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The diffraction of light by sound waves of high frequency: Part II

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1. Introduction

In the first* of this series of papers, we were concerned with the explanation of the diffraction effects observed when a beam of light traverses a medium filled by sound waves of high frequency. For simplicity, we confined our attention to the case in which a plane beam of light is normally incident on a cell of the medium with rectangular cross-section and travels in a direction strictly perpendicular to the direction along which the sound waves are propagated in the medium. By taking into account the corrugated form of the wave-front on emergence from the cell, the resulting diffraction-effects were evaluated. This treatment will be extended in the present paper to the case in which the light waves travel in a direction inclined at a definite angle to the direction of the propagation of the sound waves. The extension is simple, but it succeeds in a remarkable way in explaining the very striking observations of Debye and Sears† who found a characteristic variation of the intensity of the higher orders of the diffraction spectrum when the angle between the incident beam of light and the plane of the sound waves was gradually altered.

We shall first set out a simple geometrical argument by which the changes in the diffraction phenomenon which occur with increasing obliquity can be inferred from the results already given for the case of the normal incidence. An analytical treatment then follows which confirms the results obtained geometrically.

*C V Raman and N S Nagendra Nath, *Proc. Indian Acad. Sci.* **2**, 406–412 1935.
†P Debye and F W Sears, *Proc. Natl. Acad. Sci.* (*Washington*), **18**, 409, 1932.

2. Elementary geometrical treatment

The following diagrams illustrate the manner in which the amplitude of the corrugation in the emerging wave-front alters as the incidence of light on the planes of the sound waves is gradually changed. In the diagrams, the planes of maximum and minimum density caused by the sound waves at any instant of time are indicated by thick and thin lines (e.g. *AB* and *CD*) respectively. The paths of the light rays are represented by dotted lines in figures 1(b), (c) and (d). As we are mainly interested in the calculation of the phase-changes which the incident wave undergoes before it emerges from the cell, the bending of the light rays within the medium may, in virtue of Fermat's well-known principle, be ignored without a sensible error, *provided* the total depth of the cell is not excessive.

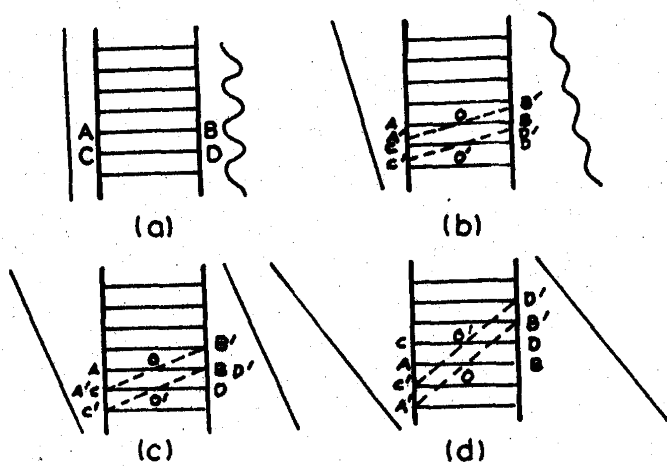

Figure 1

Considering the variation in the refractive index to be simply periodic, the neighbouring light-paths with maximum and minimum optical lengths *AB* and *CD* respectively, in the case of normal incidence, are shown in figure 1(a). The lines *AB* and *CD* are separated by $\lambda^*/2$ where λ^* is the wavelength of the sound waves. The difference between the maximum and the minimum optical lengths gives a measure of the corrugation of the wave-front on emergence. Considering now a case in which the light rays make an angle ϕ with the planes of the sound waves, we may denote the maximum and the minimum optical lengths by $A'B'$ and $C'D'$ respectively. These would be symmetrically situated with respect to *AB* and *CD* and would tend to coincide with them as ϕ is decreased. The optical length of $A'B'$ is *less* than that of *AB*, for the refractive index at any point except at *O* is less than the constant maximum refractive index along *AB*, ϕ being small. On the other hand, the optical length of $C'D'$ is *greater* than that of CD, for the refractive index is minimum along CD. A simple consideration of the above shows that the difference between the optical lengths of $A'B'$ and $C'D'$ is less than that between those of AB and CD. As this difference gives twice the amplitude of the corrugation of the emerging wave-front, it follows, in the case shown in figure 1(b), that the amplitude of the corrugation of the emerging wave-front is less than that in the case of figure 1(a).

