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Haidinger's rings in soap bubbles

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1. Introduction

The colours of soap films are the classic illustration of the interferences of light in thin plates for which the theory was first given by Thomas Young. To exhibit these colours most advantageously, it is usual to stretch a film of soap solution on a wire frame and allow it to drain by placing it in a vertical plane. Horizontal bands of colour then develop on the film which are best seen in the light reflected at its surface. These bands of colour result from the varying thickness of the film and become more conspicuous as it grows thinner. The colours seen depend both on the thickness of the film and the angle at which the light falls on it before being reflected and reaching the eye of the observer. This angle of incidence does not differ appreciably for different parts of the surface of a plane film, provided that the eye of the observer is sufficiently distant. The circumstances are, however, quite different in regard to the colours exhibited by a spherical soap bubble. In this case, it is obvious that the surface of the bubble presents itself to the eye at all angles varying from a normal to a tangential aspect. The colours of a spherical soap bubble necessarily therefore exhibit the effect of the varying obliquity of incidence of the light reflected at its surface. Indeed, in the ideal case of a spherical bubble of completely uniform thickness, the variations of colour exhibited by the surface would be determined exclusively by such variations of obliquity. The colour pattern in this ideal case would consist of a set of circular rings localised on the surface of the bubble and arranged concentrically around that diameter of the sphere which when produced meets the eye of the observer. It is the purpose of this paper to show that this effect can actually be observed and to emphasise that it is essentially similar to the well known rings of Haidinger exhibited by a plane-parallel plate in monochromatic light. Indeed, as will be shown in this paper, there is a simple geometric relation between the configuration of the interference curves of equal inclination as exhibited respectively by a spherical film and a plane-parallel one of the same thickness. The position of the observer's eye enters into this geometric relation, and when the eye is sufficiently close to the surface of the plate, the two cases become indistinguishable.

