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Continuously movable telescopes for optical interferometry

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Summary. This article discusses some consequences of continuously movable telescopes for optical astronomical image formation by aperture synthesis techniques. The aim being to bring the beams from all telescopes in an array to the same detector, with zero relative delay, with no need for variable optical delay lines. We give a simple derivation of the telescope motions in two cases. In the first, the telescopes move so as to lie on an ellipse which is continuously deforming with time as the source is tracked (Labeyrie, 1978). In the second case they lie on a straight line perpendicular to the line of sight to the star and fixed length optical fibres carry the light to the detector. In both cases the motions are relatively simple and give reasonable coverage of the (U, V) plane; in fact the coverage can be enhanced well beyond that obtained by earth rotation by additional motion of the telescopes ('hypersynthesis'). We discuss the physical meaning of the resulting formulae and their consequences for the interferometer design.

Key words: interferometry – optical – infrared – movable telescopes

1. Introduction

There is a great current interest in interferometeers for optical astronomy (here we use 'optical' to mean visible and infrared wavelengths). In contrast to high resolution radio interferometry, it is not always possible to use heterodyne receivers in the optical range, implying that some signal processing must be done at the observing wavelength. In particular, relative delays between signals from two telescopes of an interferometer must be compensated at optical wavelengths, and this delay, or path length compensation, must be nearly continuously variable to correct for the Earth's rotation. As fabrication of optical delay lines is difficult and expensive (their cost may equal that of a small telescope), it is worth thinking of other methods of path compensation. Two other possibilities are to continuously move special additional mirrors, or to move the telescopes themselves to compensate for Earth's rotation. This article considers the latter method, moving the telescopes themselves.

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Path compensation requires smooth movements of the telescopes, which imply simple motions. For example, the telescope acceleration may have a limit beyond which pointing becomes unreliable. Good servos are required to keep the telescopes pointed while they are moving laterally. A lot of engineering effort is necessary to determine the feasibility of the scheme (Labeyrie et al, 1986, have investigated related problems), and to study the merits of moving telescopes rather than building delay lines. We restrict this article to the observational consequences of such telescope motions. Our main interest is the (U, V)-plane coverage of the interferometer. We derive some simple formulae and discuss their physical meaning and consequences for interferometer design.

2. Compensation of the total delay by moving telescopes

Consider a right handed coordinate system specified by the unit vectors $(\hat{x}, \hat{y}, \hat{z})$, located at geographical latitude Λ on Earth. $\hat{z}$ points to the local zenith, $\hat{x}$ to the east and $\hat{y}$ to the north. Let $\hat{r}_*$ be the unit vector pointing to the star being observed, which is specified by the azimuth angle A and elevation angle a (which are functions of the hour angle θ), as shown in Fig. 1. Let $\vec{R}_i$ be the vector position of the ith telescope of the interferometer, specified by the rectangular coordinates (X_i, Y_i, Z_i). The allowed values of $\vec{R}_i$ are constrained by the requirement that the radiation from the star arrive at the origin O (position of a detector), via each telescope, with exactly the same phase. Consider a reference plane perpendicular to the line of sight to the star $\hat{r}_*$, and passing through the origin. This is also a surface of equal phase for the incoming radiation. Let $\vec{R}_{ip}$ be the projection of $\vec{R}_i$ onto the reference plane. Then

$$\vec{R}_{ip} = \hat{r}_* \times (\vec{R}_i \times \hat{r}_*) \tag{1}$$

The path length S travelled by the radiation from the reference plane to the detector, via the ith telescope, is S_i:

$$\begin{aligned} S_i &= |\vec{R}_{ip} - \vec{R}_i| + |\vec{R}_i| \\ &= -|(\vec{R}_i \cdot \hat{r}_*)\hat{r}_*| + |\vec{R}_i| \\ &= |\vec{R}_i| - \vec{R}_i \cdot \hat{r}_* \end{aligned} \tag{2}$$

which should be the same for all i at any instant of time (i.e., S can be a function of the hour angle θ). Thus the surface of constant S is a parabola of rotation about $\hat{r}_*$ the line of sight to the star, with the detector at the focus (Labeyrie, 1978). All telescopes must lie on this surface if their outputs are to be directly corre-

