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The dynamical theory of the motion of bowed strings

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(Plates I–III)

SUMMARY OF CONTENTS

Prefatory note

Our knowledge of the motion of bowed strings derived from the researches of Helmholtz and his successors is mainly of a kinematical character. The present investigation was prompted by a desire to attain a fuller understanding and appreciation of the distinctive *dynamical* as distinguished from the purely geometrical features of the problem. The attack can evidently be approached from two sides. One is the purely *a priori*, abstract, theoretical point of view. The other procedure is to seek out fresh experimental data on which a superstructure of dynamical theory can be built up. The paper now presented is of a preliminary character and deals principally with certain points which do not appear to have been generally recognized or emphasized. Incidentally a new kinematical method of recording the vibrations of the entire length of a bowed string in one photograph is developed and sixteen pictures obtained by this method are presented (plate III). Methods of projecting vibration-microscope figures with the lantern and of photographing them are described towards the end of the paper. Some photographs of these figures are presented in plate II.

The *modus operandi* of the violin bow

Obviously in any dynamical theory, the exact relation between the motion of the bow and that of the bowed point is of the very first importance. Helmholtz, referring to the particular case in which the motion of the bowed point is represented by the two-step zig-zags discussed by him, remarks (*vide Sensations of Tone*, English Translation, page 83) that the velocity in the forward motion "appears" to be equal to that of the bow. Later treatises state that the velocity of the forward motion is probably equal or about equal to that of the bow (e.g. Lord Rayleigh's *Theory of Sound*, I, and Barton's Text-Book of Sound, article 261). It is evident that an exact and convincing demonstration of this equality of velocities has not so far been given, and such a proof is obviously of importance, for, in a very large number of cases (*vide* researches of Helmholtz and those of Krigar-Menzel and Raps, Sitzungberichte of the Berlin Academy, 1891) the vibration-curve of the bowed point is exactly represented by a two-step zig-zag, the ratio of the forward and backward velocities bearing some relation to the position of the bowed point on the string.

The following method has been devised by me for obtaining simultaneous photographic records of the motion of the bow and of the bowed point in contact with it. A fairly long string is chosen for the experiment and a narrow slit cut in a sheet of metal is set across and immediately behind the string. The positive crater of a small arc placed in front of the string provides the necessary illumination. The string is bowed at a point as close as possible to the position of the illuminated slit. A pin is fixed transversely at the centre of the bow. A photographic plate contained in a dark slide is arranged to slide in grooves parallel to the string, and the movement is timed so that the shadow of the pin fixed to the bow passes across the illuminated slit when the photograph is being secured. We thus obtain a simultaneous record of the motion of the bow and of the vibrations of the bowed point on the string. When the apparatus has been correctly adjusted, we find that the record of the bow is absolutely parallel to that of the forward motion of the string. In some cases the two records become perfectly *coincident* (this, of course, is mere chance) and the picture becomes a most convincing demonstration of the equality of velocities. Three such records are shown in figures 1, 2, and 3, plate I.

Working by this method, we arrive at the generalization that in every case in which the motion of the bowed point is a two-step zig-zag, the velocity of the forward motion is *accurately* equal to that of the bow. This fact has obviously a bearing on the dynamical theory of the maintenance of the periodic motion.

It is a somewhat remarkable fact that the current treatises in describing the form of a bowed string at any instant during its motion (even in the simplest cases), overlook the effect of the friction of the bow on the configuration of the stretched string and thus convey a somewhat erroneous idea of the *modus operandi* of the bow. The first effect of the bow must evidently be to cause a deflection of the string in the direction of its movement into two lines meeting more or less sharply at the point of application, and a little consideration will show that the vibrations started and maintained by the bow may to some extent modify but cannot abolish this primary effect. Assuming, for the sake of clearness in ideas, that the region of application of the bow is confined to an extremely short

Figure 1

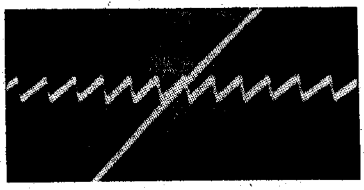

Figure 2

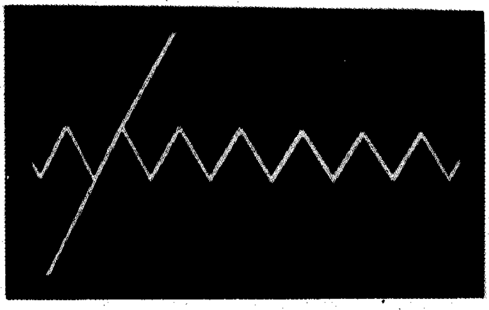

Figure 3. Simultaneous records of motion of a voilin bow and of the bowed point.

