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# The virial masses of clusters of galaxies and the effects of contamination

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**Summary**. We have used model clusters, selected from *n*-body simulations of clustering, to determine the accuracy of using virial methods for mass determination. All the clusters which have spuriously high virial masses, as a result of contamination, can be identified by using a $\chi^2$ test which compares their line-of-sight velocities with a normal distribution. For the remaining clusters, with good $\chi^2$ fits, the ratio of virial mass to the actual mass has a mean value of $0.97 \pm 0.36\,(1\sigma)$. This shows that a large degree of scatter is inherent in the virial method. We have calculated the mass-to-light ratios for three clusters (Coma, A2151 and Hercules Supercluster) in an analogous manner. Using the $\chi^2$ results from the model clusters we conclude Hercules Supercluster is seriously contaminated. For the other two clusters we derive $M/L$ ratios of $201\,M_\odot/L_\odot$ (Coma) and $168\,M_\odot/L_\odot$ (A2151). Since observed clusters are subject to additional sources of error relative to model clusters we argue that their $M/L$ ratios are uncertain by at least a factor of 3. Thus, their $M/L$ ratios could be accounted for by 'massive halo' galaxies and intracluster gas.

## 1 Introduction

Recent work on 'missing mass' in clusters of galaxies seems to suggest that this mass is not contained in the galaxies (White 1976) nor is it in the form of an intracluster gas of either H I (Gott, Wrixon & Wannier 1973) or H II (Lea *et al.* 1973). This has led to speculation of more exotic forms of matter such as massive neutrinos (e.g. Schramm & Steigman 1981). However, a persistent doubt has remained as to the possibility of contamination, the projection of foreground and background galaxies on to the cluster, causing the virial mass to be overestimated. Yahil & Vidal (1977) suggested the possibility of determining the amount of contamination by applying statistical tests to the velocity distribution of the cluster galaxies. From theoretical considerations they proposed several tests and concluded that contamination was not significant in most clusters. The opposite conclusion was reached by Turner *et al.* (1979) who used a 1000-body simulation of galaxies in an expanding universe to test

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the accuracy of virial mass determinations. They found that contamination was important in rich clusters; however, this result was derived from a limited sample of clusters. In view of the importance of 'missing mass' to our understanding of galaxy formation and evolution and the large-scale structure of the universe, we thought it worthwhile to re-examine this conclusion by analysing the clustering in one of the larger $n$-body simulations.

## 2 Method

Aarseth, Gott & Turner (1979) generated several 4000-body models of galaxies in an expanding universe. We chose the model with $\Omega = 0.1$, $n = -1$ where $\Omega$ is the ratio of the mean density to the closure density of the universe and $n$ defines the initial mass fluctuation spectrum. This model was found to give the best fit to the observed correlation and multiplicity functions of galaxies (Gott, Turner & Aarseth 1979; Bhavsar, Gott & Aarseth 1981).

The end-point of each simulation is a sphere of radius $R$, the surface of which is expanding with velocity $\dot{R}$. This contains 4000 points of mass $m$, position coordinates $x$, $y$, $z$ and velocity components $\dot{x}$, $\dot{y}$, $\dot{z}$. From this complete information the 4000 points were projected so they present the view that an observer at the surface of the sphere would have. This

