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Evidence for evolving elongated pulsar beams

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Summary. Assuming the Radhakrishnan-Cooke magnetic pole model of pulsar radiation, we analyse the total change in the position angle of the linear polarisation in 16 pulsars, using the reliable observational data of Backer and Rankin (1980). We find that pulsar beams are on the average highly elongated, with the ratio R of the North–South to East–West dimensions (directions referred to the rotation axis) being 3.0 ± 0.4. The estimate of R increases further if we allow for reasonable selection effects and correct for the other approximations in our analysis. Taking an enlarged sample of 30 pulsars, whose data are less reliable but adequate, it appears that R evolves rapidly with pulsar period P, approximately as $P^{-0.65}$. However, R seems to be independent of the period derivative $\dot{P}$ and shows only weak variation with pulsar age and strength of magnetic field. These results are nearly independent of the assumed value of the angle between the rotation and magnetic axes. From some other data we conclude that the central hollow portion of the pulsar beam does not evolve with P.

The evolution of R with P naturally accounts for the P dependence of interpulse occurrence. Coupled with the constant size of the central hollow, it might explain the P dependence of the complexity of integrated profiles. For a mean R of 3.0, the pulsar "beaming fraction" f is ~ 0.53, much larger than the value currently assumed (0.2); hence the present estimates of the number of pulsars in the Galaxy and pulsar birth rate may be high. The evolution of f with P makes much more certain the recent observation that there is a deficit (now estimated to be $\sim$ an order of magnitude) of short P (<0.5 s) pulsars in the Galaxy. Current theories of pulsar electrodynamics and radio beams may need to be revised to account for our new results.

Key words: Pulsar – pulsar beams – polarisation – polar cap – hollow cone

I. Introduction to the problem

Polarisation has played a key role in our understanding of radio pulsars. Based on their pioneering observations on the Vela pulsar (PSR 0833-45), Radhakrishnan and Cooke (1969) proposed a model of pulsar radiation the main elements of which underlie most current pulsar theories. In the RC model the source of the radiation is believed to be in the vicinity of a magnetic pole. Charged particles are accelerated along the open field lines emanating from the polar regions (Goldreich and Julian, 1969) and these emit radio frequency curvature radiation in the direction of their motion. Since curvature radiation is beamed tangential to the magnetic field, the radiation forms a hollow conical beam (Komesaroff, 1970) directed radially outward from the star and centred on the magnetic axis, and different parts of the pulse are emitted from different parts of the polar cap. Curvature radiation is polarised parallel to the plane of curvature of the magnetic field and hence the polarisation angle variation within the pulse maps the orientation of the projected magnetic field at various points in the line of sight within the pulsar beam. On the basis of the simple polarisation angle variation observed in PSR 0833-45, Radhakrishnan and Cooke (1969) proposed that the magnetic field at the radiation source is essentially dipolar so that the field lines are radial when projected on a plane perpendicular to the magnetic axis (Fig. 1). Many later polarisation studies [Manchester and Taylor (1977) and notably Backer and Rankin (1980) who could eliminate the complications from orthogonal modes] have strongly confirmed the RC picture.

Most current theories of pulsar electrodynamics implicitly assume that the pulsar beam is a cone of circular cross-section, particularly when they are dealing with a dipolar magnetic field geometry. We discuss here a strong inconsistency between this assumption and the currently available polarisation data on pulsars. Figure 1 shows that, if 2θ is the total polarisation angle swing across the pulse, then for a beam of circular cross-section,

$$\cos\theta = |y/r|, \tag{1}$$

where y is the latitude off-set between the line of sight and the magnetic pole and r is the radius of the beam[1]. If pulsars are oriented randomly with respect to Earth, then we expect $|y/r|$ to be uniformly distributed between 0 and 1 (neglecting selection and spherical effects for the moment). Table 1 lists values of 2θ for 16 pulsars (from the observations of Backer and Rankin, 1980) and shows a serious discrepancy. Equation (1) implies that half the pulsars should have 2θ values greater than 120° ($=2\cos^{-1}0.5$) whereas only 3 out of the 16 pulsars show this. Further, 4 out of 16 pulsars ought to have 2θ swings less than 82°.8 but Table 1 lists 11, 2 pulsars should have $2\theta < 57°.9$ but there are 8, only 1 pulsar should have $2\theta < 40°.7$ but there are 6, etc. There is clearly a massive discrepancy between the observations and the predictions

1 The "radius" depends on the definition of the boundary of the beam. Throughout this paper we define the periphery as that point at which the 400 MHz radio flux falls to 10% of the pulse peak

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Table 1. Data for 16 pulsars obtained from Backer and Rankin (1980)

Group A				Group B			
PSR	$2\theta^{\circ}$	W_{10}°	$\|y/Y\|$ for $R=3.0$	PSR	$2\theta^{\circ}$	W_{10}°	$\|y/Y\|$ for $R=3.0$
1237+25	175	15.2	0.02	2303+30	56.2	7.9	0.53
0525+21	152	20.8	0.08	0950+08	51.0	30.6	0.58
0301+19	136	18.3	0.13	0611+22	40.4	14.2	0.68
2020+28	97.2	18.2	0.28	1929+10	32.7	22.2	0.75
1133+16	89.6	12.1	0.32	1604−00	27.0	16.9	0.81
0823+26	79.1	10.6	0.38	1933+16	25.7	12.1	0.83
0834+06	61.4	9.4	0.49	1944+17	21.6	31.4	0.87
2016+28	58.0	13.8	0.52	0540+23	21.0	21.8	0.88

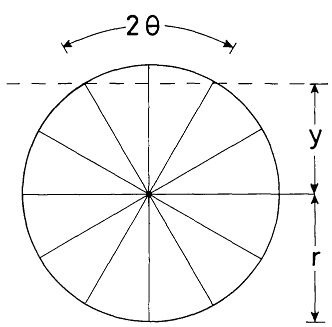

Fig. 1. Schematic representation of a circular pulsar beam of radius r. In the RC model, the projected magnetic field lines are taken to be radial, as shown. The dashed line is the path taken by a typical line of sight at the offset y from the beam centre. The total swing 2θ of the position angle of the linearly polarised component of the radio radiation is determined purely by the ratio $|y/r|$ (neglecting the spherical nature of the problem)

of the simple RC model. It is obvious that no ordinary selection effect can explain the differences (we discuss this question in greater detail later). For instance the data in Table 1 suggest that *half* the observed pulsars correspond to lines of sight intersecting the outer *one-eighth* of the circular beam. This is very unlikely since all the available evidence (including the arguments in Sect. V here) point to a beam luminosity (and pulsar visibility) that falls away from the centre.