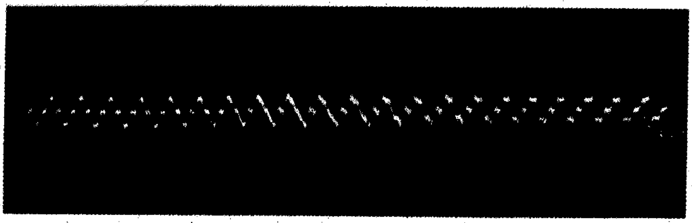

Figure 4. Record from moving slit apparatus (see plate II).

Plate I

length of the string, and that θ is the angle between the two lengths of string or either side of it, it follows (with a reservation) that the value of θ at any instant during the vibration is given by the equation,

$$\text{Frictional force acting on string} = \text{Tension} \times \sin(\pi - \theta).$$

The reservation is that we must exclude the instants at which θ suffers an infinitely sudden increase or decrease followed by an infinitely sudden change in the opposite direction owing to the passage of a crest of the wave through the bowed point. On account of the (ordinarily) very short duration of these 'discontinuous' changes and the fact that the forces acting on the string are finite, no appreciable energy passes into the string at these instants. If the value of θ is taken as constant

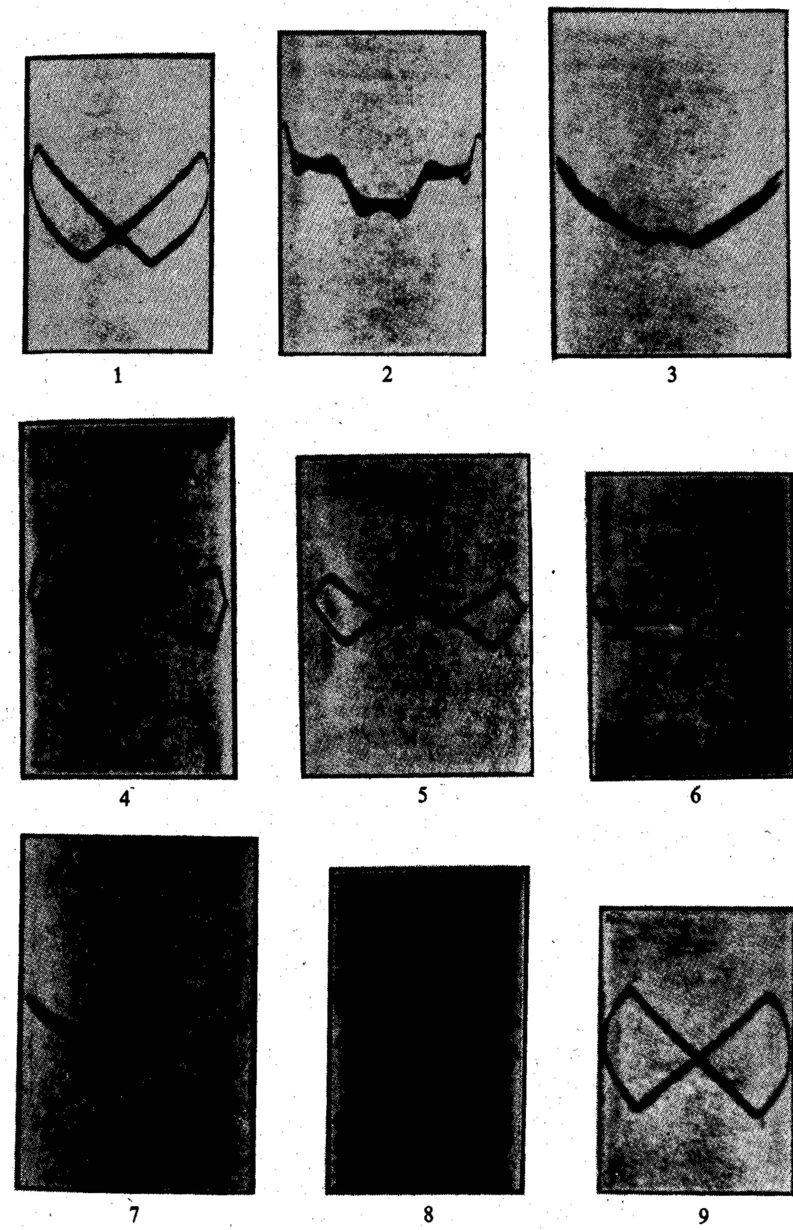

Plate II. Some photographs of vibration-microscope figures of bowed strings.

throughout the rest of the motion, we arrive at the result that there is apparently nothing to enable the vibrations of the string to be maintained in the presence of dissipative forces. In other words, we find that the expression for the motion of the string given by Helmholtz's formulae are quite incapable of explaining the maintenance of the motion. The inference to be drawn is that the kinematical expressions for the motion are incomplete, and certain additional dynamical terms must be added to enable an accurate idea of the motion to be obtained. These dynamical terms involve a certain periodic variation of the angle θ.

From purely *á priori* considerations, we thus arrive at the following conclusions: the form of the bowed string at any instant in the simplest case considered by Helmholtz is in general that of *three* straight lines meeting at sharp angles (not two, as is generally supposed). Further, two of these straight lines always meet at the bowed point, and the angle between them suffers a small periodic variation which is distinct from the sudden increase