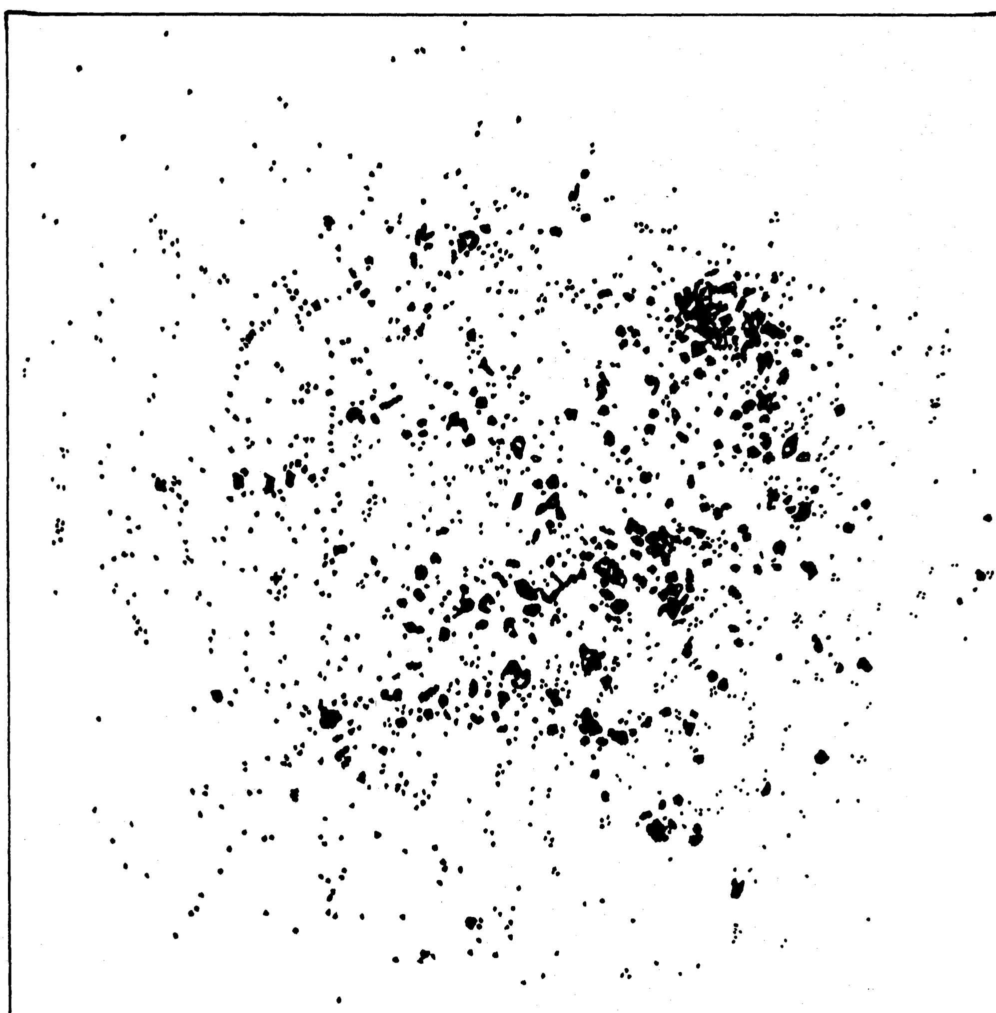

Figure 1. Projection along the $x$-axis showing the view that an observer at the surface of the sphere would have. The axes are analogous to right ascension and declination and both axes run from $-90°$ to $+90°$.