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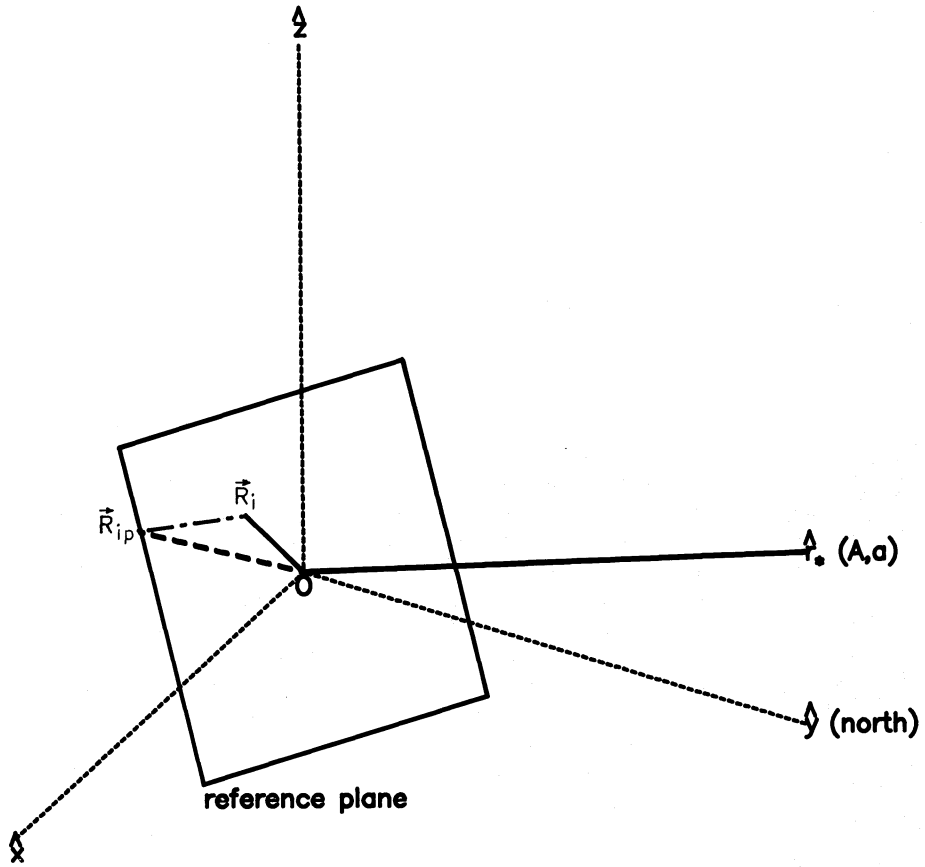

Figure 1. Geometry of telescope arrangement. $\hat{r}_*$ is the unit vector along the line of sight to the star (at azimuth A and elevation a). $\vec{R}_i$ is the position vector of the ith telescope while $\vec{R}_{ip}$ is its projection onto the reference plane, which is perpendicular to $\hat{r}_*$ and passes through the origin; the reference plane is thus a surface of equal phase for the incoming radiation. The detector is at the origin O of the coordinate system. $\vec{R}_i$ must lie on a parabola of rotation about the axis $\hat{r}_*$, with the focus at the origin, if the signals from all telescopes are to be directly correlated.

lated. In rectangular coordinates the allowed values of X_i, Y_i and Z_i obey the relation

$$[X^2(\theta) + Y^2(\theta) + Z^2(\theta)]^{1/2} - [X(\theta)\cos a \sin A + Y(\theta)\cos a \cos A + Z(\theta)\sin a] = S(\theta) \quad (3)$$

Practical considerations will limit the usable (X, Y, Z) space allowed by Eq. (3). For example, it is more difficult to raise telescopes above ground than it is to move them about in a plane, which may enforce the constraint $Z(\theta) = 0$. Furthermore the optical beam may be brought to the detector via an underground tunnel to avoid further perturbations to the phase by warm air currents near the ground; this would severely limit the allowed combinations of $X(\theta)$ and $Y(\theta)$ in Eq. (3). In the next section we discuss a specific geometry of telescopes and the corresponding (U, V) coverage.

2.1. Radial arrangement of pairs of telescopes on the ground

Consider a pair of telescopes labelled 1 and 2, moving along a radial track on the ground ($Z(\theta) = 0$), whose azimuth angle is A_t (not more than two telescopes can lie on a straight line and satisfy Eq. (3)). Let the distances of the telescopes from the detector (i.e., the origin) be R_1 and R_2 respectively. The corresponding set of rectangular coordinates (X_1, Y_1) and (X_2, Y_2) must each satisfy Eq. (3), in addition to satisfying the constraint of the track:

$$Y_{1,2}(\theta) = X_{1,2}(\theta)\cot A_t \quad (4)$$

The resulting solutions for the coordinates are

$$X_1(\theta) = \frac{S(\theta)\sin A_t}{1 - \cos a \cos(A - A_t)}$$
$$Y_1(\theta) = \frac{S(\theta)\cos A_t}{1 - \cos a \cos(A - A_t)}$$
$$X_2(\theta) = \frac{-S(\theta)\sin A_t}{1 + \cos a \cos(A - A_t)}$$
$$Y_2(\theta) = \frac{-S(\theta)\cos A_t}{1 + \cos a \cos(A - A_t)} \quad (5)$$

The $\hat{x}$ and $\hat{y}$ components of the corresponding baselines are $\Delta X(\theta)$ and $\Delta Y(\theta)$:

$$\pm\Delta X(\theta) = X_1(\theta) - X_2(\theta) = \frac{2S(\theta)\sin A_t}{1 - [\cos a \cos(A - A_t)]^2}$$
$$\pm\Delta Y(\theta) = Y_1(\theta) - Y_2(\theta) = \frac{2S(\theta)\cos A_t}{1 - [\cos a \cos(A - A_t)]^2} \quad (6)$$

where the $\pm$ refers to the two possible conventions in choosing the baseline components. The corresponding $U(\theta)$ and $V(\theta)$ components of the projected baseline are derived from the relations:

$$U(\theta) = \Delta X(\theta)\cos\theta - \Delta Y(\theta)\sin\Lambda\sin\theta + \Delta Z(\theta)\cos\Lambda\sin\theta$$
$$V(\theta) = \Delta X(\theta)\sin\delta\sin\theta + \Delta Y(\theta)[\cos\delta\cos\Lambda + \sin\delta\sin\Lambda\cos\theta] + \Delta Z(\theta)[\cos\delta\sin\Lambda - \sin\delta\cos\Lambda\cos\theta] \quad (7)$$