Figure 1(c) illustrates a case when the maximum optical length is just equal to the minimum optical length. This occurs when the direction of the incident beam is inclined to the planes of the sound-wave-fronts at an angle α_1 given by $\tan^{-1}(B'B/OB) = \tan^{-1}(\lambda^*/2)/(L/2) = \tan^{-1}(\lambda^*/L)$. That the optical lengths of $A'B'$ and $C'D'$ in figure 1(c) are equal follows by a very simple geometrical consideration. Thus, when light rays are incident on the sound waves at an angle $\tan^{-1}(\lambda^*/L)$, the amplitude of the corrugation of the emerging wave-front vanishes, i.e. a plane incident beam of light remains so when it emerges from the medium. This result would also be true whenever $\alpha_n = \tan^{-1}(n\lambda^*/L)$, $N \neq 0$. The case when $n = 2$ is illustrated in figure 1(d). In all these cases the diffraction effects disappear. As the corrugation vanishes when ϕ is α_{n+1} or α_n, there is an intermediate direction which makes an angle β_n with the sound waves giving the maximum corrugation if light travels along that direction. We can take $\beta_0(=0)$ to represent the case when the incident beam of light is parallel to the sound waves.

Thus, we have deduced that the corrugation of the emerging wave-front is maximum when the direction of light is parallel to the sound waves $[\beta_0(=0)]$, decreases steadily to zero as the inclination ϕ between the incident light and the sound waves is increased to α_1, increases to a smaller maximum as ϕ increases from α_1 to β_1, decreases to zero as ϕ increases from β_1 to α_2, increases to a still smaller maximum as ϕ increases from α_2 to β_2, and so on.

As the variation of the refractive index is simply periodic along the direction normal to the sound-wave-fronts, it follows that the optical length of the light path is also simply periodic along the same direction when the incident light rays are parallel to the sound waves. This means that the corrugation of the emerging wave-front is also simply periodic. When the incident light rays are incident at an angle ϕ to the sound waves, the optical length of the light path would be simply periodic in a direction perpendicular to the light rays. This means that the emerging wave-front would be tilted by the angle ϕ about the line of the propagation of the sound waves and that its corrugation would be simply periodic along the same line.

We have shown in our previous paper that a simply periodic corrugated wave is equivalent to a number of waves travelling in directions which make angles, denoted by θ, with the direction of the incident beam given by

$$\sin\theta = \pm\frac{n\lambda}{\lambda^*} \quad n(\text{an integer}) \geqslant 0 \tag{1}$$

where λ is the wavelength of the incident light. In view of the results obtained in the previous paragraph, the formula (1) would also hold good when the incident light is a small angle with the sound waves.

The relative intensities of the various diffraction spectra which depend on the amplitude of the corrugation should obey a law similar to the one in the case of the normal incidence.

Thus, we find that the results in the case of an oblique incidence would be similar to those of the normal incidence with the amplitude of the corrugation modified. Hence, we deduce, in virtue of the statement I, the following results, assuming the results, in the case of normal incidence, obtained in our earlier paper.

The diffraction spectrum will be most prominent when $\phi = 0$. The intensity of the various components wander when ϕ is increased. When ϕ increases from zero to α_1, the number of the observable orders in practice decreases and when $\phi = \alpha_1$ all the components disappear except the central one which will attain maximum intensity. This does not mean that the intensities of all the orders except the central one decreases to zero monotonically as ϕ varies from zero to α_1, but some of them may attain maxima and minima in their intensities before they attain the zero intensity when $\phi = \alpha_1$. This is obvious in virtue of the property that the intensity of the nth component depends on the square of the Bessel function J_n. As ϕ increases from α_1 to β_1 the intensity of the central component falls and the other orders are reborn one by one. As ϕ increases from β_1 to α_2, the number of observable orders decreases and when $\phi = \alpha_2$ all the orders vanish except the central one which will attain the maximum intensity and so on.

