

Evolution of the multipolar magnetic field in isolated neutron stars

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ABSTRACT

The evolution of the multipolar structure of the magnetic field of isolated neutron stars is studied assuming the currents to be confined to the crust. We find that, except for multipoles of very high order ($l \geq 25$), the evolution is similar to that of a dipole. Therefore no significant evolution is expected in the pulse shape of isolated radio pulsars because of the evolution of the multipole structure of the magnetic field.

Key words: magnetic fields – stars: neutron – pulsars: general.

1 INTRODUCTION

Strong multipole components of the magnetic field have long been thought to play an important role in the radio emission from pulsars. Multipole fields have been invoked for the generation of electron–positron pairs in the pulsar magnetosphere. For example, the Ruderman & Sutherland (1975) model requires that the radius of curvature of the field lines near the stellar surface should be of the order of the stellar radius to sustain pair production in long-period pulsars. This is much smaller than the expected radius of curvature of the dipole field. Such a small radius of curvature could be a signature of either an extremely offset dipole (Arons 1998) or a multipolar field (Barnard & Arons 1982). Further, soft X-ray observations of pulsars show non-uniform surface temperatures which can be attributed to the presence of a quadrupolar field (Page & Sarmiento 1996).

Magnetic multipole structure at and near the polar cap is also thought to be responsible for the unique pulse profile of a pulsar (Vivekanand & Radhakrishnan 1980; Krolik 1991; Rankin & Rathnasree 1995). The recent estimates that there should be several tens of sparks populating the polar cap is also best explainable if multipole fields dictate the spark geometry near the surface (Deshpande & Rankin 1998; Rankin & Deshpande 1998). Significant evolution in the structure of the magnetic field during the lifetime of a pulsar may therefore leave observable signatures. If the multipoles grow progressively weaker in comparison with the dipole then one can expect pulse profiles to simplify with age and vice versa.

The evolution of the magnetic fields in neutron stars in general is still a relatively open question. During the last decade, two major alternative scenarios for the field evolution have emerged. One of these assumes that the field of the neutron star permeates the whole star at birth, and its evolution is dictated by the interaction between superfluid vortices (carrying angular momentum

and superconducting fluxoids (carrying magnetic flux) in the stellar interior. As the star spins down, the outgoing vortices may drag and expel the field from the interior, leaving it to decay in the crust (Srinivasan et al. 1990). In a related model, plate tectonic motions driven by pulsar spindown drag the magnetic poles together, reducing the magnetic moment (Ruderman 1991a,b,c).

The other scenario assumes that most of the field is generated in the outer crust after the birth of the neutron star (Blandford, Applegate & Hernquist 1983). The later evolution of this field is governed entirely by the ohmic decay of currents in the crustal layers. The evolution of the dipole field carried by such currents has been investigated in some detail in recent literature (Sang & Chanmugam 1987; Geppert & Urpin 1994; Urpin & Geppert 1995, 1996; Konar & Bhattacharya 1997, 1999). These studies include field evolution in isolated neutron stars, as well as those accreting from their binary companions. The results show interesting agreement with observations, lending some credence to the crustal picture.

In this paper, we explore the ohmic evolution of higher order multipoles in isolated neutron stars, assuming the currents to be originally confined in the crustal region. Our goal is to find whether there would be any observable effect on the pulse shape of radio emission from isolated pulsars as a result of this evolution. In Section 2 we discuss the details of the computation and in Section 3 we present our results and discuss the implications.

2 COMPUTATIONS

The evolution of the magnetic field, as a result of ohmic diffusion, is governed by the equation (Jackson 1975)

$$\frac{\partial \mathbf{B}}{\partial t} = -\frac{c^2}{4\pi} \nabla \times \left(\frac{1}{\sigma} \nabla \times \mathbf{B} \right), \quad (1)$$

where $\sigma(r,t)$ is the electrical conductivity of the medium. Following Wendell, Van Horn & Sargent (1987) we introduce a