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(the above formulae differ from those of Thompson et al (1986) in that $\hat{x}$ points east). Equation (7) can be rearranged to give the following relation between the $U(\theta)$ and $V(\theta)$ components, assuming that $\Delta Z(\theta) = 0$ always:

$$[U(\theta)]^2 + \left[\frac{V(\theta) - \Delta Y(\theta)\cos\Lambda\cos\delta}{\sin\delta}\right]^2 = [\Delta X(\theta)]^2 + [\sin\Lambda\,\Delta Y(\theta)]^2 \quad (8)$$

where δ is the declination of the source. Only one telescope can lie on a track at a given azimuth, and the total number of usable tracks is limited only by practical considerations; however they must all have the same $S(\theta)$, and their signals must be brought radially to the detector. Figure 2 shows three sample U-V tracks, for stars at $\delta = 15°$, $45°$ and $75°$ respectively, for a site at $\Lambda = 45°$, when two pairs of telescopes are used, one pair on an E-W track and one pair on a N-S track. $S(\theta)$ is set to a constant value of 1.0 (in arbitrary units), and elevations below 30° are not included.

As a consistency check one can compute the third baseline component $W(\theta)$ which is the projection along the line of sight:

$$\begin{aligned} W(\theta) &= -\Delta X(\theta)[\cos\delta\sin\theta] \\ &\quad + \Delta Y(\theta)[\sin\delta\cos\Lambda - \cos\delta\sin\Lambda\cos\theta] \\ &\quad + \Delta Z(\theta)[\sin\delta\sin\Lambda + \cos\delta\cos\Lambda\cos\theta] \\ &= \Delta X(\theta)[\cos a\sin A] + \Delta Y(\theta)[\cos a\cos A] \\ &= \frac{2S(\theta)\cos a\cos(A - A_t)}{1 - [\cos a\cos(A - A_t)]^2} \end{aligned} \quad (9)$$

This is the extra distance that the radiation has to travel to reach, say, telescope 2 with respect to telescope 1; this then must be equal to the difference in paths from telescope 1 and telescope 2 to the detector:

$$\begin{aligned} R_1 - R_2 &= [(X_1)^2 + (Y_1)^2]^{1/2} - [(X_2)^2 + (Y_2)^2]^{1/2} \\ &= \frac{S(\theta)}{1 - \cos a\cos(A - A_t)} - \frac{S(\theta)}{1 + \cos a\cos(A - A_t)} \\ &= \frac{2S(\theta)\cos a\cos(A - A_t)}{1 - [\cos a\cos(A - A_t)]^2} \end{aligned} \quad (10)$$

which is the same as in Eq. (9).

2.2. *A single radial track on the ground*

The telescope motions in Eq. (5) are not 'simple', in the sense that large accelerations might be necessary at some hour angles; neither are the corresponding $U - V$ plots of Eq. (8) easy to interpret. The need to simplify the telescope motions suggests the following choice of $S(\theta)$:

$$S(\theta) = S_0[1 - [\cos a\cos(A - A_t)]^2] \quad (11)$$

where S_0 is a constant length. Then the telescope motions reduce to

$$\begin{aligned} X_1(\theta) &= S_0\sin A_t[1 + \cos a\cos(A - A_t)] \\ Y_1(\theta) &= S_0\cos A_t[1 + \cos a\cos(A - A_t)] \\ X_2(\theta) &= -S_0\sin A_t[1 - \cos a\cos(A - A_t)] \\ Y_2(\theta) &= -S_0\cos A_t[1 - \cos a\cos(A - A_t)] \end{aligned} \quad (12)$$

which yields constant baseline components:

$$\begin{aligned} \pm\Delta X(\theta) &= 2S_0\sin A_t \\ \pm\Delta Y(\theta) &= 2S_0\cos A_t \end{aligned} \quad (13)$$

The corresponding $U - V$ plot is an ellipse:

$$[U(\theta)]^2 + \left[\frac{V(\theta) - 2S_0\cos A_t\cos\Lambda\cos\delta}{\sin\delta}\right]^2 = 4S_0^2[\sin^2 A_t + \sin^2\Lambda\cos^2 A_t] \quad (14)$$

Thus a pair of telescopes, restricted to a radial track on the ground at azimuth A_t, can be moved in a 'simple' manner so as to maintain zero path difference between their signals. Note that this solution works for one radial track only, since $S(\theta)$ in Eq. (11) is a function of A_t. Alternatively the telescopes can be fixed and the detector moved in a complementary manner, to achieve the same result (Labeyrie et al, 1986).

3. Compensation of partial delay by moving telescopes

Now suppose that the light from each telescope reaches the detector through optical fibres, each of the same length. The telescopes should then be moved so that a plane wave arrives at the telescopes with zero relative delay. This implies that the component $W(\theta)$ of the projected baseline, which is defined in Eq. (9), must always be zero. In general there are many types of telescope motions which will satisfy the above criterion. We will discuss two such telescope motions and the corresponding $U - V$ coverage.