It is possible to argue that, because of the large discrepancy discussed above, the RC model is entirely wrong in all respects. However, compelling observational evidence has accumulated in favour of many aspects of the model (e.g., Manchester and Taylor, 1977). Here we show that all that is needed is to abandon the circular beam hypothesis. The observations are consistent with the key features of the RC model such as (a) radiation from magnetic poles, (b) dipolar field geometry, and (c) polarisation position angle related to projected field direction, provided we allow for an elongated beam cross-section [in addition to eliminating orthogonal modes as suggested by Backer and Rankin (1980)]. To obtain numerical estimates of the elongation we assume the shape of the beam to be an ellipse which is the most direct generalisation of a circle (this leads to a conservative estimate of the beam elongation as discussed in Sect. III). Section II discusses the simple relations connecting 2θ and some of its averages (like $\langle\cos\theta\rangle$) to aspects of the beam geometry such as the line of sight offset y and the elongation parameter R ($\equiv$major axis/minor axis) of the elliptic beam. In Sect. III we analyse the data in Table 1 and find that $R=3.0\pm0.4$. We discuss the question of selection effects and errors from the various approximations we have made and conclude that R is probably even higher. We note that Jones (1980) has also studied pulsar polarisation data using a different analysis (related to our arguments in Sect. V here) and concludes that $R=2.5$.

Kundt (1982) suggested that the elongation R may evolve during a pulsar's life. We investigate this possibility in Sect. IV with an expanded data base of 30 pulsars. We find a statistically significant variation of the elongation with pulsar period P, R being large at short P and decreasing to ~ 1 (i.e., a circular beam) at long P. In Sect. V we analyse some of the spherical effects which have been neglected in the previous sections and show that all our results carry through unaffected. Since the analysis here makes use of a different observational parameter viz., the maximum rate of change of the polarisation angle with pulse longitude $|d\theta/d\varphi|_{\max}$, it is satisfying that there is good consistency with the earlier calculations. Finally in Sect. VI we discuss possible implications of our results. An evolving elongated beam seems to naturally explain some of the observed correlations in pulsars such as the variation of interpulse frequency and profile complexity with P and the fading away of pulsars as a function of $\dot{P}P^{-5}$. Further, the value of R affects the beaming fraction f, the fraction of pulsars in the Galaxy that are beamed towards Earth. Since current statistical studies of pulsars (number in the Galaxy, birth rate, model of pulsar evolution, etc.) are based on the period independent value of $f\sim 0.2$ (derived assuming circular beams), a reassessment may be called for [we note that Kundt (1977, 1981) has suggested that f may be ~ 0.5–1.0 if pulsar beams turn out to be elongated]. Also, the large beam elongation coupled with its P dependence, if confirmed with larger data samples, could improve our understanding of the physics of pulsars and their radio radiation.

II. Theory

We assume that pulsar beams are elliptical in shape with the principal axes oriented North–South and East–West with respect to the rotation axis i.e., parallel to the local lines of constant longitude and constant latitude respectively. Let the dimension of the semi-axis in the North–South direction be Y and let it be Y/R in the perpendicular direction (Fig. 2). We are interested in estimating the elongation parameter R, which we take to be the same in all pulsars in a given sample.

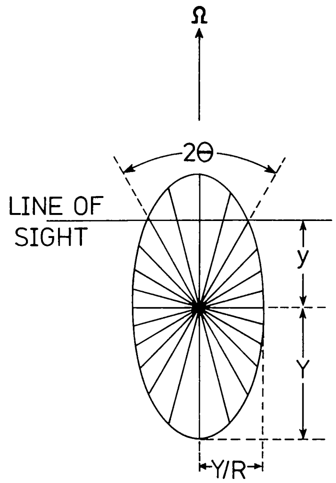

Fig. 2. An elliptic pulsar beam similar to Fig. 1. The beam elongation is characterized by the parameter R, the ratio of the semi-major axis (Y) to the semi-minor axis (Y/R). The beam elongation is along the direction of the local longitude; Ω is the projected direction of the rotation axis. The line of sight goes East-West with respect to the pulsar and is parallel to the minor axis of the beam. For a given R, the polarisation swing 2θ is a monotonic function of the relative offset $|y/Y|$

As the pulsar rotates, the line of sight to Earth traces an East–West line on the pulsar beam i.e., a line of constant latitude (Fig. 2). If y is the offset between the line of sight and the magnetic pole, then the total polarisation swing 2θ for an elliptic beam in the RC model can be obtained from

$$\cos\theta = R|y/Y|/[1+(y/Y)^2(R^2-1)]^{1/2}. \quad (2)$$

We are here neglecting spherical effects which are treated in detail in Sect. V. Let us assume that there is equal probability of observing pulsars anywhere within the range $0 \leqq |y/Y| \leqq 1$ (we discuss the error from this approximation in the next section). Let us divide the pulsars into two groups: group A pulsars with $0 \leqq |y/Y| < 0.5$ and group B with $0.5 \leqq |y/Y| < 1.0$. From Eq. (2) the mean value of $\cos\theta$ in these two group is

$$\langle\cos\theta\rangle_A = [(3+R^2)^{1/2}-2]/(R-1/R), \quad (3)$$

$$\langle\cos\theta\rangle_B = [2R-(3+R^2)^{1/2}]/(R-1/R). \quad (4)$$

The mean value of $\cos^2\theta$ is similarly

$$\langle\cos^2\theta\rangle_A = \frac{1}{Q} - \frac{2}{RQ^{3/2}}\tan^{-1}(RQ^{1/2}/2), \quad (5)$$

$$\langle\cos^2\theta\rangle_B = \frac{1}{Q} - \frac{2}{RQ^{3/2}}[\tan^{-1}(RQ^{1/2})-\tan^{-1}(RQ^{1/2}/2)], \quad (6)$$

$$Q = 1-1/R^2. \quad (7)$$

The variance $\sigma^2_{A,B}$ on the distribution of $\cos\theta$ in groups A and B is given by

$$\sigma^2_{A,B} = [\langle\cos^2\theta\rangle_{A,B} - \langle\cos\theta\rangle^2_{A,B}]. \quad (8)$$

To estimate R from the available data we order the pulsars in decreasing magnitude of the polarisation angle swing[2] 2θ and divide them into two equal groups – high swing and low swing. We can identify the high swing pulsars with group A and the low swing pulsars with group B. The misfit factor

$$S = n(\Delta\langle\cos\theta\rangle_A/\sigma_A)^2 + n(\Delta\langle\cos\theta\rangle_B/\sigma_B)^2, \quad (9)$$

where $\Delta\langle\cos\theta\rangle$ is the difference between the expected values of $\cos\theta$ and the values computed from the observations and n is the number of pulsars in group A (or B), is clearly a function only of the assumed R. We estimate R by locating the minimum value of S and determine the 1σ bounds by identifying the points at which $S = S_{\min}+1$.