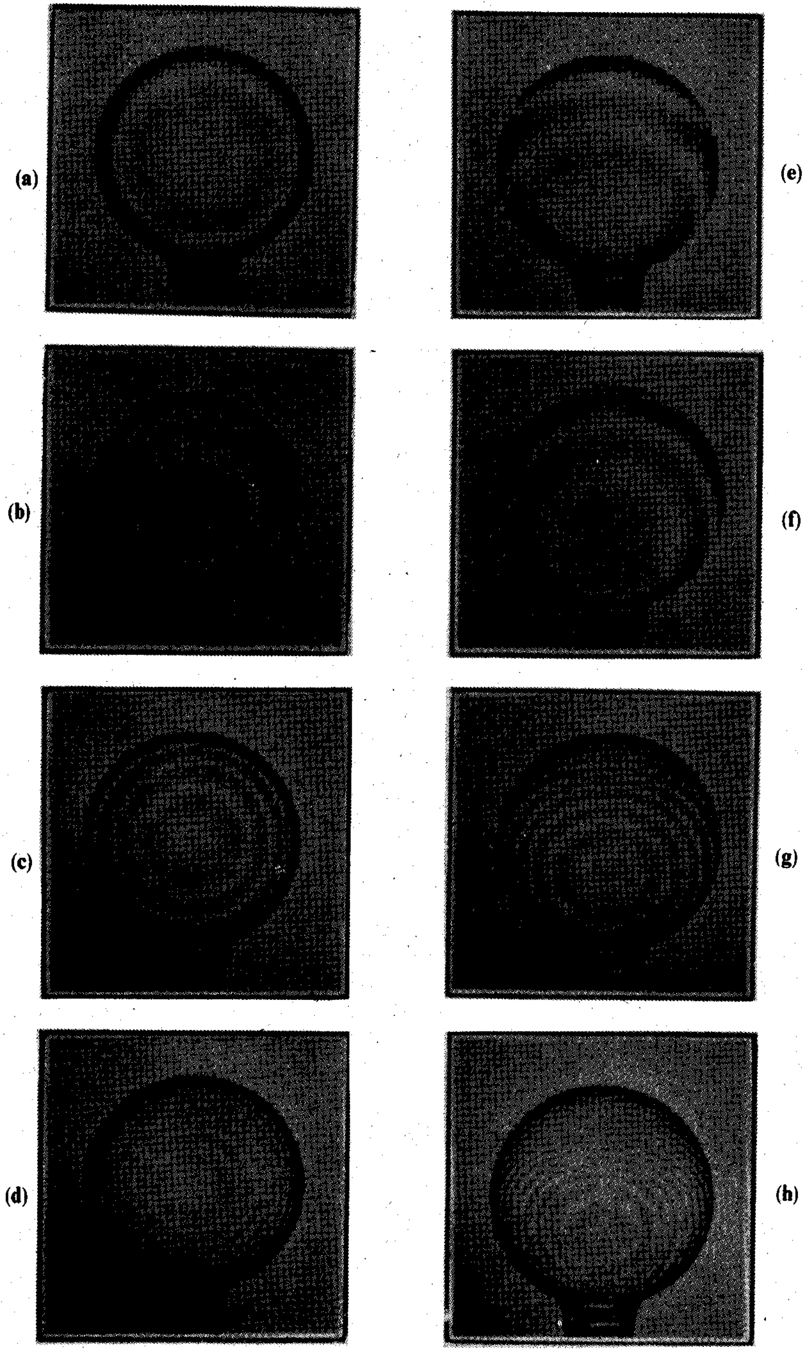


Plate I. Figures a–h. Interference figures in soap bubbles by transmitted light.

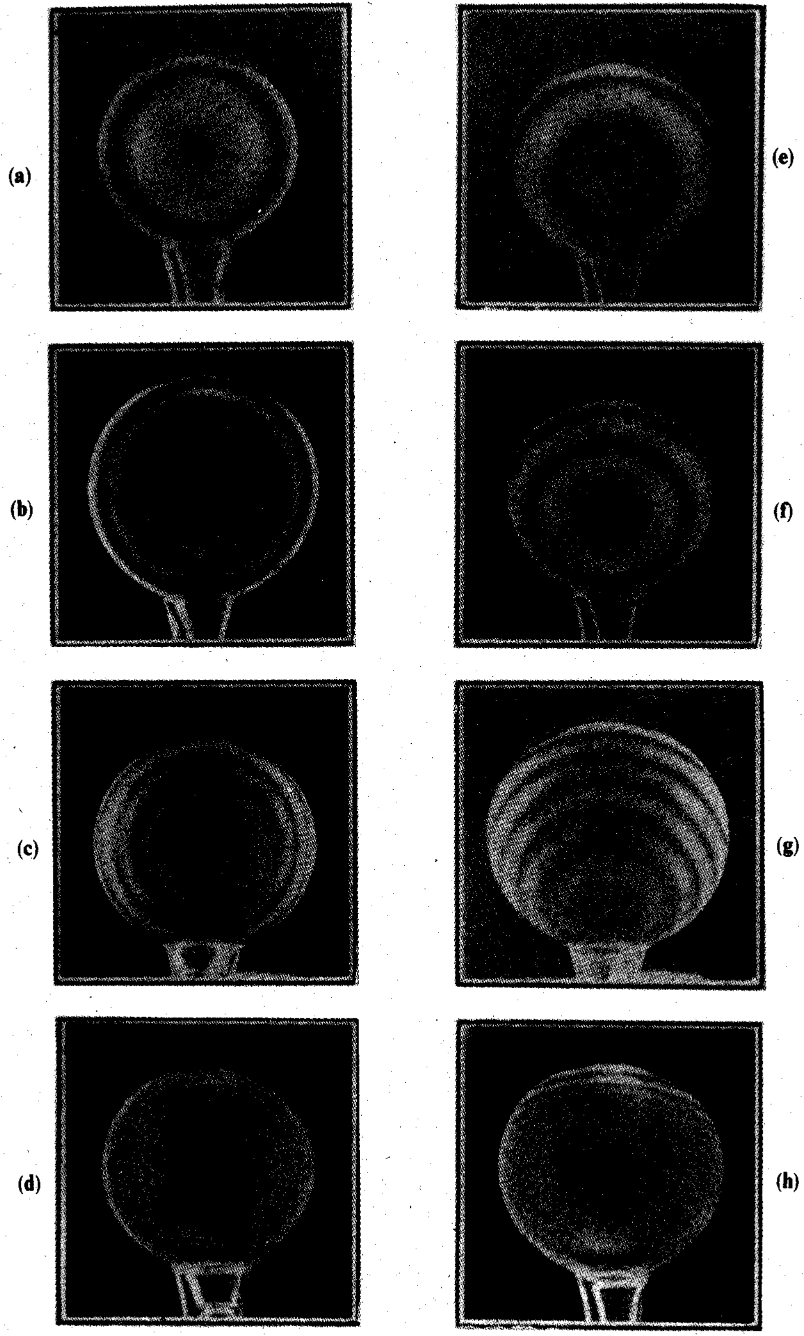


Plate II. Figures a–h. Interference figures in soap bubbles by reflected light.