3.1. *Two-dimensional motion of telescopes on the ground*

Suppose that a pair of telescopes moves not on rail tracks, as in the previous section, but instead is free to move in two dimensions upon a platform (Labeyrie et al, 1986). As in Sect. 2.1, let the pair of telescopes have baseline components $\Delta X(\theta)$ and $\Delta Y(\theta)$, with $\Delta Z(\theta)$ being zero always. To maintain $W(\theta) = 0$, the baseline components must obey the relation

$$\Delta X(\theta)[\cos\delta\sin\theta] = \Delta Y(\theta)[\sin\delta\cos\Lambda - \cos\delta\sin\Lambda\cos\theta] \quad (15)$$

Although there are many solutions to Eq. (15), they can be reduced by specifying constraints on the U, V components. From Eqs. (7) and (9),

$$V(\theta) = \frac{\Delta Y(\theta)\cos\Lambda}{\cos\delta} \quad (16)$$

Thus by choosing the $V(\theta)$ coverage as a function of the hour angle θ, one fixes the Y motion of the baseline. This uniquely fixes the X motion through Eq. (15). From Eqs. (16), (15) and (7) we obtain

$$\Delta X(\theta) = \frac{V(\theta)[\sin\delta\cos\Lambda - \cos\delta\sin\Lambda\cos\theta]}{\cos\Lambda\sin\theta} \quad (17)$$

$$U(\theta) = \frac{-V(\theta)[\cos\delta\sin\Lambda - \sin\delta\cos\Lambda\cos\theta]}{\cos\Lambda\sin\theta} \quad (18)$$

A study of the last three equations suggests a particularly simple set of solutions:

$$V(\theta) = V_0\sin\theta \quad (19)$$

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$$\Delta Y(\theta) = \frac{V_0 \cos\delta \sin\theta}{\cos\Lambda} \quad (20)$$

$$\Delta X(\theta) = \frac{V_0(\sin\delta\cos\Lambda - \cos\delta\sin\Lambda\cos\theta)}{\cos\Lambda} \quad (21)$$

$$\left[\frac{\Delta X(\theta) - V_0\sin\delta}{\sin\Lambda}\right]^2 + [\Delta Y(\theta)]^2 = \left[\frac{V_0\cos\delta}{\cos\Lambda}\right]^2 \quad (22)$$

$$\left[\frac{U(\theta) - V_0\cos\delta\tan\Lambda}{\sin\delta}\right]^2 + [V(\theta)]^2 = V_0^2 \quad (23)$$

where V_0 is a constant, equal to the maximum amplitude of the V motion. From Eqs. (19) to (23), if we choose to move the Y component of the baseline sinusoidally, then we have to move the X component cosinusoidally, so that the vector in the $X - Y$ space rotates over the ellipse in Eq. (22). The centre of the ellipse lies on the X axis by an offset given by the maximum amplitude of the Y excursion one has chosen. For such a motion the projected baseline also traces an ellipse in the $U - V$ plane (Eq. 23) whose centre is displaced along the U axis by another predetermined amount, $V_0 \tan\Lambda/\tan\delta$. Figure 3 shows sample $U - V$ plots for sources at three different declinations (as in Fig. 2) for an array of two telescopes at latitude $\Lambda = 45°$.

One can verify that such a solution exists by visualising the baseline geometry. Physically the condition $W(\theta) = 0$ corre-