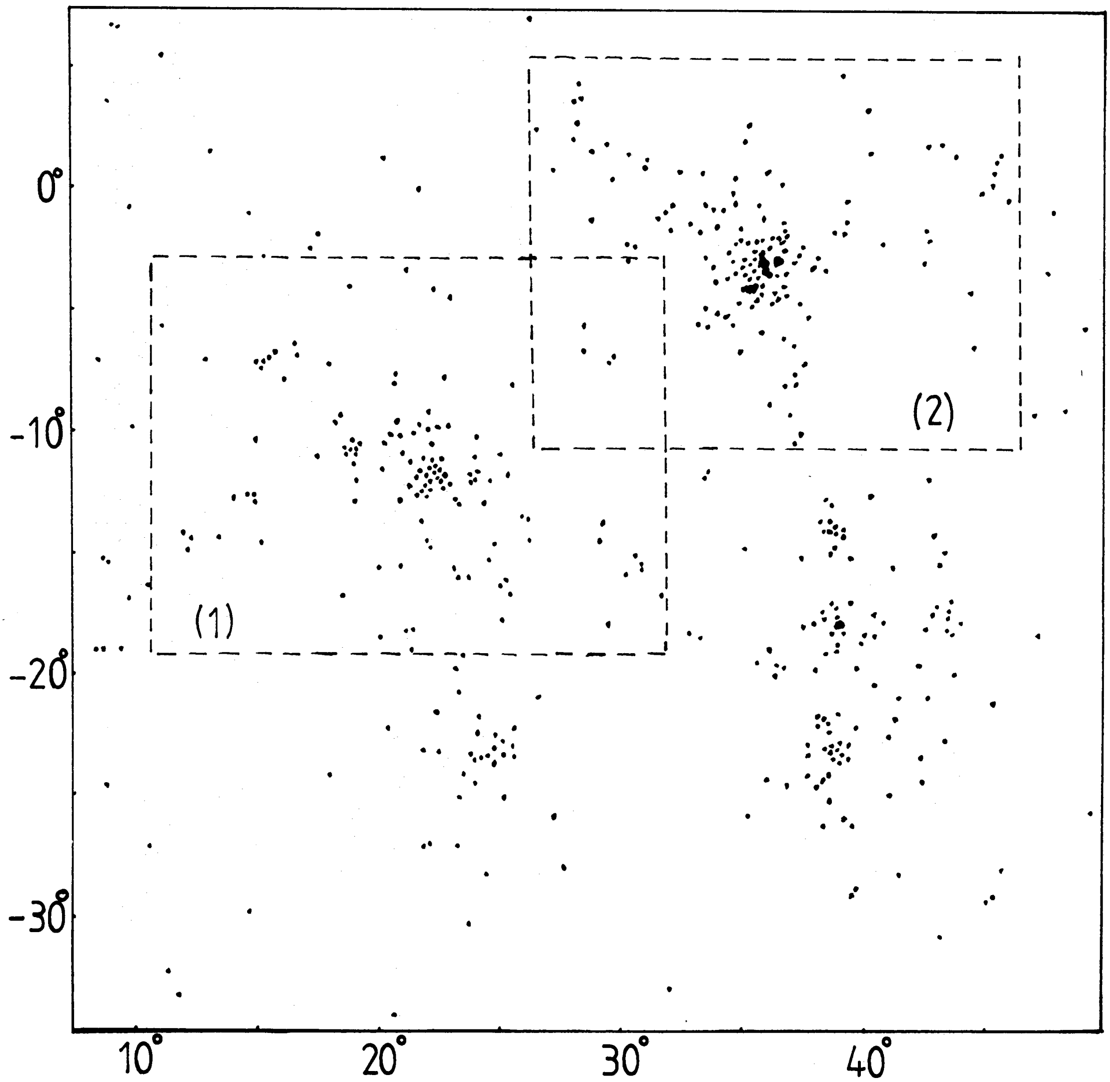


**Figure 2.** Enlargement of a region from Fig. 1 with the estimated boundaries of two clusters marked.

is shown in Fig. 1. It can be seen that the general appearance is rather 'lumpy' with widespread clustering and voids, as is the observed data (see, for instance, Davis *et al.* 1982).

From these projections, regions of interest were selected and plotted on an enlarged scale and from these enlargements the clusters were identified by eye. This is illustrated in Fig. 2. Having identified the clusters in projection, the individual galaxies were plotted as points along the suppressed third space coordinate and foreground and background galaxies eliminated, again by eye. This gave the final sample of galaxies that we define as the true cluster. In addition, the galaxies identified in projection were plotted as points along the redshift axis. Obvious foreground and background galaxies were eliminated. For 'borderline' galaxies we adopted a cut-off at three standard deviations from the mean velocity. This gave the final sample of galaxies that we define as the apparent cluster. In Fig. 3 we show the two clusters identified in Fig. 2, plotted as points along both the suppressed space axis and the redshift axis.

It can be seen from Fig. 3 that, as might be expected, the use of redshifts as a third space coordinate can be quite misleading. It can also be seen from Figs 2 and 3 that even when the three space coordinates are used to identify the clusters there is still some uncertainty in the location of the boundaries.