followed by the sudden decrease caused by the passage of a 'crest' of the wave through the bowed joint. This small periodic variation is necessary to enable the maintenance of the motion to be explained and involves a modification of the usual kinematical expression for the motion at all points on the string, small no doubt, but theoretically of great importance. The most promising method of verifying the above-stated conclusions would appear to be to obtain a series of shadow photographs of the form of the string, at and near the bowed point, on a falling plate under periodic illumination secured by electric sparks of short duration, following one another at intervals of a tenth or a twelfth of the period of vibration of the string. Experiments by this method are under contemplation.

Another line of reasoning leads to conclusions precisely similar to those stated above. It is known that by bowing in a suitable manner near the end of the string, or better still near its middle point, it is possible to elicit the octave of the string without any admixture of the fundamental motion. In this case, the centre is a node, and to admit of the passage of the energy requisite for the maintenance of the vibrations of the second half of the string there must be a small motion at the node, the phase of which for each periodic component differs from that of the large motion elsewhere by quarter of an oscillation.* The kinematically deduced expressions obviously fail to account for the existence of this small motion and must therefore be modified accordingly. This small motion appears to be distinctly present and is shown in record no. 5 in plate III, obtained by a method to be described below.

The preceding discussion when carried a step further enables us to deduce from purely *á priori* considerations some of the possible forms of vibration of bowed strings. It is obvious from dynamical considerations that in steady motion the velocity of the bowed point at any instant in the direction of bowing cannot

*See "The Small Motion at the Nodes of a Vibrating String," section II, Bulletin no. 6.