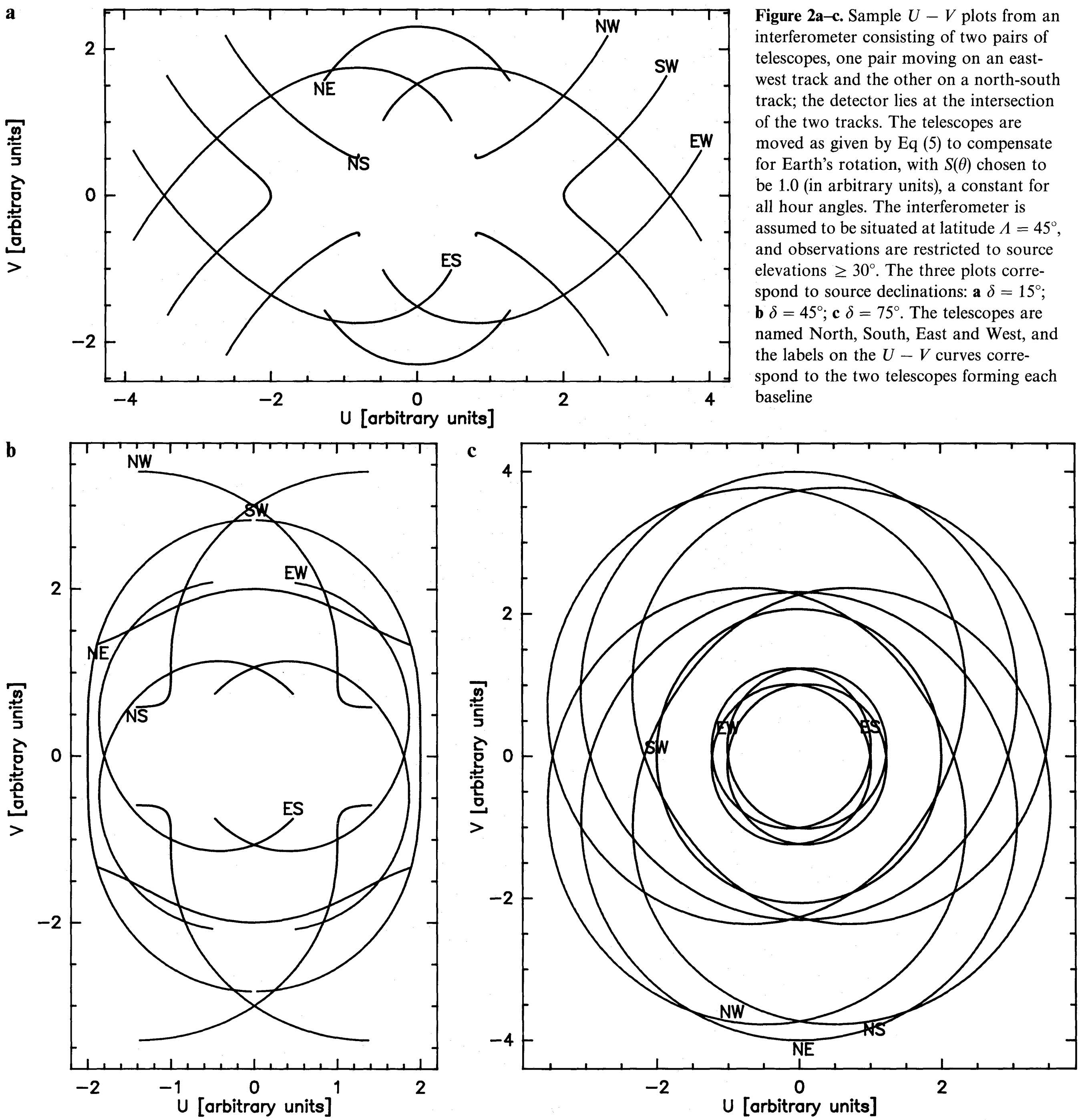

Figure 2a–c. Sample $U - V$ plots from an interferometer consisting of two pairs of telescopes, one pair moving on an east-west track and the other on a north-south track; the detector lies at the intersection of the two tracks. The telescopes are moved as given by Eq (5) to compensate for Earth's rotation, with $S(\theta)$ chosen to be 1.0 (in arbitrary units), a constant for all hour angles. The interferometer is assumed to be situated at latitude $\Lambda = 45°$, and observations are restricted to source elevations $\geq 30°$. The three plots correspond to source declinations: **a** $\delta = 15°$; **b** $\delta = 45°$; **c** $\delta = 75°$. The telescopes are named North, South, East and West, and the labels on the $U - V$ curves correspond to the two telescopes forming each baseline

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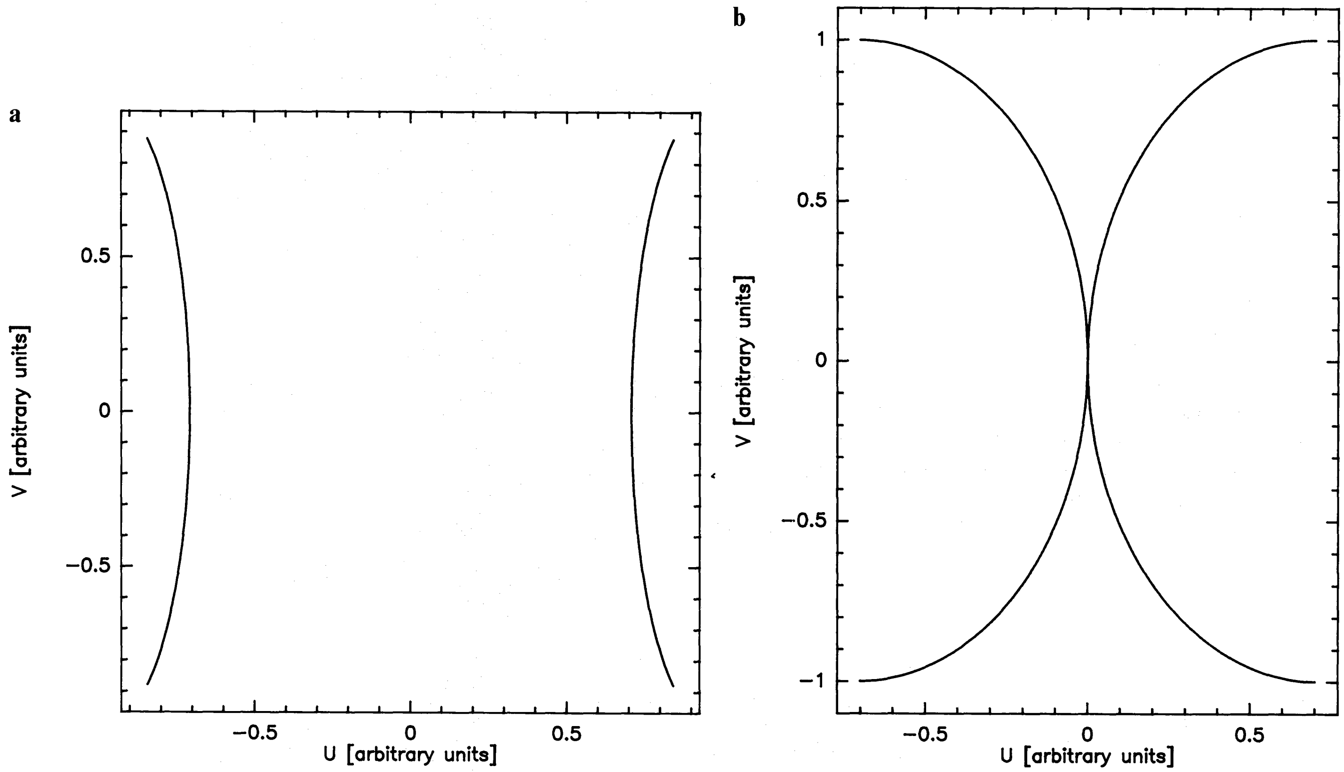