2. Methods of observation

The interference figures of soap bubbles in transmitted light, though not so striking as those exhibited by reflection, are in a sense the more easily observed. All that is necessary is to view the bubble against an illuminated background. Using monochromatic light (a mercury lamp with a green ray filter), the interference curves may be observed and photographed with very brief exposures, and the changes which occur with time may therefore be readily followed and recorded. To observe the complete interference figures of a soap bubble as seen by reflection, it is necessary to place the bubble inside an illuminated enclosure. This may conveniently be done by surrounding the bubble by a cover of translucent material, e.g., paper, and lighting up the latter from outside. It is necessary, of course, to provide the enclosure with two apertures, one to serve as a dark background for the bubble, and the other as an opening through which it may be observed or photographed. Plates I and II reproduce photographs obtained in this way. Plate I illustrates soap bubbles as seen in transmitted light and plate II as observed by reflection. Figures (a) to (d) in both plates illustrate the Haidinger rings exhibited by soap bubbles of uniform thickness (different in each case); while figures (e) to (h) in the two plates illustrate the interference curves of non-uniform bubbles. The form of the interferences in the latter case is evidently determined jointly by variations of thickness and of obliquity.

3. Behaviour of spherical soap-films

Optical observation as described above enables us readily to follow both the configuration of thickness in a spherical bubble when it is first blown and also the changes that occur in it with lapse of time. It is quite possible, using suitable technique, to blow bubbles of almost ideal perfection in regard to uniformity of thickness. This is largely a question of avoiding any superfluity of liquid in the first instance and of adjusting the rate of blowing to the quantity of liquid used. The use of a capillary tube with its ends ground flat appears to be the most convenient method of securing both these aims, and bubbles blown with its aid exhibit, in the first instance, a perfect system of Haidinger rings. Soon after the bubble is blown, however, the natural flow of the liquid downwards within the film causes the lower levels to gain in thickness at the expense of the upper. This non-uniformity of thickness reveals itself immediately in interference figures of which the form is no longer independent of the direction in which the bubble is observed. When the bubble is seen horizontally, the ring-system ceases to be centred round the line joining the eye and the bubble and gradually moves downwards making the pattern unsymmetrical. Successive rings as they go down become incomplete arcs meeting the edge of the bubble. Finally, the innermost ring touches the bottom, and the interference curves above gradually straighten out and stretch more or less horizontally across the bubble, reaching and cutting its periphery. A distinct curvature, however, remains as an indication of the effect of varying obliquity on their form. The complete sequence of changes observed and its progress with lapse of time naturally depend upon the initial thickness and size of the bubble and the viscosity of the soap solution used. The changes are naturally the more rapid, the greater the thickness of the bubble in the first instance and the less the viscosity of the solution used.

Any initial excess of the liquid used in blowing a bubble reveals itself in a striking non-uniformity of its thickness. The contour lines of constant thickness, are of course, horizontal circles, and these congregate densely in the lower levels of the bubble; their position is indicated by the interference lines of equal thickness which appear crowded together in this region and indeed seem to bear no relation to the more widely distributed curves seen in the upper parts of the bubble. Viewed from a point vertically above or below it, such a non-uniform bubble presents a symmetrical aspect, the interference curves being horizontal circles. These, however, are crowded together in the lower parts of the bubble and very wide apart in the upper levels.

It may be pointed out that as the sequence of changes described above for a spherical soap bubble may be readily photographed and measured, it offers the possibility of a quantitative test of the theory that viscous flow of liquid within the film is the cause of its thinning. According to Willard Gibbs (*Scientific Papers*, p. 307, *et seq.*), the changes occurring within a soap-film are due not so much to the action of gravity as to the suction exercised by the ring of liquid formed along the line where the film meets its solid supports. Indeed it appears that in the case of a plane film this suction is the agent principally responsible for its thinning down. It would, therefore, seem to be important to investigate whether this is the case also for a spherical bubble when the perimeter of its support is reduced to the absolute minimum necessary.