3. Analytical treatment

In the following, we employ the same notation as in our earlier paper. The optical length of a path in the medium parallel to the direction of the incident light making an angle ϕ with the sound waves may be easily calculated. It is

$$\int_0^{L\sec\phi} \mu(s)\,\mathrm{d}s$$

or

$$\mu_0 L\sec\phi - \mu\int_0^{L\sec\phi} \sin b(x - s\sin\phi)\,\mathrm{d}s.$$

Integrating we obtain the integral as

$$\mu_0 L\sec\phi - \frac{\mu}{b\sin\phi}\{\sin(bL\tan\phi)\sin bx + [\cos(bL\tan\phi) - 1]\cos bx\}.$$

The last term can be written as

$$-A\sin bx + B\cos bx$$

where

$$A = \frac{\mu}{b\sin\phi}\sin(bL\tan\phi)$$

$$B = -\frac{\mu}{b\sin\phi}[\cos(bL\tan\phi) - 1].$$

Thus the optical length of the path can be written as

$$\mu_0 L\sec\phi - \sqrt{(A^2+B^2)}\sin b\left(x - \tan^{-1}\frac{B}{A}\right).$$

Ignoring the constant phase factor, the optical length is

$$\mu_0 L\sec\phi - \frac{2\mu}{b\sin\phi}\sin\left(\frac{bL\tan\phi}{2}\right)\sin \mathrm{bx}.$$

If the incident light is

$$\exp\left[2\pi i\nu\left(t - \frac{x\sin\phi}{c}\right)\right]$$

when it arrives at the face of the cell, it will be

$$\exp\left[\frac{2\pi i}{\lambda}\left(ct - x\sin\phi - \int_0^{L\sec\phi}\mu(s)\mathrm{d}s\right)\right]$$

when it arrives at the face from which it emerges.

The amplitude of the corrugated wave at a point on the screen whose join with the origin has its x-direction-cosine l, depends on the evaluation of the diffraction integral

$$\int_{-p/2}^{p/2}\exp\left[\frac{2\pi i}{\lambda}\left\{(l-\sin\phi)x + \frac{2\mu}{b\sin\phi}\sin\left(\frac{bL\tan\phi}{2}\right)\sin bx\right\}\right]\mathrm{d}x.$$

The evaluation of the integral and the discussion of its behaviour with respect to l may be effected in the same way as in our earlier paper. Maxima of the intensity due to the corrugated wave occur in directions making angles, denoted by θ, with the direction of the incident beam when

$$\sin(\theta+\phi) - \sin\phi = \pm\frac{n\lambda}{\lambda^*}\quad n(\text{an integer}) \geqslant 0. \tag{1}$$

The relative intensity of the mth order to the nth order is given by

$$\frac{J_m^2(v)}{J_n^2(v)} \tag{2}$$

where

$$v = \frac{2\pi}{\lambda}\cdot\frac{2\mu}{b\sin\phi}\sin\left(\frac{bL\tan\phi}{2}\right)$$

$$= \frac{2\pi\mu L}{\lambda}\sec\phi\frac{\sin t}{t} \text{ where } t = \frac{bL\tan\phi}{2} = \frac{\pi L\tan\phi}{\lambda^*}.$$

The expression for the relative intensities in our earlier paper can be obtained from (2) by making $\phi \to 0$ when $v \to (2\pi\mu L/\lambda) = v_0$. So the expression for the relative intensities

$$J_m^2(v_0)/J_n^2(v_0) \tag{3}$$

in the case of normal incidence will change to

$$J_m^2(v)/J_n^2(v)$$

where

$$v = v_0 \sec\phi\frac{\sin t}{t} \tag{4}$$

and

$$t = \frac{\pi L\tan\phi}{\lambda^*}.$$

Even if ϕ be small so that $\sin\phi \approx \tan\phi \approx \phi$, it is *not* justifiable to write $\sin t \approx t$ *unless* $\pi L\phi/\lambda^*$ is also small to admit the approximation. As $\pi L/\lambda^*$ is sufficiently large we should expect great changes in the diffraction phenomenon even if ϕ be a fraction of a degree. v vanishes when

$$t = n\pi \quad n(\text{an integer}) > 0,$$

that is, when $L\tan\phi = n\lambda^*$,

or

$$\phi = \tan^{-1}\frac{n\lambda^*}{L}, \; n(\text{an integer}) > 0,$$

confirming the same result obtained geometrically. Whenever v vanishes, it can be seen that the amplitude of the corrugation of the wave-front also vanishes. The statement I in section 2 and the consequences with regard to the behaviour of the intensity among the various orders can all be confirmed by the expression (3).