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vector potential $\mathbf{A} = (0, 0, A_\phi)$ assuming the field to be purely poloidal, such that

$$S(r, \theta, t) = -r \sin \theta A_\phi(r, \theta, t),$$

where $S(r, \theta, t)$ is the Stokes' stream function. S can be separated in r and θ in the form

$$S(r, \theta, t) = \sum_{l=1} R_l(r, t) \sin \theta P_l^1(\cos \theta),$$

where $P_l^1[\cos(\theta)]$ is the associated Legendre polynomial of degree one and R_l is the multipole radial function. From equation (1) we obtain

$$\frac{\partial^2 R_l}{\partial x^2} - \frac{l(l+1)}{x^2} R_l = \frac{4\pi R_*^2 \sigma}{c^2} \frac{\partial R_l}{\partial t}, \quad (2)$$

where $x \equiv r/R_*$ is the fractional radius in terms of the stellar radius R_* . The solution of this equation, with the boundary conditions

$$\begin{aligned} \frac{\partial R_l}{\partial x} + \frac{l}{x} R_l &= 0, & \text{as } x \rightarrow 1, \\ R_l &= 0, & \text{at } x = x_c, \end{aligned} \quad (3)$$

for a particular value of l , gives the time evolution of the multipole of order l . Here, the first condition matches the correct multipole field in vacuum at the stellar surface and the second condition makes the field vanish at the core–crust boundary (where $r = r_c$, the radius of the core) to keep the field confined to the crust. We assume that the field does not penetrate the core in the course of evolution, as the core is likely to be superconducting.

2.1 Crustal physics

The rate of ohmic diffusion is determined mainly by the electrical conductivity of the crust. The conductivity of the solid crust is given by

$$\frac{1}{\sigma} = \frac{1}{\sigma_{\text{ph}}} + \frac{1}{\sigma_{\text{imp}}},$$

where σ_{ph} is the phonon scattering conductivity, which we obtain from Itoh et al. (1984) as a function of density and temperature, and the impurity scattering conductivity σ_{imp} is obtained from the expressions given by Yakovlev & Urpin (1980).

We construct the density profile of the neutron star in question using the equation of state of Wiringa, Fiks & Fabrocini (1988) matched to those of Negele & Vautherin (1973) and Baym, Pethick & Sutherland (1971) for an assumed mass of $1.4 M_\odot$. As conductivity is a steeply increasing function of density, and because the density in the crust spans eight orders of magnitude, the conductivity changes sharply as a function of depth from the neutron star surface. Thus the deeper the location of the current distribution, the slower the decay.

Another important factor in determining the conductivity is the temperature of the crust. In the absence of impurities, the scattering of crustal electrons come entirely from the phonons in the lattice (Yakovlev & Urpin 1980) and the number density of phonons increases steeply with temperature. The thermal evolution of the crust therefore plays an important role in the evolution of the magnetic field.

The thermal evolution of a neutron star has been computed by many authors, and it is clearly seen that the inner crust

($\rho > 10^{10} \text{ g cm}^{-3}$) quickly attains an isothermal configuration after birth. In the outer regions of the crust, the temperature follows an approximate relation,

$$T(\rho) = \left(\frac{\rho}{\rho_b}\right)^{1/4} T_i, \quad \rho \lesssim \rho_b, \quad (4)$$

where T_i is the temperature of the isothermal inner crust and ρ_b is the density above which the crust is practically isothermal. As the star cools, a larger fraction of the crust starts becomes isothermal, with ρ_b being approximately given by

$$\rho_b = 10^{10} \left(\frac{T_i}{10^9}\right)^{1.8}. \quad (5)$$

Relations (4) and (5) above have been obtained by fitting to the radial temperature profiles published by Gudmundsson, Pethick & Epstein (1983). For the time evolution of T_i we use the results of Urpin & van Riper (1993) for the case of standard cooling (the crustal temperature T_m in their notation corresponds to T_i above).

A third parameter that should be considered in determining conductivity is the impurity concentration. The effect of impurities on the conductivity is usually parametrized by a quantity Q , defined as $Q = 1/n \times \sum_i n_i (Z - Z_i)^2$, where n is the total ion density, n_i is the density of impurity species i with charge Z_i and Z is the ionic charge in the pure lattice (Yakovlev & Urpin 1980). In the literature Q is assumed to lie in the range 0.0–0.1. However, statistical analyses indicate that the magnetic field of isolated pulsars does not undergo significant decay during the radio pulsar lifetime (Bhattacharya et al. 1992; Hartman et al. 1997; Mukherjee & Kembhavi 1997). It has been shown (Konar 1997) that to be consistent with this, values of Q in excess of 0.01 are not allowed in the crustal model.