Since all the results are based on the specific elliptic shape assumed for the beam, this assumption can be checked with the corresponding (10%) pulse widths W_{10}. For an ellipse (of any R), the mean widths in groups A and B should satisfy

$$\langle W_{10}\rangle_A/\langle W_{10}\rangle_B = \frac{0.9566(2Y/R)}{0.6144(2Y/R)} = 1.56. \quad (10)$$

In comparison, a rectangular beam would have $\langle W\rangle_A/\langle W\rangle_B = 1.0$ while a "diamond"-shaped beam would have a ratio of 2.0, these results being again independent of R.

III. Estimate of beam elongation

The estimation of 2θ from polarisation observations is generally complicated by the presence of orthogonal radiation modes (Manchester et al., 1975; Backer et al., 1976) and the attendant discontinuous flipping of the mean polarisation angle. However, Backer and Rankin (1980) have shown that, when good data are organised in the form of histograms of the polarisation angle at various longitudes across the pulse, it is quite easy to follow the angle variation of a single mode.

We have estimated 2θ for 16 pulsars (at 430 MHz) from the histograms given by Backer and Rankin (1980). We eliminated two pulsars from their work – PSR 1919+21 because they find an unusual polarisation angle variation which is not easily interpreted, and PSR 1541+09 because the polarisation is very weak. For each pulsar we obtained the mean polarisation angle of one of the orthogonal modes as a function of longitude. (In those cases where the other mode is also strong we combined the data on both modes with a suitable constant angle offset between them.) We fitted the observed angle variations to the RC model (Manchester and Taylor, 1977; also see Narayan and Vivekanand, 1982b for details of the fitting procedure) and obtained the polarisation angle swings 2θ listed in Table 1. We note that the accuracy would have been quite adequate even if we had estimated 2θ directly by eye from the published data. For PSR 1237+25 we have taken 2θ to be 175° (instead of a small angle as suggested by the data of Backer and Rankin) because several other studies (e.g., Bartel et al., 1982) show that the line of sight to Earth passes very close to the magnetic pole.

The pulsars in Table 1 have been listed in the order of decreasing 2θ and have been classified into two groups, A and B, of eight pulsars each as discussed in Sect. II. Table 2 shows the mean value of $\cos\theta$ for groups A and B from which we deduce that $R = 3.0 \pm 0.4$. This is an extremely large and unexpected elongation. We note that circular beams with $R=1$ are quite clearly ruled out (as was already evident from the arguments in Sect. I).

Before considering the meaning and consequences of the large estimate of R, it is necessary to discuss possible sources of error and any selection effects which could invalidate our results. Our analysis assumes a uniform distribution of $|y/Y|$ in the range 0 to 1. Since the pulsar luminosity may be expected to vary as a

2 2θ is measured between the two points where the pulse intensity drops to 10% of its peak value; these points define the pulse width W_{10}

Table 2. Summarized results for pulsars of groups A and B in Table 1

Group	Observed ⟨cosθ⟩	Expected ⟨cosθ⟩		⟨W_{10}⟩	Computed ⟨|y/Y|⟩		Expected ⟨|y/Y|⟩	Observed ⟨cotθ⟩	Expected ⟨cotθ⟩
		$R=3.0$	$R=1.0$		$R=3.0$	$R=1.0$			$R=4.8$
A	0.567	0.549±0.089	0.250±0.051	14.8±3.8	0.28	0.57	0.25±0.05	0.911	1.20±0.24
B	0.949	0.951±0.013	0.750±0.051	19.6±7.9	0.74	0.95	0.75±0.05	3.66	3.60±0.24

Table 3. Data for 30 pulsars, grouped into three period ranges of 10 pulsars each

Group A						Group B					
PSR	P(s)	2θ	W_{10}	$\|d\theta/d\varphi\|^{-1}_{max}$	Type	PSR	P(s)	2θ	W_{10}	$\|d\theta/d\varphi\|^{-1}_{max}$	Type
a) $P<0.388$s											
2020+28	0.3434	97.2	18.2	0.103	*C*	1929+10	0.2265	32.7	22.2	0.67	*S*
0833−45	0.0892	85	19.0	0.17	*S*	1933+16	0.3587	25.7	12.1	0.48	*S*
1556−44	0.2570	65	19.6	0.11	*S*	0540+23	0.2460	21.0	21.8	1.0	–
0950+08	0.2531	51.0	30.6	0.50	*S*	0531+21	0.0331	~20	50	~1.0	*S*?
0611+22	0.3349	40.4	14.2	0.33	–	1642−03	0.3876	20	6.6	0.11	*S*
b) $0.388\text{s}<P<1.2\text{s}$											
0329+54	0.7145	180	16.1	0.036	*S*?	2016+28	0.5580	58	13.8	0.20	*S*
1508+55	0.7396	180	12.1	0.053	*S*?	2021+51	0.5291	50	21.8	0.29	*S*
1133+16	1.1879	89.6	12.1	0.103	*C*	1154−62	0.4005	40	27.0	0.50	*S*
0823+26	0.5307	79.1	10.6	0.071	–	1604−00	0.4218	27.0	16.9	0.59	–
1240−64	0.3884	60	19.5	0.24	*S*	1944+17	0.4406	21.6	31.4	1.4	–
c) $1.2\text{s}<P$											
1237+25	1.3824	175	15.2	0.017	*C*	0628−28	1.2444	90	37.6	0.22	*S*
0525+21	3.7455	152	20.8	0.033	*C*	2319+60	2.2564	75	24.7	0.100	*C*
2045−16	1.9615	145	17.1	0.027	*C*	0834+06	1.2738	61.4	9.4	0.077	*C*
0301+19	1.3876	136	18.3	0.056	*C*	2303+30	1.5759	56.2	7.9	0.13	–
1700−32	1.2117	135	19.0	0.025	*S*	1919+21	1.3373	25	11.3	0.083	*C*

function of $|y/Y|$ this cannot be strictly true. Narayan and Vivekanand (1982b) in a study on PSR 0950+08 found that the pulse intensity for an outer cut of the beam is significantly less than for an intermediate cut. In another study (Narayan and Vivekanand, 1982a) they analysed the interpulses of a number of pulsars and concluded that there is evidence for a monotonic fall off of radio flux with the latitude offset $|y/Y|$. We make yet another independent study in Sect. V which again suggests a falling luminosity (possibly in a gaussian fashion). A fall-off (rather than an increase) in intensity towards the beam edge is also intuitively appealing. When this selection effect is present, smaller values of $|y/Y|$ will be over-represented, which tends to increase 2θ and decrease the estimated values of $\langle\cos\theta\rangle_{A,B}$ in the sample. Therefore our estimated value of 3.0 must be lower than the true value of R.