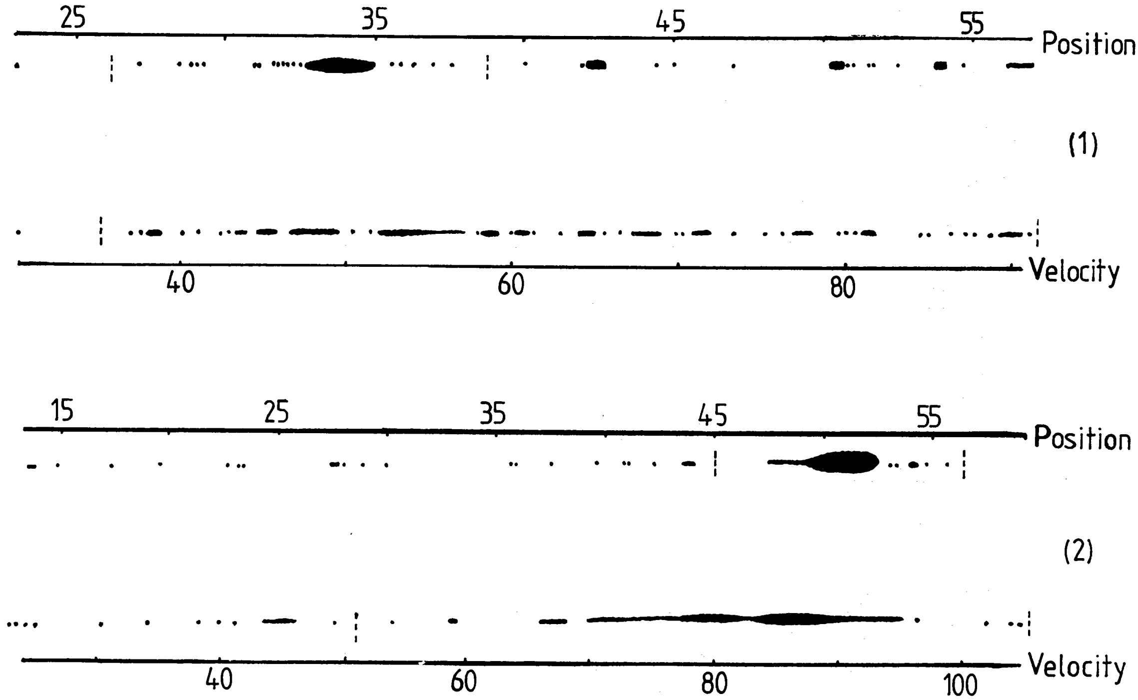


Figure 3. The clusters identified in Fig. 2 plotted as points along the suppressed third space axis and along the redshift axis. These axes are in model units of distance and velocity. The vertical axis is arbitrary. Using the redshifts in place of the third space coordinate causes the upper cluster to be seriously contaminated ($M_{\mathrm{vt}}/M = 3.94$). The lower cluster is defined equally well by the redshifts or the third space coordinate and is relatively uncontaminated ($M_{\mathrm{vt}}/M = 1.32$).

Having selected the clusters, both apparent and true, their virial masses were calculated using the standard definition

$$M_{\mathrm{vt}} = \frac{\langle V^2 \rangle \langle R \rangle}{G} \tag{1}$$

where $M_{\mathrm{vt}}$ is the virial mass of the cluster and $\langle V^2 \rangle$ and $\langle R \rangle$ are the three-dimensional galaxy velocity dispersion and virial radius respectively. These are conventionally defined as

$$\langle V^2 \rangle = \frac{\alpha}{\sum_i m_i} \sum_i m_i (V_i - \bar{V})^2$$

$$\langle R \rangle = \beta \left( \sum_i m_i \right)^2 \left( \sum_i \sum_{j>i} \frac{m_i m_j}{r_{ij}} \right)^{-1} \tag{3}$$

where $m_i$ is the mass of the $i$th galaxy, $V_i$ the line-of-sight velocity of the $i$th galaxy, $\bar{V}$ the mean line-of-sight velocity of all the galaxies, $r_{ij}$ the separation on the sky between the $i$th and $j$th galaxy and the summations $i$ and $j$ run over all galaxies. $\alpha$ and $\beta$ are deprojection factors to convert the observables ($V_i$ is in one dimension, $R_{ij}$ is in two dimensions) to the full three-dimensional qualities. It is conventionally assumed that there is spherical symmetry and the velocities are isotropic hence $\alpha = 3$ and $\beta = \pi/2$.