2.2 Numerical scheme

To solve equation (2) we assume the multipole radial profile used by Bhattacharya & Datta (1996; see also Konar & Bhattacharya 1997). This profile contains the depth and the width of the current configuration as input parameters and we vary them to check the sensitivity of the result to these. We solve equation (2) numerically using the Crank–Nicholson method of differencing. We have modified the numerical code developed by Konar (1997) and used by Konar & Bhattacharya (1997) to compute the evolution of multipolar magnetic fields satisfying the appropriate boundary conditions given by equation (3).

3 RESULTS AND DISCUSSION

In Figs 1 and 2 we plot the evolution of the various multipole components of the magnetic field, assuming the same initial strength for all, with time as a result of pure diffusion in an isolated neutron star. It is evident from the figures that, except for very high multipole orders ($l \gtrsim 25$) the reduction in the field strength is very similar to that of the dipole component. For a multipole of order l there would be 2^l reversals across the stellar surface. For typical spin periods, the size of the polar cap bounded by the base of the open field lines is ~ 0.01 per cent of the total surface area. To contribute to the substructure of the pulse, therefore, the required multipoles must have a few reversals in the polar cap, which demands that the multipole order must be five or more. On the other hand, if the multipole order were very large

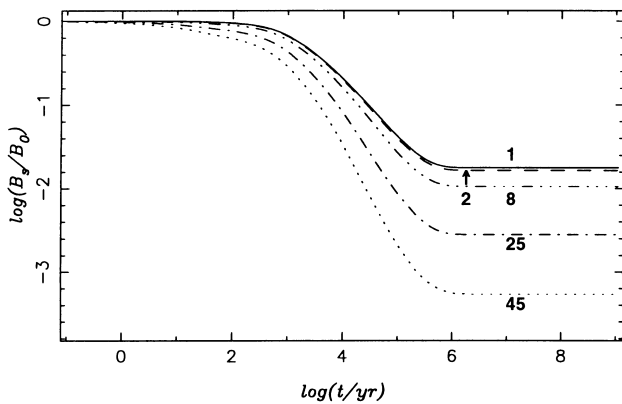


Figure 1. The evolution of the surface magnetic field for various multipoles resulting from pure diffusion. The numbers next to the curves correspond to respective orders of multipole. All the curves correspond to $Q = 0.0$ and a depth of current concentration at $x = 0.98$, i.e. a density of $\rho = 10^{11} \text{ g cm}^{-3}$.

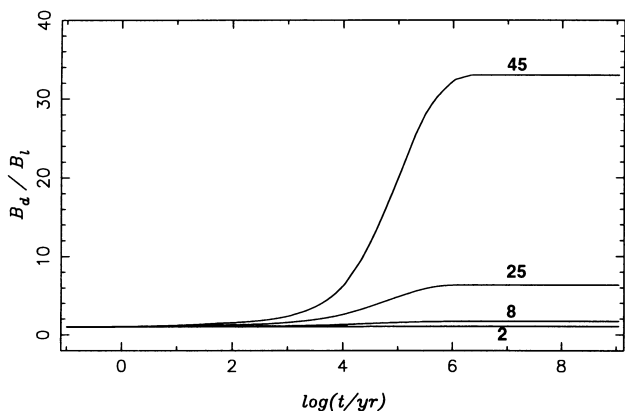


Figure 2. The ratio of the dipole surface field to the multipole field is plotted as a function of age. The numbers next to the curves correspond to respective orders of multipole. All the curves correspond to $Q = 0.0$ and a depth of current concentration at $x = 0.98$ i.e. a density of $\rho = 10^{11} \text{ g cm}^{-3}$.

($l > l_{\text{max}} \sim 20$) the fine structure would be so small that it would be lost in the finite time resolution of observations. Therefore, l values in the range 5 to l_{max} would be the major contributors to the observed structure of the pulse profile. However, as seen from Figs 1 and 2, multipoles of such orders evolve similarly to the dipole. Therefore no significant evolution is expected in the pulse shape as a result of the evolution of the multipole structure of the magnetic field. As discussed before, multipole orders contributing to the required field line curvature for pair production are low, most prominently a quadrupole. As the evolution of these low orders is also very close to the dipole, the radii of curvature of the field lines on the polar cap are not expected to change significantly in the lifetime of a radio pulsar.