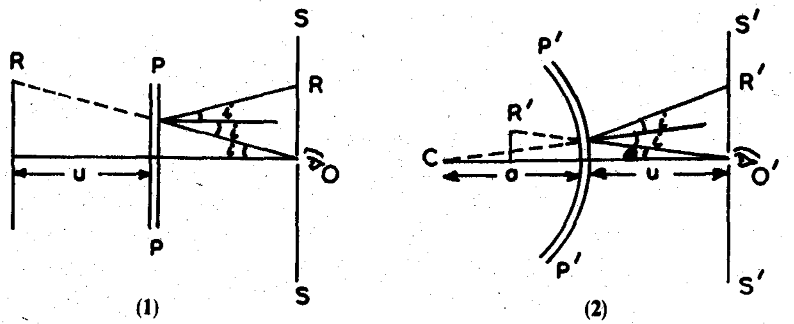
4. The crossed air-jets method

A simple and efficacious technique has been devised by the authors which enables a soap bubble to be reduced to uniform thickness even when it is initially non-uniform and by which the uniformity of thickness thus obtained can be maintained. A soap bubble is blown and allowed to sit on the circular end of a vertical glass tube. Two *very gentle* currents of air are blown upwards from two glass tubes placed below the level of the support of the bubble and displaced from it in two directions exactly a right angle apart. The effect of the air currents impinging at an angle on the surface of the bubble is to set up an upward drift of the liquid within the film in two directions 90° apart. The resulting circulation soon results in the thickness of the film becoming the same everywhere. The continued flow of the two air currents with just sufficient force to counteract the tendency of downward movement ensures the thickness of the bubble remaining uniform in spite of its steady thinning by evaporation. By using a mechanical blower with a control stop-cock, the currents of air can be made sufficiently weak to accomplish this purpose without causing the bubble to oscillate on its support and without setting up any visible vortex motion on its surface. The success of the device is most clearly exhibited by its optical results. The interference pattern takes the form of perfect circles and becomes identical from whatever aspect the bubble may be viewed. The adoption of this device has been found very useful in obtaining photographs of the Haidinger rings, especially those seen in reflection, for which somewhat longer exposures are necessary than in the case of the transmitted system of rings. The photographs reproduced in the two plates illustrating the paper were actually obtained in this way.

5. Optical characters of the interferences

As is to be expected, the transmitted system of rings exhibit less striking contrasts between the maxima and minima of illumination than the interferences observed by reflection. Towards the margin of the bubble, however, the minima of illumination in the rings observed in transmission are seen to become much more conspicuous (plate I, and on the extreme edge of the bubble they become broad and very dark, the maxima, at the same time, becoming bright and narrow lines cutting through them. The reverse is seen to be the case in the reflected system of rings, the minima becoming sharp dark lines cutting across the broad and bright maxima. These effects, of course, are due to the effect of multiple reflections within the film becoming sensible at oblique incidences.

Reference may appropriately be made here to the photographs of soap bubbles between crossed nicols observed in monochromatic light by transmission



(1) (2)

Figures 1 and 2

obtained by one of us and published some years ago in the *Indian J. Phys.* (1929, 4 plate X, facing page 390). Striking differences will be noticed between the effects there reproduced and the photographs of the Haidinger rings in transmitted light which illustrate the present paper. Apart from the appearance of a black cross, it is noticed the rings as seen by transmission between crossed nicols, resemble rather the rings as seen in reflection without them and illustrated in the present paper. The explanation for this will be clear from the theoretical discussion of the results by Prof. K S Krishnan in the paper appearing in the issue of the *Indian J. Phys.* quoted above; indeed Krishnan definitely drew attention to the feature just mentioned.

6. Angular diameters of the rings

We may now refer to the point already indicated in the introduction, namely that the interference figures exhibited by a spherical film of uniform thickness are essentially similar to the well known rings of Haidinger exhibited by a plane-parallel plate. This will be clear on a consideration of the geometric relationship between the configuration of the rings in the two cases.

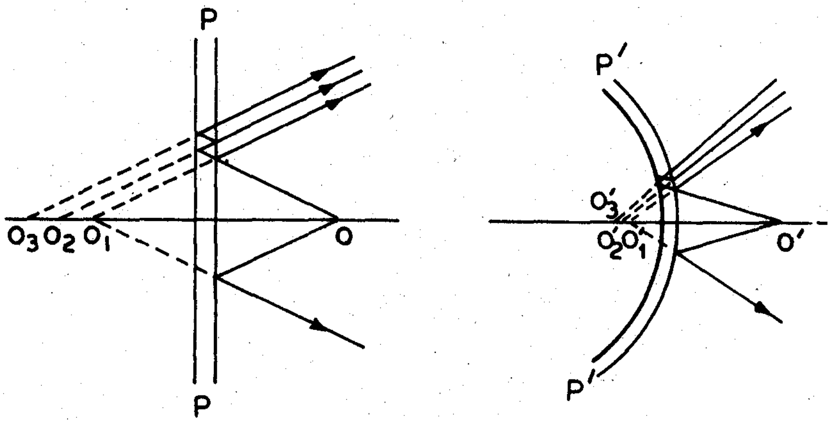
Figures 1 and 2 represent respectively the arrangements (similar in both cases) for observing the interference figures in a plane-parallel plate PP and a curved plate P′P′ of the same thickness. An opaque diffusing screen SS or S′S′ which is illuminated on the side facing PP or P′P′ serves as the source of light. The eye of the observer is placed behind an aperture O or O′ in the screen and views the illuminated surface of the latter as seen reflected at the faces of the plate PP or P′P′. A ray of light from a point R or R′ on the screen reaches the eye after reflection at an angle of incidence i on the surface of the plate PP or P′P′ which is the same in both cases. It will however be seen after such reflection at different angles to the axial ray in the two cases, namely, i in one case, and θ in the other. If a is the radius of curvature of the plate and u is the distance between it and the observer's eye, it can be deduced from the diagram that