## 3 Results

We identified 26 clusters containing between 48 and 273 galaxies and we show our results in Fig. 4 which is the frequency distribution of $M_{vt}/M$. Figure 4(a) is for the apparent clusters (where redshifts are used to eliminate contaminants) and Fig. 4(b) is for the true clusters (where the third space coordinate is used to eliminate contaminants). It can be seen that the distribution for the apparent clusters has a long-tail. This is entirely due to contamination.

A test for contamination is to compare the line-of-sight velocity distribution to a normal distribution (Yahil & Vidal 1977).

A form of this test is a modified $\chi^2$ (Freund 1972) defined by

$$\chi^2 = \sum_i \frac{(f_i - e_i)^2}{e_i} \tag{4}$$

where $f_i$ is the actual number in an interval $i$, $e_i$ the expected number in the interval $i$ and $i$ runs over the total number of intervals. The intervals were defined in terms of $\sigma$, the standard deviation of the velocity distribution, and we used seven intervals, $\pm 1\sigma$ or greater, $\pm 0.6$–$1.0\,\sigma$, $\pm 0.2$–$0.6\,\sigma$, $-0.2$ to $+0.2\,\sigma$. The number of degrees of freedom is $i-2$ and the appropriate values of $\chi^2$ can be found in standard tables.

When this test was applied to the 26 clusters, all five of those with seriously inflated values of $M_{vt}/M$ were identified at the 0.95 significance level. In addition, one cluster with a reasonable value of $M_{vt}/M$ was erroneously identified.

When the contaminated clusters were removed then the distribution of $M_{vt}/M$ for the remaining apparent clusters has a mean value of 0.97 and a standard deviation of 0.36. This compares with the distribution of $M_{vt}$/M for the true clusters which has a mean of 0.98 and a standard deviation of 0.34. Whilst it is gratifying that the mean $M_{vt}/M$ is so close to unity there is significant scatter. Since this occurs even under the ideal conditions of the model universe, this scatter must be regarded as inherent in the virial method. The most probable reasons for its occurrence are:

(i) Departures from spherical symmetry and velocity isotropy invalidating the de-projection factors used in equations (2) and (3).

(ii) Non-satisfaction of the virial theorem. White (1976) shows that the gravitational potential energy of a cluster oscillates about its virial equilibrium value. As the cluster evolves dynamically such oscillations are gradually damped out.

(iii) Uncertainty in assigning cluster membership to 'borderline' galaxies.