It is possible that the pulse strength does not fall monotonically with increasing $|y/Y|$ but peaks at some intermediate value (the hollow cone model of the pulsar beam might suggest this). To first order this is not expected to affect the estimate of R. Also the data show no evidence for this effect. Firstly, the probability of detecting a pulsar should increase with increasing $|y/Y|$ for group A and decrease with increasing $|y/Y|$ for group B. Consequently, at the optimum value of R (=3.0), the observed value of $\langle\cos\theta\rangle_A$ should be larger than the expected value while for group B the trend should be the other way. Table 2 shows that there is practically no evidence for this. Secondly, the values of $|y/Y|$ of the 16 pulsars, computed using the optimum value of R (Table 1), should peak around $y/Y\sim0.5$. Again there is no evidence for this. The mean values of $|y/Y|$ in groups A and B shown in Table 2 are entirely consistent with the values 0.25 and 0.75 expected for a uniform distribution.

Another point concerns the pulse width. The 2θ values in Table 1 correspond to the widths W_{10} (10% of peak intensity) which are almost equal to the full pulse widths. One could instead use other measures of width such as W_{50}, the width corresponding to 50% of the intensity peak, or W_{eq}, the equivalent width. Both these are smaller than W_{10} and hence would lead to smaller values of 2θ. Therefore, using these measures of width would only increase the deduced elongation R beyond our estimate of 3.0.

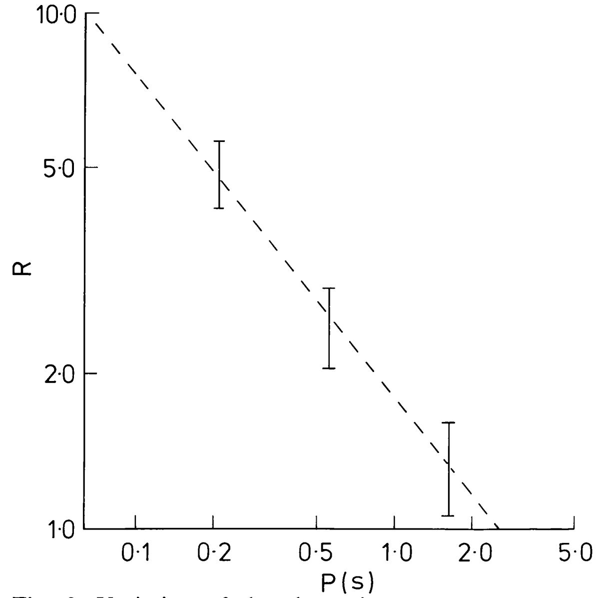

Fig. 3. Variation of the elongation parameter R with pulsar period. The estimated R, along with the 1σ error limits, are plotted against the geometric-mean periods of the pulsars in the three period ranges discussed in Sect. IV. The dashed line is the visually estimated best fit and corresponds to $R=1.78\,P^{-0.63}$

Finally we consider the error from our assumption of an elliptic shape for the beam. From Table 2 we see that $\langle W_{10}\rangle_A/\langle W_{10}\rangle_B=0.76\pm0.36$. The expanded data in Sect. IV also suggest a similar value. This is not consistent with the value of 1.56 expected for an elliptic beam (Sect. II), but agrees with the value of 1.0 for a rectangular beam. Now for a rectangular beam of axial ratio R (=North–South dimension/East–West dimension), $\cot\theta$ is uniformly distributed between 0 and R and hence

$$\langle\cot\theta\rangle_A=0.25R\pm0.5R/(12n)^{1/2}, \quad (11)$$

$$\langle\cot\theta\rangle_B=0.75R\pm0.5R/(12n)^{1/2}. \quad (12)$$

From the observed values of $\langle\cot\theta\rangle_{A,B}$ in Table 2 we conclude that, if the beam has a rectangular rather than an elliptic shape, then $R=4.8^{+1.1}_{-0.7}$, which is greater than 3.0. The beam may even be "butterfly" shaped (as in Fig. 6) in which case R would be still greater.

One other possible source of error in our analysis is the neglect of spherical effects. This is considered in some detail in Sect. V where we show that none of our conclusions is affected.

From the detailed discussion of various effects given above it seems that our estimate of 3.0 for the beam elongation is really a lower bound. The true value must be larger and could even be significantly larger. Jones (1980) analysed pulsar polarisation data by a technique similar to our arguments in Sect. V and concluded that $R=2.5^{+1.3}_{-0.7}$. Since he used half-maximum intensity widths W_{50}, which are $\sim0.6W_{10}$ (for the pulsars in Table 1), the corresponding R from our calculations is $R=5.0\pm0.7$. Jones' value would appear to be an underestimate.

Before closing this section a note of caution is necessary concerning the statistical reliability of our results. We have used sixteen observational numbers to estimate R. It would be reassuring if our conclusions could be confirmed by a larger sample of pulsars. Meanwhile, considering the fairly tight 1σ limits which we obtain, we believe the results of this paper can be accepted with reasonable confidence.

IV. Evolution of beam elongation

Kundt (1982) made the interesting suggestion that the elongation parameter R could evolve during the life of a pulsar. To investigate this possibility we use an expanded sample of pulsars. In addition to the 16 pulsars of Table 1 we now add another 13 pulsars for which Manchester and Taylor (1977, Tables 2 and 3) quote reliable values of 2θ (at ~400 MHz). We also include PSR 0329+54 for which the multifrequency observations of Bartel et al. (1982) clearly indicate $2\theta=180°$.

