here that the curve for $z=0$ shows that the fraction of gas above 10^6 K is of the order of 10 per cent.

We note here that the mass fractions from the simulation of CO99 are much larger than our results. Because the simulations use different techniques and employ different resolutions, it is not obvious what these discrepancies owe their existence to and if they are of much importance.

5 DISCUSSIONS

The equation of state for low-density gas in Fig. 2 depends on the particular form of the time-scale for passing of the shock wave in equation (3). as is evident from the analytical solution (equation (10). In reality, modelling the efficiency of shock heating might add additional parameters and the resulting equation would therefore have some scatter.

In Fig. 3, we also plot the limits from the observations of the soft X-ray background. After the subtraction of the discrete sources, it now appears that a flux of 4 keV cm^{-2} s^{-1} sr^{-1} keV^{-1} at 0.25 keV can be taken as an upper limit to any possible contribution from diffuse matter in the IGM (e.g. Wu et al. 1999). We apply this limit to our result for IGM at $z=0$. If the gas at temperature T [with the corresponding density $n(T)$, as given in Fig. 2 for $z=0$] has a filling factor ϵ_T, then the flux at 0.25 keV is proportional to flux (keV/cm^2 s sr keV) $\propto \epsilon_T n(T)^2 T^{-0.5} \exp(-0.25\,\mathrm{keV}/k_B T) \times (c/H_0)$ and this allows us to calculate ϵ_T as a function of T. As ϵ_T is found to decrease very quickly with T, we can approximate $\epsilon(>T) \sim \epsilon_T$ and use it to put a limit on the mass fraction of gas with temperatures larger than a given value T. This limit is shown as a dashed arrow in Fig. 3 for gas with $n_b/\bar{n}_b = 100$ at $T=T_{max}$ for $z=0$. The limit for lower-density gas is less restrictive. The above limit is obviously uncertain to the extent that the appropriate path length differs from the approximate c/H_0.

In this calculation we have used a metallicity of 0.01 solar abundance, which is the metallicity found in the Lyα absorption systems at high redshift, and is relevant for the diffuse gas considered here. It is possible that the metallicity of this diffuse gas increases in time, as found in the numerical simulation by Cen & Ostriker (1999). It is, however, not yet known from observations how this metallicity evolves in time, and its dependence, if any, on the density and clumpiness of the gas. Any metallicity will make the above limit more stringent; in other words, it would make the contribution of the diffuse shock-heated IGM gas towards to the soft X-ray background much more prominent. As the detailed numerical simulation of the soft X-ray background radiation by Croft et al. (2001) shows, the contribution of the (enriched) shocked-heated diffuse gas is comparable to the upper limit of the unresolved X-ray emission in the soft band.

Given the relation between density and temperature, we can estimate the resulting Sunyaev–Zel'dovich distortion on the cosmic microwave background. Defining f_{los} to be the fraction of the line of sight going through hot filamentary structures, one can estimate the integrated Compton y parameter as

$$y \sim 10^{-6}\left(\frac{n_b/\bar{n}_b}{10^2}\right)\left(\frac{T_e}{10^6\,\mathrm{K}}\right)\left(\frac{c/H_0}{10^3\,\mathrm{Mpc}}\right)\left(\frac{f_{los}}{0.03}\right). \quad (10)$$

The (comoving) volume fraction of 1σ peaks from Bardeen et al. (1986) is of the order of 3 per cent. Another way of estimating this is to use the fact that the gas with density $\gtrsim n_b$ occupies a length-scale $[\lambda/(1+z_c)](n_b/\bar{n}_b)^{-3/2}$. The Compton y-parameter is approximated as

$$y \sim \frac{2\lambda}{(1+z_c)}(n_b/\bar{n}_b)^{-3/2}\frac{n_b k_B T_e}{m_e c^2}\sigma_T, \quad (11)$$

where σ_T is the Thomson cross-section. For the case at $z=0$, y for a single structure is found to be of the order of $\sim 4\times10^{-8}$ for $n_b/\bar{n}_b \sim 100$. There will be structures, however, of the order of $[(c/H_0)/L_{ln}] \times f_{los} \sim 30(f_{los}/0.03)$ in one line of sight, and so the total distortion will amount to $y \sim 10^{-6}$.

6 SUMMARY

We have applied the Zel'dovich approximation to estimate the heating of the diffuse intergalactic medium by shocks associated with 1σ density peaks in structure formation at different redshifts. We are able to reproduce the equation of state of the warm–hot IGM found in recent numerical simulations. We estimate the baryon fraction of the gas above 10^6 K at the present epoch to be at least $\sim 0.1\Omega_b$. The integrated Sunyaev–Zel'dovich distortion from the diffuse IGM filaments amounts to $y \sim 10^{-6}$.

ACKNOWLEDGMENTS

BN acknowledges joint support from the Indian National Science Academy and the Royal Society, UK, and thanks the Astrophysics Department of the University of Oxford for hospitality. We thank the anonymous referee for detail comments on the paper.

NOTE ADDED IN PROOF

We thank L. Hernquist for drawing our attention to the fact that the predictions of Cen & Ostriker (1999) that we summarize in Fig. 1 are not in agreement with the results of other authors, notably those of Refregier et al. (2000), Croft et al. (2001), Springel, White & Hernquist (2001) and Refregier & Teyssier (2000) who have estimated the evolution of the density-weighted temperature of the IGM and have invariably obtained a much lower value (typically a factor ~2–3 by $z \sim 0$). Hernquist (private communication) informs us that revised estimates by Cen & Ostriker (private communication) for the temperature evolution, correcting for an error in the integration of the thermal energy equation, now actually lie below the other results.

Hernquist (private communication) has also informed us of an error in Springel et al. (2001) in estimating both the Compton y-parameter and the angular power spectrum of CMB anisotropies, resulting from the omission of a factor h in constructing maps of these quantities, so that $\langle y \rangle$ should be reduced from the value quoted by 0.67 and the power spectrum should be reduced by h^2 (see Springel et al. 2001). We note, however, that this error did not effect the simulations performed by Springel et al., so their estimate for the evolution of the density-weighted temperature is unchanged.

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