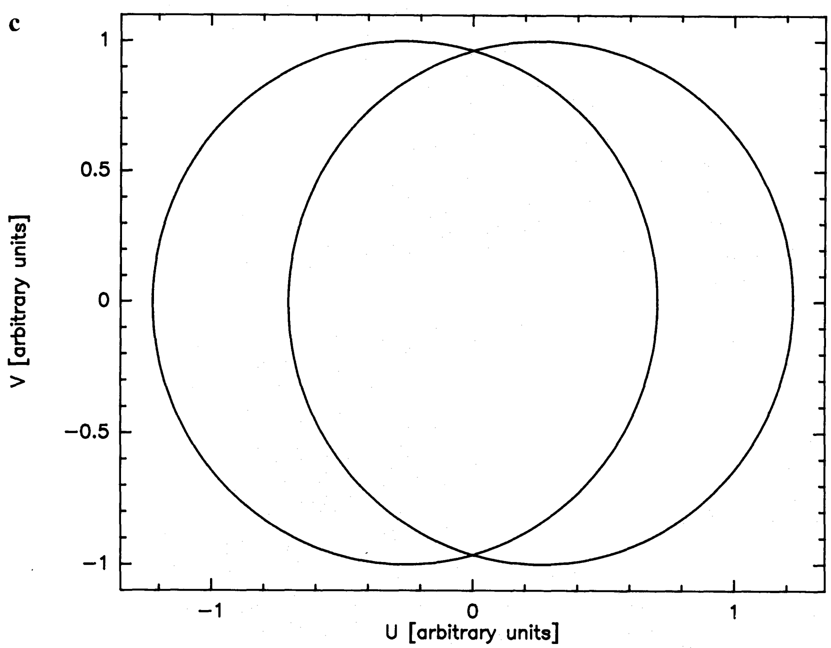

Figure 3a–c. Sample $U-V$ plots from an interferometer consisting of two telescopes, both moving in two dimensions on a platform, so as to lie on a straight line passing through the detector, which lies at the origin. The telescopes are moved as given by Eqs. (20) and (21), with $V_0 = 1.0$ (in arbitrary units), to partially compensate for Earth's rotation; light is carried from each telescope to the detector via fixed length optical fibres. The interferometer is assumed to be situated at latitude $\Lambda = 45°$, and observations are restricted to source elevations $\geq 30°$. The three plots correspond to source declinations: **a** $\delta = 15°$; **b** $\delta = 45°$; **c** $\delta = 75°$

sponds to keeping the baseline perpendicular to the line of sight $\hat{r}_*$, i.e., it lies in the $U-V$ plane. But the baseline must also lie in the $X-Y$ plane since the Z-component motion is not allowed. So the solution is to keep the telescopes along the line of intersection between the two planes, which rotates continuously as the hour angle increases. As a consistency check, one can independently obtain the angle that this line makes with, say, the north (Y) direction. This must equal the inverse tangent of $\Delta X(\theta)/\Delta Y(\theta)$, as we have verified.

3.2. Telescopes fixed on ground–only Z motion allowed

We now discuss the case where both the X and Y components of the baseline are fixed and only the Z component is allowed to vary. This can be done by raising or lowering one or both of the telescopes. From an engineering point of view, there appears to be no great difficulty in raising a telescope's height by about one metre (D. Plathner, 1987). It can be verified from Eq. (9) that given fixed values of $\Delta X(\theta)$ and $\Delta Y(\theta)$, $W(\theta)$ can be maintained

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to be zero by moving the Z component as:

$$\Delta Z(\theta) = \frac{\Delta X(\theta)\cos\delta\sin\theta - \Delta Y(\theta)[\sin\delta\cos\Lambda - \cos\delta\sin\Lambda\cos\theta]}{\sin\delta\sin\Lambda + \cos\delta\cos\Lambda\cos\theta} \quad (24)$$

The corresponding U and V components can be shown to be:

$$V(\theta) = \frac{\Delta Y(\theta)\cos\Lambda + \Delta Z(\theta)\sin\Lambda}{\cos\delta} \quad (25)$$

$$U(\theta) = \frac{\Delta X(\theta) - V(\theta)\sin\delta\sin\theta}{\cos\theta} \quad (26)$$

Thus one obtains some $U - V$ coverage by moving the Z component of the baseline only. The utility of Eqs. (25) and (26) can be seen by setting $\Delta X(\theta)$ and $\Delta Y(\theta)$ to values that satisfy Eq. (22). That is, one must imagine the X and Y components to have been moving along the ellipse of Eq. (22), and then stopping at hour angle $\theta = \Theta$, when the projected baselines were $U(\Theta)$ and $V(\Theta)$. Then Eq. (25) becomes:

$$V(\theta) = V(\Theta) + \frac{\Delta Z(\theta)\sin\Lambda}{\cos\delta} \quad (27)$$

Thus the $U - V$ coverage obtained from a pure $X - Y$ motion of the baseline can be improved at each U and V by the Z motion alone; how much one actually improves depends upon the hour angle, declination and geographic latitude.