Tables 3a, b, c give the data on the above 30 pulsars grouped into three ranges of P, the 10 pulsars in each range being divided into two groups as before. The values of W_{10} for the new pulsars are from Manchester and Taylor (1981) except for PSR 0531+21 and PSR 1508+55 whose widths were estimated directly from the original observations of Manchester (1971). The values of 2θ for the 16 Backer-Rankin pulsars and PSR 0329+54 correspond to W_{10}. The 2θ values for the other 13 pulsars are the estimates of Manchester and Taylor (1977); it is not clear to what width they correspond. Since the Manchester-Taylor estimates of 2θ for most of the Backer-Rankin pulsars agree very well with ours, we presume they have considered either W_{10} itself or something close to it. Barring 2 pulsars (viz., PSR 0531+21 and PSR 1919+21) for which 2θ is a little uncertain, we believe the data we use here are quite reliable, though not as uniform as the sample in Table 1.

Using the method discussed in Sect. II we obtain the following estimates of R in the three period ranges

$$R=4.9\pm0.7,(P<0.388\,\text{s});\quad 2.5\pm0.5,(0.388\,\text{s}<P<1.2\,\text{s});\quad 1.3\pm0.3,(1.2\,\text{s}<P). \quad (13)$$

As Fig. 3 shows, there is a large and statistically significant variation of R with P. By an eyeball fit we estimate that R varies approximately as

$$R\sim1.8P^{-0.65}. \quad (14)$$

The coefficient could be larger than 1.8 because of the various selection effects and other approximations discussed in Sect. III. However, the variation with P seems to be real and unlikely to have arisen from statistical fluctuations. Once again we caution that each P range contains only 10 pulsars (which is, however, reasonably adequate since we estimate only one number viz., R).

A few of the values of 2θ in Table 3 are suspect. For PSR 0531+21, the optical data of Kristian et al. (1971) suggest $2\theta\sim90°$. Also, Backer and Rankin (1980) have proposed models for PSR 0823+26 and PSR 1919+21 with $2\theta=61°$ and $87°$, respectively, for the dominant mode. If we use these values of 2θ instead of the numbers listed in Table 3, we estimate R to be $4.1+0.6$, $2.5+0.5$, and $1.1+0.2$, respectively, in the three period ranges. The changes are small and the period variation continues to be prominent.

Since the evolution of beam elongation is an important result, we investigate whether R varies with any of the other pulsar parameters e.g., the period derivative $\dot P$, the characteristic age $\tau=P/2\dot P$ or the magnetic field $B_{12}\sim(10^{15}P\dot P)^{1/2}$. We find

$$R=3.1\pm0.6(10^{15}\dot P<1.5);\quad 2.1\pm0.4(1.5<10^{15}\dot P<6.0);\quad 2.9\pm0.5(6.0<10^{15}\dot P)$$

$$R=3.5\pm0.6(10^{-6}\tau<2.75\,\text{yr});\quad 2.2\pm0.3(2.75\,\text{yr}<10^{-6}\tau<5.25\,\text{yr});\quad 2.3\pm0.6(5.25\,\text{yr}<10^{-6}\tau) \quad (15)$$

$$R=3.6\pm0.5(B_{12}<1.0);\quad 2.3\pm0.8(1.0<B_{12}<2.0);\quad 2.1\pm0.3(2.0<B_{12}).$$

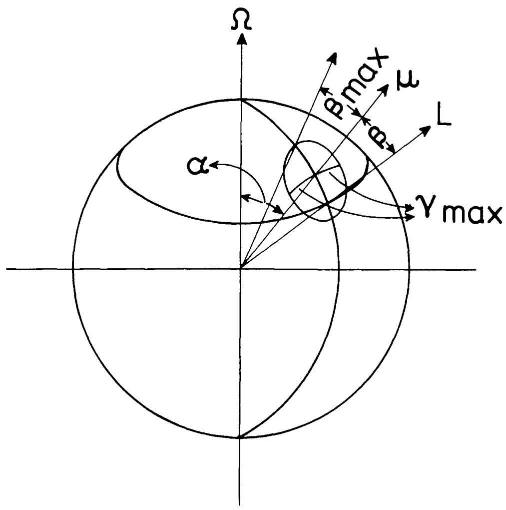

Fig. 4. Illustration of the spherical geometry of the pulsar beam. The spherical neutron star spins around the rotation axis Ω. The magnetic dipole moment μ makes an angle α with respect to Ω. The pulsar beam is a cone centred on the magnetic pole, having an elliptic cross-section, with angular axial dimensions $2\beta_{max} \times 2\gamma_{max}$. The line of sight L at a latitude offset β from μ traces a line of constant latitude $\alpha+\beta$

There is virtually no variation of R with $\dot{P}$ while the weak variation with τ and B_{12} is probably due to the variation with P. We thus conclude that the beam elongation is primarily a function of the pulsar period and has essentially no dependence on other pulsar parameters.

V. Spherical effects

In the analysis so far we have assumed a planar geometry. In reality, the beam is the cross-section of a cone attached to a rotating spherical neutron star. We now make an analysis including the spherical effects. As might be anticipated, the earlier results continue to hold. However, since the analysis here is different and also makes use of a different set of observational data viz., $|d\theta/d\varphi|_{max}$, it confirms that our picture is internally consistent.

Let α be the angle between the magnetic and rotation axes and β the latitude offset between the line of sight and the magnetic axis (Fig. 4). Let β_{max} be the maximum offset at which the pulsar is visible; hence the North–South dimension of the beam is $2\beta_{max}$. Let the maximum angular East–West dimension of the beam be γ_{max} (which corresponds to $2Y/R$ in Fig. 2). Assuming an elliptic shape for the beam, the East–West dimension $\gamma(\beta)$ at any offset β is given by

$$[\gamma(\beta)/\gamma_{max}]^2+[\beta/\beta_{max}]^2=1\,, \qquad |\beta|\leqq\beta_{max}\,. \tag{16}$$

The pulse width $W_{10}(\beta)$ measured in degrees of longitude is given by

$$W_{10}(\beta)=\gamma(\beta)/\sin(\alpha+\beta)\,.$$

Using the spherical weighting factor $\frac{1}{2}\sin(\alpha+\beta)d\beta$ for the probability of occurrence of a given offset between β and $\beta+d\beta$ we calculate the mean pulse width $\langle W_{10}\rangle$ to be

$$\langle W_{10}\rangle=\frac{1}{2\mathscr{P}}\int_{-\beta_{max}}^{\beta_{max}}[\gamma(\beta)/\sin(\alpha+\beta)]\sin(\alpha+\beta)d\beta$$
$$=\pi\gamma_{max}\beta_{max}/(4\sin\alpha\sin\beta_{max})\,, \tag{17}$$

where, taking $\beta_{max}\leqq\alpha$, we have

$$\mathscr{P}=\frac{1}{2}\int_{-\beta_{max}}^{\beta_{max}}\sin(\alpha+\beta)d\beta=\sin\alpha\sin\beta_{max}\,. \tag{18}$$

The mean value of $|\sin\beta|$ over all lines of sight is similarly

$$\langle|\sin\beta|\rangle=\frac{1}{2\mathscr{P}}\int_{-\beta_{max}}^{\beta_{max}}|\sin\beta|\sin(\alpha+\beta)d\beta=\frac{1}{2}\sin\beta_{max}\,. \tag{19}$$

In the above results we have not considered variation of luminosity with $|\beta|$.