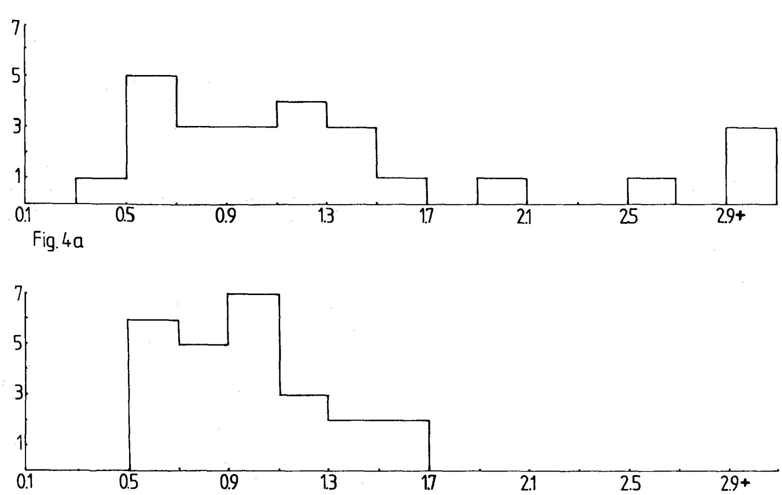


**Figure 4.** Frequency distributions of $M_{vt}/M$ for the apparent clusters (a) and for the real clusters (b).

## 4 Application to observations

A search of the literature showed three clusters, Coma, Hercules Supercluster and A2151, that were sufficiently well observed to be comparable with the model clusters. The boundaries of these clusters are far from clear. The radius of Coma is estimated to be between 100′ and 360′ (Rood *et al.* 1972). Following Tift & Gregory (1976) we chose 3°. For Hercules Supercluster and A2151 we adopted the boundaries used by Tarenghi *et al.* (1979). The uncertainty in these definitions may be judged from the excellent plates of the region shown by Burbidge & Burbidge (1959).

The individual galaxies were then plotted as points on the redshift axis and foreground and background galaxies removed using the same procedure as was used for the model clusters. Finally, the velocity dispersions, virial radii, virial masses and $\chi^2$ values were calculated using equations (1)–(4) modified, where appropriate, such that weightings were by luminosity instead of mass. The results are shown in Table 1.

For $\chi^2 > 11.07$ the velocity distribution differs from a normal distribution at the 0.95 confidence level. Thus, on the basis of the results from the previous section, Hercules Supercluster is contaminated and the mass-to-light ratio is seriously in error. Coma and A2151 both have accepable $\chi^2$ values and therefore at the $1\sigma$ level their virial masses are within 50 per cent of the correct value.

However, for real clusters, as opposed to model ones, there are three additional sources of uncertainty. First, for the model clusters we found the mass-weighted virial radii and velocity dispersions are lower than the unweighted quantities. For the real clusters there is no significant difference between the weighted and unweighted quantities, suggesting that the real clusters are less dynamically evolved than their model counterparts. Assuming that they are at least gravitationally bound, their gravitational potential energy may be further from its virial equilibrium value (White 1976). Secondly, the model clusters contain only one component, the galaxies. The real clusters contain at least two components, the galaxies and

**Table 1.**

| Cluster | Number of galaxies | $\langle V^2\rangle^{1/2}$ (km s$^{-1}$) | $R$ (Mpc) | $M_{\mathrm{vt}}$ ($M_\odot$) | $L_{\mathrm{p}}$ ($L_\odot$) | $M_{\mathrm{vt}}/L_{\mathrm{p}}$ | $\chi^2$ |
|---|---|---|---|---|---|---|---|
| Coma | 93 | 885 | 2.36 | $2.03\times10^{15}$ | $1.01\times10^{13}$ | 201 | 8.24 |
| Hercules (A2151) | 39 | 883 | 1.36 | $1.16\times10^{15}$ | $6.92\times10^{12}$ | 168 | 8.07 |
| Hercules Supercluster | 117 | 1243 | 4.43 | $7.49\times10^{15}$ | $2.42\times10^{13}$ | 310 | 12.83 |

*Sources:*

Position and redshifts: Coma, Tift & Gregory (1976) and references therein; Hercules, Tarenghi *et al.* (1979) and references therein.

Luminosities: magnitudes, to the limit $M_{\mathrm{p}} \leq 15.7$, were summed from Zwicky & Herzog (1963) for those cluster galaxies with redshifts. This gave Coma 9.10 mag, Hercules Supercluster 9.84 mag and A2151 11.20 mag; the latter two include a correction factor of 4/3 since not all galaxies have redshifts. The following corrections were then applied:

(a) Galactic absorption Coma 0.19 mag, Hercules 0.32 mag.

(b) Conversion from isophotal to total magnitudes of 0.58 mag (Coma) and 0.50 mag (Hercules). These corrections based on Huchra's (1976) estimate of 0.65 mag for early-type galaxies and 0.33 mag for late-type galaxies.