To test the sensitivity of these results to the impurity concentration of the crust and the density at which the initial current is concentrated, we have evolved models with various values of these parameters. The results are displayed in Figs 3 and 4, where we plot the ratio of the dipole to higher multipoles at an age of 10^7 yr. It is seen that the results are insensitive to

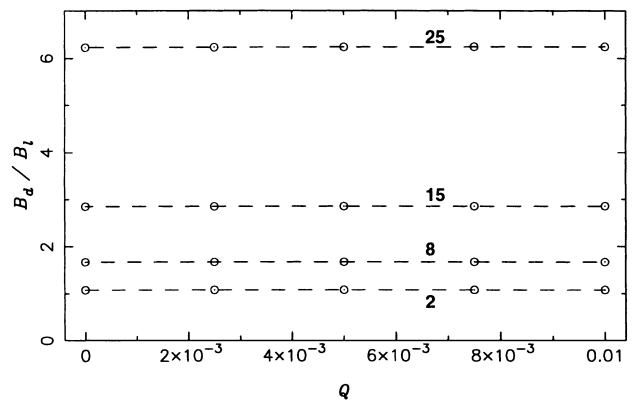


Figure 3. The ratio of the dipole surface field to that of the multipoles at 10^7 yr as a function of Q . The numbers next to the curves correspond to respective orders of multipole. All curves correspond to a depth of $x = 0.98$, corresponding to a density of $\rho = 10^{11} \text{ g cm}^{-3}$, at which the initial current is concentrated.

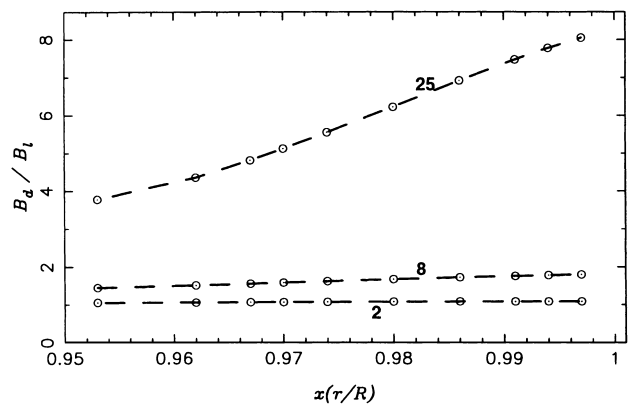


Figure 4. The ratio of the dipole surface field to that of the multipoles at 10^7 yr as a function of depth. The points marked in the plots here correspond to densities $\rho = 10^{13.5}, 10^{13}, 10^{12.5}, 10^{12}, 10^{11.5}, 10^{11}, 10^{10.5}, 10^{10}, 10^{9.5}$ and 10^9 g cm^{-3} . The numbers next to the curves correspond to respective orders of multipole. All curves correspond to $Q = 0$.

these parameters, particularly for low orders of multipoles of interest.

Krolik (1991) and Arons (1993) conjectured, that except for multipoles of order $l \geq R_*/\Delta r$, the decay rates would be similar because of the finite thickness Δr of the crust over which the current is confined. The evolution plotted in Fig. 1 assumes that $\Delta r = 1.2 \text{ km}$, for which $R_*/\Delta r \sim 8$. However it is seen from Figs 1 and 2 that significant decay occurs only for $l \geq 25$, much greater than $R_*/\Delta r$. This is most likely caused by a steep increase in conductivity towards the interior.

In conclusion, our results indicate that for a crustal model of the neutron star magnetic field there will be no significant change in the multipolar structure with age. This fact seems to be corroborated by observations: studies identifying multiple components in pulse profiles (Kramer et al. 1994) show that the number of components does not vary with the age of the pulsar. Thus the evolution of the multipolar structure of the magnetic field is unlikely to leave any observable signature on pulsar emission. This is in contrast to the predictions from the plate tectonics model

of Ruderman (1991a,b,c) which suggests a major change in the field structure with pulsar spin evolution.

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