Now, the rate of change of polarisation angle with pulse longitude $|d\theta/d\varphi|$ reaches its maximum value when the line of sight is closest to the magnetic pole. At this point we have [Manchester and Taylor (1977), Eq. (10-25)].

$$|d\theta/d\varphi|_{max}=|\sin\alpha/\sin\beta|\,. \tag{20}$$

Combining with Eq. (19) we see that

$$\sin\beta_{max}=2\sin\alpha_{eff}\langle|d\theta/d\varphi|_{max}^{-1}\rangle\,, \tag{21}$$

where we have written the relation for an effective α_{eff}.

Table 3a–c list $|d\theta/d\varphi|_{max}^{-1}$ for all the 30 pulsars. For the Backer-Rankin pulsars we have used our least squares fits of the polarisation angle variation, while for the others (including PSR 0329+54 and PSR 1237+25) we give the values of Manchester and Taylor (1977). From Eq. (21), which makes no assumption on the beam shape, and assuming α_{eff} to be 60°, we obtain

$$2\beta_{max}=101^{\circ}{}^{+49}_{-31}(P<0.388\,\mathrm{s})\,; \quad 74^{\circ}{}^{+37}_{-30}(0.388\,\mathrm{s}<P<1.2\,\mathrm{s})\,;$$
$$15^{\circ}{}^{+4}_{-4}(1.2\,\mathrm{s}<P)\,. \tag{22}$$

Although the analysis made here is not as sensitive as the methods of the earlier sections, we note that the rapid evolution of the North–South beam dimension with P which we found in Sect. IV persists. Moreover, by the structure of Eq. (21) it is clear that the actual value of α_{eff} that we assume is unimportant as far as the variation of β_{max} is concerned. Also, the above analysis does not make any assumption on the beam shape.

Using Eq. (17), which is valid for an elliptic beam, we can estimate γ_{max} from W_{10}. The beam elongation R is then given by

$$R=2\beta_{max}/\gamma_{max}\,. \tag{23}$$

Figure 5 shows the variation of (deduced) R with α_{eff} in the three period ranges. Since R is seen to be insensitive to α_{eff}, the evolution of beam elongation with period cannot be explained by invoking a period evolution of α_{eff}.

Finally, we consider the variation of luminosity with $|\beta|$. Let us divide the pulsars in each of Table 3a–c into two groups – Group A' containing the 5 pulsars with the lowest values of $|d\theta/d\varphi|_{max}^{-1}$ and group B' having the pulsars with high $|d\theta/d\varphi|_{max}^{-1}$ (groups A' and B' are different from the earlier groups A and B). A comparison of Eqs. (18) and (21) shows that for a given α_{eff}, if there is no luminosity selection effect, the mean value of $|d\theta/d\varphi|_{max}^{-1}$ over the range 0 to any β is directly proportional to the corresponding probability $\mathscr{P}$. Thus $|d\theta/d\varphi|_{max}^{-1}$ is uniformly distributed between 0 and $(\sin\beta_{max})/(2\sin\alpha_{eff})$. Therefore

$$K=\frac{\langle|d\theta/d\varphi|_{max}^{-1}\rangle_{B'}}{\langle|d\theta/d\varphi|_{max}^{-1}\rangle_{A'}}=\frac{0.75(\sin\beta_{max}/2\sin\alpha_{eff})}{0.25(\sin\beta_{max}/2\sin\alpha_{eff})}=3\,. \tag{25}$$

From the data in Table 3a–c we estimate K to be 4.4 ± 1.4, 6.6 ± 3, and 4.0 ± 1.2 in the three period ranges, leading to a weighted mean value

$$K=4.4\pm0.9\,. \tag{26}$$

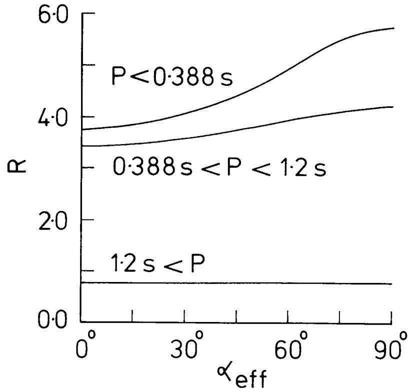

Fig. 5. Variation of the estimated beam elongation R with the mean angle α_{eff} between the rotation and magnetic axes. It is reasonable to expect that $30° < \alpha_{eff} < 60°$. Note that R is fairly insensitive to α_{eff} in all the three period ranges. The actual values of R shown here are different from (but statistically consistent with) those in Fig. 3 because the method of analysis as well as the data used are not the same

A value of $K>3$ implies that there is a crowding of lines of sight close to the magnetic pole and a spreading out farther away. Thus we find that the beam luminosity in pulsars falls with increasing $|\beta|$. This agrees with other independent studies (Narayan and Vivekanand, 1982a, b). The value of 4.4 for K is consistent with the value of 3.9 expected for a gaussian variation, though other forms are also possible (for instance, $K=5.5$ for an exponential variation and 3.8 for a triangular fall-off). However, the analysis made here virtually rules out any question of the luminosity *increasing* outwards from the magnetic pole, which, as discussed in Sect. I, is the only way to reconcile the data with a circular beam. Thus, by the arguments in Sect. III, all our estimates of the beam elongation R are underestimates of the true value.

VI. Discussion

In the present study we have shown that the large discrepancy, between the observed distribution of the polarisation angle swing 2θ (and the angle gradient $|d\theta/d\varphi|_{max}$) and the currently accepted version of the RC model coupled with circular beams, can be reconciled if we assume that pulsar beams are highly elongated. Taking the beam shape to be elliptical we estimate (Sect. III) the mean elongation (=North-South dimension/East-West dimension) to be 3.0 ± 0.4. It is significant that we have been unable to identify a single plausible selection effect which could suggest that the true $R<3.0$.