(c) Faint galaxies i.e. $M_{\mathrm{p}} > 15.7$ – Coma a factor 100/60, Hercules 100/35. These corrections are obtained by applying a standard luminosity function derived from the Coma luminosity function (Gregory & Tift 1976).

(d) A distance modulus of Coma 34.81 mag, Hercules 35.85 mag.

(e) $M_{\mathrm{ZW}\odot} = 5.48$.

intracluster gas. Limber (1959) and Smith (1980), using $n$-body models with various distributions of unseen mass, found that the virial theorem over-/underestimated the total mass in proportion to (i) the mass of the unseen component and (ii) whether its spatial distribution was more/less centrally condensed than that of the galaxies. As regards intracluster gas, Lea *et al.* (1973) found that in order to fit the X-ray flux from Coma cluster they had to assume a spatial distribution of the gas somewhat less centrally condensed than that of the galaxies. Their model also predicted that the mass of the gas was of order the mass of the galaxies. This suggests that if intracluster gas is the only significant dark component, then it will have little effect on the virial mass determination. Thirdly, complete redshift information is available for the model clusters. This is not the case for real clusters. Aarseth & Saslaw (1972) using $n$-body models, showed that where only a sample of redshifts are available, selection effects lead to a bias of redshifts towards the brighter galaxies, which tends to underestimate the total mass. This conclusion assumes there has been sufficient dynamical evolution to cause mass-segregation. As discussed earlier, there seems little evidence for this in real clusters and so the effect may not be significant.

Since for real clusters the results are usually quoted as mass-to-light ratios it is necessary also to consider the uncertainties in their luminosities. As shown in Table 1 these are calculated by summing the luminosities of the observed (brighter) galaxies and then assuming a form for the luminosity function to derive a correction for the fainter galaxies. These corrections are substantial, i.e. 40 per cent of the luminosity of Coma is due to this correction and 65 per cent of A2151. The standard luminosity function for even the most studied cluster, Coma, is uncertain (see, for instance, Abell 1977). There is also uncertainty in the galaxy magnitudes themselves (Huchra 1976).

Finally, the mass-to-light ratio is directly proportional to the Hubble constant. A value of 75 km s$^{-1}$ mpc$^{-1}$ was used in deriving all the quantities in Table 1.

Combining these uncertainties leads to a formal error of at least a factor of 3. Thus, although the cluster mass-to-light ratios imply the existence of dark matter the amount of this matter could be accounted for by intracluster gas and 'massive halo' galaxies. The derived mass-to-light ratio of Coma (predominantly composed of early-type galaxies) is higher than A2151 (predominantly late-type galaxies). However, the result is inconclusive since the difference is within the errors.

## 5 Conclusions

By studying clustering in $n$-body simulations of galaxies in an expanding universe we have shown that:

1. Those clusters that are seriously contaminated by foreground or background galaxies can be identified by a $\chi^2$ test comparing their velocity distribution to a normal distribution.
2. The remaining clusters have a mean value for the statistic $M_{\rm vt}/M$ that is close to unity; however, there is significant scatter. This scatter appears to be intrinsic to virial methods of mass determination.

We have applied these results to three real clusters. We find that Hercules Supercluster is seriously contaminated. For the other two clusters we derive mass-to-light ratios of 201 $M_\odot/L_\odot$ (Coma) and 168 $M_\odot/L_\odot$ (A2151) with a formal error of at least a factor of 3. The case for 'missing mass' is unproven since at the lower level these mass-to-light ratios can be accounted for by 'massive halo' galaxies and intracluster gas.

It would obviously be desirable to improve the accuracy of virial mass determinations since the method is of such general applicability. A possible approach to this could be to use

$n$-body simulations to experiment with different cluster selection procedures, e.g. varying density enhancement criteria, varying velocity cut-offs, etc. Also it may be possible to improve our understanding of clusters by constructing multi-component $n$-body simulations to include both gas and galaxies. At the present moment, $n$-body models show satisfactory agreement with observation as regards correlation functions and multiplicity functions; however, the model clusters appear to be more dynamically evolved than their observed counterparts.

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