In the numerical case when $L = 1$ cm, and $\lambda^* = 0{\cdot}01$ cm, the amplitude of the corrugation vanishes $\tan\alpha_1 = 0{\cdot}01$ or $\alpha_1 = 0°\,34'$. This means that as ϕ varies from 0° to 0° 34′, the relative intensities of the various orders wander according to (2) till when $\phi = 0°$ 34′, all the orders disappear except the central one which attains maximum intensity. This does not mean that the intensities of all the orders except the central one decrease monotonically to zero but they *may possess* several maxima and minima before they become zero. The intensity of the nth order depends on the behaviour of $J_n^2\,[v_0 \sec\phi\,(\sin(\pi L \tan\phi/\lambda^*)/(\pi L \tan\phi/\lambda^*))]$ under the above numerical conditions as ϕ varies from 0° to 0° 34′. As ϕ just exceeds 0° 34′, all the orders are reborn one by one till a definite value of ϕ after which they again fall one by one and when $\phi = 1°\ 8'$, all the orders disappear except the central one.

The numerical example in the above paragraph shows the delicacy of the diffraction phenomenon. If the wavelength is quite small, the diffraction phenomenon will be present in the case of the strictly normal *incidence* as the relative intensity expression (3) does not depend on λ^* but will soon considerably change even for slight variations of ϕ as the relative intensity expression (4) depends on λ^*. One should be very careful in carrying out the intensity measurements in the case of normal incidence, for even an error of a few minutes of arc in the incidence will affect the intensities of the various orders.

4. Comparison with the experimental results of Debye and Sears

Debye and Sears make the following statement in their paper: "Fixing the attention on one of the spectra *preferably of higher order*, one can observe that it attains its maximum intensity if the trough is turned through a small angle such that the primary rays are no longer parallel to the planes of the supersonic waves. Different settings are required to obtain highest intensities in different orders. If the trough is turned continuously in one direction, starting from a position which gave the highest intensity to one of the orders, the intensity decreases steadily, goes through zero, increases to a value much smaller than the first maximum, decreases to zero a second time and goes up and down again through a still smaller maximum." This statement very aptly describes the behaviour of the function

$$J_n^2\left[v_0 \sec\phi \frac{\sin(\pi L \tan\phi/\lambda^*)}{(\pi L \tan\phi/\lambda^*)}\right]$$

as ϕ alters under the conditions imposed in the above statement. The zeroes and the maxima of the intensity of the nth order, as a function of ϕ, correspond to the zeroes and the maxima of the above function.

5. Summary

The theory of the diffraction of light by sound waves of high frequency developed in our earlier paper is extended to the case when the light beam is incident at an angle to the sound wave-fronts, both from a geometrical point of view and an analytical one. It is found that the maxima of intensity of the diffracted light occur in directions which make definite angles, denoted by θ, with the direction of the incident light given by

$$\sin(\theta+\phi)-\sin\phi=\pm\frac{n\lambda}{\lambda^*},\quad n(\text{an integer})\geqslant 0$$

where λ and λ^* are the wavelengths of the incident light and the sound waves in the medium. The relative intensity of the mth order to the nth order is given by

$$J_m^2\left(v_0\sec\phi\frac{\sin t}{t}\right)\Big/J_n^2\left(v_0\sec\phi\frac{\sin t}{t}\right)$$

where $v_0=(2\pi\mu L/\lambda)$, $t=(\pi L\tan\phi/\lambda^*)$, ϕ is the inclination of the incident beam of light to the sound waves, μ is the maximum variation of the refractive index in the medium when the sound waves are present and $L\sec\phi$ is the distance of the light path in the medium. These results explain the variations of the intensity among the various orders noticed by Debye and Sears for variations of ϕ in a very gratifying manner.