In Sect. IV we have shown that R evolves with the pulsar period P, as was first suggested by Kundt (1982). We estimate that R may be as large as 5 or more at short periods and seems to decrease systematically to essentially $R \sim 1$ (circular beam) at long periods. The straight line fit in Fig. 3 gives $R \propto P^{-0.65}$. In an investigation of the effect of the spherical geometry of the problem (Sect. V), we find that neither the value of R nor its evolution with P is significantly affected by variations in α_{eff}, the unknown angle between the rotation and magnetic axes. Another result we obtain is that R varies primarily with P and seems to be independent of $\dot{P}$, τ or B.

For the 16 pulsars in Table 1 we obtain $\langle W_{10} \rangle_A / \langle W_{10} \rangle_B = 0.76 \pm 0.36$ while for the three groups of pulsars in Table 3a–c the mean ratio is $0.84 \pm \sim 0.2$. This suggests that the pulsar beam may be somewhat rectangular or may even flare out away from the centre.

In Sect. V we find that there is a crowding of lines of sight near the magnetic pole and a thinning out farther away. This suggests that the luminosity falls with increasing $|\beta|$, in agreement with earlier independent studies (Narayan and Vivekanand, 1982a, b).

Pulsar beams are believed to be hollow cones (Komesaroff, 1970) because of the frequent occurrence of double and multiple component pulse profiles. Table 3a–c give the pulse types of many of the pulsars as identified by Manchester and Taylor (1977, Tables 2 and 3), using the morphological classification scheme of Taylor and Huguenin (1971). The single component profiles (Type S) are commonly considered to be outer cuts of the beam while the doubles and multiples (Type C, for complex) are believed to be inner cuts intersecting the hollow part. The maximum values of $|d\theta|d\varphi|_{max}^{-1}$ among the C type profiles in Table 3a–c are 0.103, 0.103, and 0.100, respectively. These values are remarkably equal suggesting that the North-South dimension of the central hollow region does not evolve with P. Taking α_{eff} to be 60°, Eq. (20) shows that the inner North-South diameter of the inner hollow cone is $\sim 10°$. Further, from Manchester and Taylor (1977, Table 22) we find the mean component separation in 22 C type pulsars to be 10°.8. Again taking α_{eff} to be 60° and allowing for averaging over an elliptic beam (requiring factors of sin 60° and $\pi/2$) we estimate the East-West peak to peak distance to be $\sim 15°$; the corresponding diameter of the hollow region should hence be $\sim 10°$. We thus reach the interesting conclusion that the central hollow region of the pulsar beam is essentially circular and apparently does not evolve with pulsar period. This result is independent of α_{eff} since the estimates of both the North-South and East-West dimensions scale essentially as $\sin \alpha_{eff}$.

Figure 6 summarizes the picture of pulsar beams that emerges from our results. Note that at short periods the beam is extremely large, extending $\sim 100°$ in latitude.

One of the hitherto unsolved problems in pulsars is the occurrence of interpulses in only short period pulsars. For instance, among the 30 pulsars in Table 3a–c, interpulses are observed in 3 pulsars viz., PSR 0950+08, PSR 1929+10, and PSR 0531+21, in the short period group, in 2 pulsars viz., PSR 0823+26 and PSR 1944+17 (Hankins, 1980), in the medium P group (but both have fairly short periods, <0.6 s), and in none of the long period pulsars. The evolution of beam elongation could be a natural explanation for this observation. Figure 7 shows our estimate of the fractional rate of occurrence of an interpulse, f', as a function of P. We have assumed that R has the variation shown in Eq. (14), that α is uniformly distributed on a sphere and that interpulses can occur either from the opposite pole to the main pulse or the same pole [as in our model of PSR 0950+08, Narayan and Vivekanand (1982b)]. From Fig. 7 we estimate that 3.1 ± 1.2 out of the 10 pulsars in Table 3a should have interpulses, the corresponding numbers for Table 3b and c being 1.3 ± 1.1 and 0.6 ± 0.8, respectively. While the good quantitative agreement with the actual numbers observed may be fortuitous, we believe our explanation is qualitatively correct.

Another interesting observation in pulsars is that Type C profiles occur predominantly in long period pulsars. Vivekanand and Radhakrishnan (1980) suggested that pulse intensity could be modulated across the polar cap due to evolving irregularities in either the surface relief or the local magnetic field. Our results here give an alternative explanation. We find that the outer North-South dimension of the beam has a strongly variable elongation while the dimension of the inner hollow region is essentially independent of P. Thus, only a small fraction of short P pulsars (typically 1 or 2 out of 10) will intersect the central hollow region to produce a complex profile while we typically expect about 3

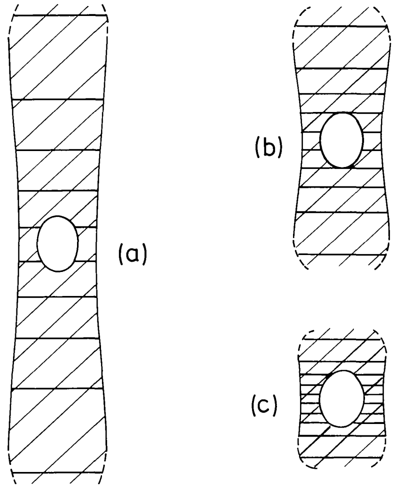

Fig. 6a–c. Mean pulsar beams, drawn to scale, corresponding to the period ranges **a** $P<0.388$ s, **b** $0.388\,\mathrm{s}<P<1.2$ s, and **c** $1.2\,\mathrm{s}<P$. Radio radiation is received from the hatched regions. The horizontal lines are idealized lines of sight spaced at 10% probability intervals and have been calculated assuming a gaussian variation of probability. The outer extremities of the beams have been represented by dashed lines since the beams may not have a firm boundary but probably fade away gradually. The dumb-bell beam shape is suggested by the statistics of pulse width data. While the beam elongation reduces dramatically with increasing period, the hollow region in the centre seems to be essentially period-independent. If complex pulse profiles are identified with lines of sight which intersect the central hollow, then only 1 or 2 pulsars out of 10 in **a** are expected to have such profiles while in **b** and **c** we expect ~ 3 and ~ 6 out of 10, respectively