$$\theta/i = a/(a + u).$$

In other words, the system of interference rings produced by the curved plate is geometrically similar to that for the plane-parallel plate, but appears reduced in size; the ratio of the angular diameters of the rings in the two cases is the same as the angular diameters of an object at the position of the observer's eye as seen by reflection in the flat and curved plate respectively bear to each other. If the plate is concave instead of being convex towards the eye, we have merely to consider u and a in the formula as of opposite sign, but the relation as stated above remains unaltered. It will be noticed that when u is sufficiently small in relation to a, in other words, when the eye is sufficiently near to them, a curved plate and a plane one exhibit interference figures which are identical. On the other hand when the distance between the eye and the plate is sufficiently great, the angular diameters of the rings for a curved plate are inversely proportional to such distance, in other words, the rings appear as of fixed linear dimensions relatively to the plate. In such a case, also, it makes no difference whether the plate is convex or concave towards the eye of the observer. The Haidinger rings on the front and rear surfaces of a spherical soap bubble appear, therefore, coincident to an observer at a sufficient distance from it. But this would not be the case when the eye is placed near the bubble.

7. Localisation of the rings

As is well known, the Haidinger rings due to a plane-parallel plate may be seen with an extended source of light through a telescope of any aperture focussed for infinity, whereas, as we have seen, the interference figures of a curved plate vary in their size and location with the position of the observer's eye, and accordingly demand a limited aperture of observation, though an extended source of light may be used. It appears, therefore, worth-while to emphasise the essential similarity between the two cases by considering an arrangement in which the interference rings instead of being virtual are received on a screen. This may be done by merely reversing the roles of the aperture and the screen in figures 1 and 2 above. The aperture O is illuminated and the light diverging from it falls on the plate PP or P'P' and after reflection at its surfaces is received on the screen SS or S'S' which becomes the surface on which the interference pattern is actually formed and observed. It will be obvious from figures 1 and 2 that the interference figures of the plane and curved plates seen on the screen would then be completely



(3) (4)

Figures 3 and 4

identical. Further, as the position of the screen SS or S′S′ is quite arbitrary, it is clear that the Haidinger rings are not localised at any particular position in the field but are observable everywhere to the right of the plate PP or P′P′. The reason for this will be seen better from figures 3 and 4 in which O or O′ is a point source of light on one side of the plate PP or P′P′ and O_1, O_2, O_3, etc., or O'_1, O'_2, O'_3 are its virtual images formed by successive reflections at the two surfaces. As these sources are coherent amongst themselves, they must give rise to a system of interference rings symmetric about the line $O\ O_1\ O_2 \cdots$ or $O'\ O'_1\ O'_2 \cdots$ which extends from the surface of the plate to an infinite distance on the right side. The special feature which distinguishes the case of the plane-parallel plate from that of the curved one is the circumstance arising from purely geometrical considerations that in the former case, any movement of the source O either along the line $O\ O_1$ or perpendicular to it leaves the positions of the interference maxima and minima *at infinity* undisturbed.

Summary

By suitable arrangements which are described, it is possible to obtain soap bubbles which are perfectly uniform in thickness and to maintain them in that state. Such bubbles exhibit by transmission or reflection, interference figures consisting of concentric rings which are essentially of the same physical nature as the Haidinger rings due to a plane-parallel plate, besides being geometrically similar to them in configuration. Sixteen photographs are reproduced showing the interference figures of soap bubbles of uniform thickness as also of the changes which occur in them when the films are allowed to drain. The question of the localisation of the interference figures of a curved plate is discussed.