4. 'Hypersynthesis'

Finally, we discuss the possibility of imposing additional motions on the telescopes to achieve greater efficiency of $U - V$ mapping for bright sources. We use the term 'hypersynthesis' (suggested by P. Lena) to designate synthesis with this additional dimension of motion, beyond that required for earth-rotation supersynthesis. In principle hypersynthesis implies choosing a suitable form of $S(\theta)$, and intuitively, it is obvious that $S(\theta)$ must oscillate at a rate faster than the rotation rate of earth; then each physical baseline would trace, in the $U - V$ plane, a curve oscillating within an annular band; the faster the oscillation of $S(\theta)$, the more complete is the $U - V$ coverage within this band. As an illustrative case we choose

$$S(\theta) = S_0[1 + \alpha\cos(N_c\theta)] \quad (28)$$

in Eq. (3), where α is the relative amplitude of oscillation and N_c is its frequency. The product αN_c must be subject to an upper limit since the additional velocities of telescopes are directly proportional to it. Figure 4 shows the $U - V$ coverage obtained using a four telescope interferometer (having the same general conditions as in Fig. 2(a)), using Eq. (28) with $S_0 = 1.0$, $\alpha = 0.2$ (oscillation amplitude equal to 20% of the baseline), and $N_c = 50$ oscillations per 24-hour period. In Fig. 4, it is easy to recognise the corresponding tracks in Fig. 2(a) which have now been 'widened'. If we adopt the total length of a $U - V$ track as an index of efficiency of mapping, Fig. 4 shows that all but four tracks of Fig. 2(a) have been mapped with an efficiency in excess of 2 (or even 3). This would lead to a significant reduction in the required time of observation for a bright source. The price one pays for this extra efficiency is approximately a factor of 2 increase in the maximum velocities of telescopes. Thus in Fig. 4 a telescope at 100 m distance from the detector would have to be moved with a velocity of 0.1 m/s when the source is at an elevation of 30°; however we do not consider this to be a significant problem.

5. Discussion

We have shown in Sect. 2 that telescopes can be moved in such a manner that the total phase of the radiation arriving at a common detector, via the telescopes, is a constant. The telescopes have to be placed on a surface which is a parabola of rotation about the line of sight $\hat{r}_*$ (Eq. 3). The semi latus rectum of the parabola, $S(\theta)$, can vary as any convenient function of the hour angle θ.

However all solutions allowed by Eq. (3) may not be practical. For example, the path of the radiation to the detector should

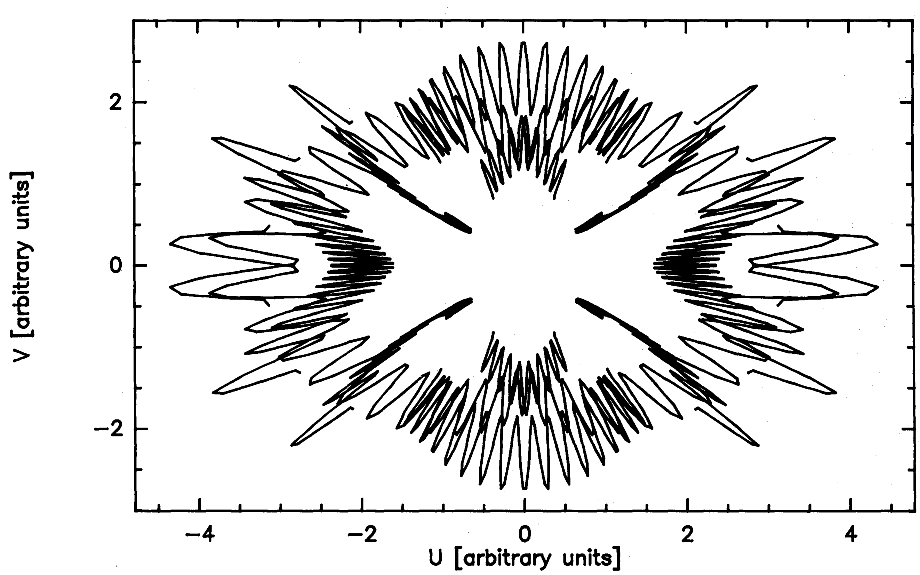

Figure 4. Sample $U - V$ plots from the interferometer in Fig. 2(a), when $S(\theta) = 1.0\,[1 + \alpha\cos(N_c\theta)]$, and is no longer a constant. α and N_c have been chosen to be 0.2 and 50 respectively, so that the product is equal to 10. Choosing a larger value of N_c, and therefore a correspondingly smaller value of α, will reduce the mapping efficiency and increasing the redundancy of the $U - V$ coverage

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ideally be a straight line. This would minimize the number of optical elements in the path, improving the efficiency, and would also simplify the construction of tunnels for bringing light to the detector. As raising and lowering of telescopes is a relatively difficult exercise, the easiest arrangement appears to be that of telescopes moving along radial tracks, with the detector at the origin.