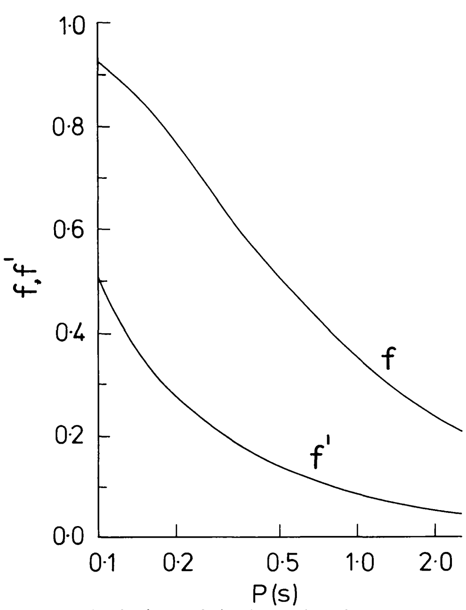

Fig. 7. Variation of the beaming fraction f and the fractional rate of interpulse occurrence f' with pulsar period, corresponding to the variation of R shown in Fig. 3. f is evaluated as $\langle\Omega_{\mathrm{PSR}}\rangle/4\pi$, where $\langle\Omega_{\mathrm{PSR}}\rangle$ is the total solid angle over which the rotating pulsar is visible, suitably averaged over all values of α. f' is similarly $\langle\Omega_I\rangle/\langle\Omega_{\mathrm{PSR}}\rangle$, where $\langle\Omega_I\rangle$ is the solid angle over which interpulses are visible (including both single and double pole interpulses)

Type C pulsars out of 10 in the medium P pulsars and more than half the long period pulsars should be Type C. These numbers are generally in agreement with the actual observations (Table 3, where we take pulsars labelled S? to be Type C).

Our results might also be compatible with the occurrence of drifting subpulses on only long P pulsars. If this phenomenon is caused by the circular motion of the emission zone around the magnetic pole (Ruderman and Sutherland, 1975), then it is reasonable to believe that it could occur in a nearly circular beam like Fig. 6c but it would be rather difficult to envisage in elongated beams as in Fig. 6a or b.

An important parameter in statistical studies of pulsars is the fraction of pulsars which beam towards Earth. Assuming circular beams and using the mean pulse width of $\sim 15°$–$20°$ it has been estimated that $f\sim 0.2$. Although this value has been widely used in numerous studies, it has been recognized that it is extremely uncertain. The results of this paper now give us a better idea of the North-South dimension of the beam. Figure 7 shows the variation of f with P implied by our results, assuming that α is uniformly distributed and that R varies according to Eq. (14). Note that $f\sim 1$ for short period pulsars. We recall that Kundt (1977, 1981) has for long been suggesting that pulsar beams may be elongated ("banana"-like) and that f may therefore be much higher than the commonly accepted value of 0.2.

The variation of f with P could be important for statistical studies of pulsar evolution. An important requirement for these studies is that one should identify a range of P (and $\dot{P}$ since one normally employs the $P-\dot{P}$ diagram) where there is no birth or death of pulsars. Our results show that pulsars are dying at all values of P due to beam shrinkage. Until this effect can be properly quantified [Eq. (14) should be considered only as an initial estimate and has to be carefully refined with more data], all results on pulsar evolution, such as the estimate of the braking index, the decay time of the magnetic field, etc., obtained from statistical studies of pulsar data must be treated with caution. Current estimates of the number of pulsars in the Galaxy and the birthrate may also be overestimates since $\langle f\rangle$ is greater than 0.2.

A point to be noted is that beam shrinkage is a new mechanism for pulsar death which has not been considered so far. Some of the scenarios considered usually for pulsar death include decay of the magnetic dipole moment (Flowers and Ruderman, 1977) and alignment of the magnetic dipole moment with the spin axis (Jones, 1976). Lyne et al. (1975) have pointed out that a line of the form $\dot{P}P^{-5}=\mathrm{const}$ ($\sim 5\,10^{-17}$) seems to describe an empirical cut-off for pulsar activity and explained it as representing a constant field strength at the light cylinder. Ruderman and Sutherland (1975) considered the variation of the accelerating potential acting on the charged particles in their model and predicted a line of the form $\dot{P}^2P^{-5}=\mathrm{const}$. We offer yet another explanation here. The number of pulsars within observable range of a survey is proportional to the mean pulsar luminosity (assuming a two-dimensional spatial distribution) which varies as $P^{-0.86}\dot{P}^{0.38}$ (Vivekanand and Narayan, 1981). Combined with the variation of beaming fraction $f\propto R\propto P^{-0.65}$, this gives an overall variation of the form $P^{-1.51}\dot{P}^{0.38}$. This suggests that pulsar death, or rather thinning out, should be described by lines of constant $\dot{P}P^{-4}$. Of course, one does not expect the apparently abrupt pulsar cut-off that is observed. Also, the variation $R\sim 1.8\,P^{-0.65}$ probably does not continue beyond $P\sim 2.5$ s where $R\sim 1$.

Vivekanand and Narayan (1981) pointed out that there appears to be a large deficit of pulsars in the Galaxy with $P<0.5$ s. This phenomenon, which they call "injection" (since pulsars are apparently injected at large P), is a puzzle since neutron stars are expected to be born spinning rapidly with $P \sim$ ms (Manchester and Taylor, 1977). Vivekanand et al. (1982) in a later study concluded that there does not appear to be any selection effect in the pulsar surveys which could explain injection. Our present results make the problem still more severe. From the data on the 30 pulsars in Table 3a–c we estimate that $\langle f \rangle$ for the pulsars with $P < 0.5$ s is approximately 2.5 times greater than that for $P > 0.5$ s. We thus find that the number of pulsars in the Galaxy with $P < 0.5$ s is at least an order of magnitude less than expected. It is possible that injection could be explained by a large variation of $\alpha_{\rm eff}$. However, it is difficult to believe that the evolution of $\alpha_{\rm eff}$ could be as large as the required amount.

To our knowledge none of the current pulsar theories predicts the high elongation of pulsar beams, its evolution with P or the apparent lack of variation with other pulsar parameters. A reanalysis of the theories keeping these clues in mind appears worthwhile. Meanwhile, our results can be refined and possible new effects investigated if more data become available. Polarisation observations are in fact available on many more than the 30 pulsars that we have considered here. Unfortunately, the published data are not directly useful because of unquantifiable polarisation angle flips caused by the orthogonal radiation modes. In view of the interesting results that we have obtained, new high quality observations [preferably analysed by the histogram approach of Backer and Rankin (1980)] or a careful re-examination of the old data may be worthwhile.

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Note added in proof: The strong interpulse in the recently discovered millisecond pulsar (Backer et al., *Nature* **300**, 615, 1982) seems to confirm that pulsar beams are highly elongated at short periods (see the discussion by Narayan and Radhakrishnan, *Curr. Sci.* **52**, 46, 1983).

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