In Sect. 2.1 we derived the required motions of a pair of telescopes restricted to move along a radial track at a fixed azimuth A_t (Eq. 5). The corresponding track in the $U-V$ plane is given by Eq. (8), which is not simple to interpret. However it is clear that the coverage in the $U-V$ plane is reasonable (Fig. 2), and that its efficiency can be enhanced by using more telescopes. For the choice of constant $S(\theta)$, an additional telescope can be used at the origin, in conjunction with a constant delay line. Since this telescope is fixed it can be much larger; its beam can be appropriately split to provide closure phase information for the other telescopes. The telescopes would need to be moved with a maximum speed of 0.027 m/s, assuming that they are allowed to reach a maximum distance of 100 m from the detector, when the source elevation is 30° (probably the minimum elevation for useful observing).

In Sect. 2.2 we use a particular choice of $S(\theta)$ (Eq. 11) which simplifies the X, Y motions of a pair of telescopes lying on a fixed radial track; signals from telescopes on other radial tracks will not be phase coherent with the signals from the particular two telescopes. The corresponding plot in the $U-V$ plane is an ellipse, because the baseline components are of fixed length (Eq. 13). Thus in a two-telescope interferometer, the X, Y motions are simple, being sinusoids of period one day. Unfortunately this choice of $S(\theta)$ is suitable for one radial track only. To use many more pairs of telescopes, each on a different radial track, a choice of $S(\theta)$ independent of A_t is required in Eq. (6). As already mentioned, identical results can be achieved by moving the detector in a complementary manner.

The arrangement of fixed radial tracks is not easily amenable to future expansion of the interferometric array. Any tracks added later on would require a significant modification of the central laboratory where the beams are combined.

In Sect. 3.1 we discussed the required motions of telescopes when the delay from the telescope to the detector is compensated by using fixed length optical fibres. Then the telescopes are moved only to maintain them on a line perpendicular to the line of sight to a star, which is continuously rotating as θ changes. Equations (19) to (23) show that very simple telescope motions are needed to achieve the above. They are sinusoidal motions with period of one day. With a V amplitude of 100 m such motion involves a maximum speed of 0.01 m/s at mid latitudes, which is reasonable. In principle there is no limit to the number of telescopes used in the array, as long as they all lie on the rotating straight line. The $U-V$ coverage is also reasonable. For this case it appears ideal to allow telescopes the freedom to move in two dimensions on the ground. The main disadvantage of the scheme is that the light must suffer loss in the fixed length of the optical fibre even when the telescope is much closer to the detector.

One might feel that a more general constraint on the telescope motion, viz. $W(\theta) = \text{constant}$, instead of $W(\theta) = 0$, would offer a greater advantage in mapping the $U-V$ plane. We have done some calculations which show that the corresponding $U-V$ mapping is not desirable; there are large holes in the maps, and the mapping is concentrated in regions of the $U-V$ plane and not inform as required for good imaging.

In Sect. 3.2 we have shown that, if possible, the Z motion can also be used to map the $U-V$ plane. In particular it can be used to complement the $U-V$ mapping obtained by a purely $X-Y$ motion, which would be of practical use. However the X, Y and Z motions of the telescopes are not easy to compute, and are certainly not 'simple'.

Table 1 summarizes the situation described in the text, gives the formulae for telescope motions, and velocities required for some reasonable applications. The formulae refer to the velocity of a telescope at a distance R from the origin, which is the maximum distance possible in the deforming ellipse. $\dot{a}$ is the rate of change of elevation with respect to the hour angle θ (assumed to be 1.0 radians per radian for the velocity calculations), and ϖ is the rotation rate of earth (15″/s). α and N_c are assumed to be 0.2 and 50 respectively. In situation 2, V_0 refers to the maximum V component of the projected baseline. The velocities are computed for $R = V_0 = 100$ m at elevation $a = 30°$. Note that the telescope velocities scale linearly with distance.

The moving telescope method of interferometry should be especially efficient at infrared wavelengths, where observations

Table 1. Summary of telescope speeds required for path compensation

NO.	SITUATION	FORMULA	VALUE
1	Compensation of the total delay by movable telescopes, as discussed in section 2	$\dfrac{\varpi R \dot{a} \sin a}{1-\cos a}$	0.027 m/sec.
2	Compensation of partial delay by moving telescopes, and with the use of optical fibres, as in section 3	$\dfrac{\varpi V_0 \cos\delta}{\cos\Lambda}$	$\dfrac{0.007\cos\delta}{\cos\Lambda}$ m/sec.
3	'Hypersynthetic' motion of telescopes in the first situation, using eq. (28).	$\dfrac{\varpi R[\dot{a}\sin a + \alpha N_c(1-\cos a)]}{1-\cos a}$	0.1 m/sec.

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are only practical in the 'bright source' regime (Roddier, 1987), so that integration times are short. Thus information along the $U - V$ tracks is probably highly redundant, so it would be better to use extra motions of telescopes (as in Eq. (28) for example) to 'hypersynthesize' the $U - V$ coverage.

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