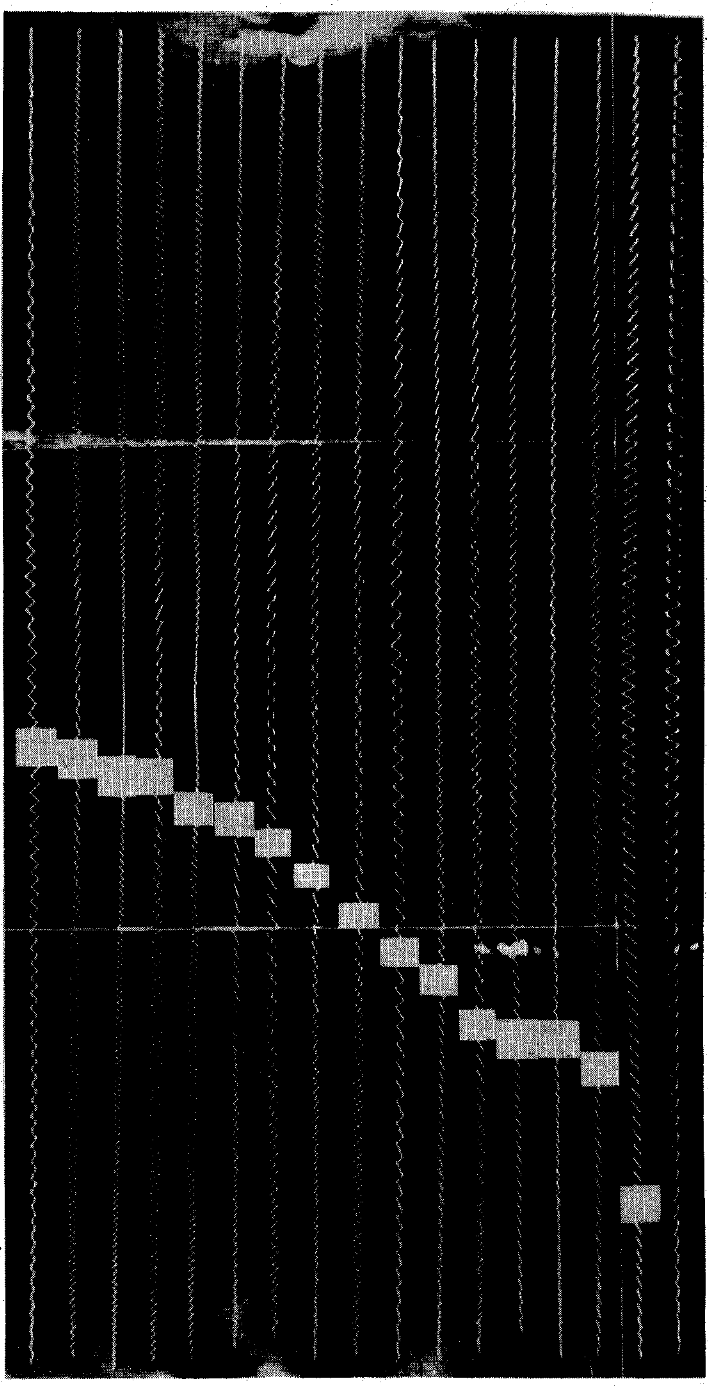

Plate III

normally exceed that of the bow itself. For, if it did, the direction of the frictional force would be reversed and the extra velocity would be quickly damped down. Again, as we have already seen, the maintenance of the motion depends upon a periodic variation of the frictional force acting on the string, and if, as is generally the case, the damping of the vibrations is very small, the motion approximates more and more closely to a type in which there is no damping, and no maintenance, in other words to a free oscillation subject to no periodic forces. The string possesses an infinite number of degrees of freedom, and it follows that, provided the pressure and velocity of bowing lie within certain limits, it would normally adjust itself as closely as possible to a type of motion in which the frictional force due to the bow is constant. Since this friction is a function of the relative velocity, periodic terms can be absent from it only when the forward and backward velocities of the bowed point are each of them constant, and the forward velocity is equal to that of the bow. Such a type of motion in the case of a stretched string is possible, provided the ratio of the forward and backward velocities is such that the motion, when analysed and expressed in a Fourier series, does not contain any harmonic, a node of which coincides exactly with the point of bowing. For example, if the node is that of the nth harmonic, the ratios of the forward and backward velocities may be one of the ratios $r/n - r$ where r is an integer less than n. The particular ratio which obtains depends of course on the pressure and velocity of bowing. These inferences are generally in accord with the results of experiment.

A new kinematical method of recording the entire motion of a bowed string

It is well known that a bowed string is capable of many complicated types of vibration. To convey a really accurate and effective idea of the motion of the string in any of these cases, it would be necessary to observe or photograph the vibration-curves of many individual points on the string, care being taken, when each photograph is secured, that the string is bowed precisely at the same point and as nearly as possible with the same pressure and velocity. When this is done, it would be necessary also to indicate the place of bowing and the point of observation against each photograph, and if possible to arrange these in some kind of geometrical order, so that the eye can realize the configuration of the string as a whole. Now it would obviously be a great gain, if the whole business could be managed in one operation. It is well known that, after a little practice, it is much easier to maintain a nearly uniform style and place of bowing for, say half a second, than to reproduce these correctly 15 or 20 times in several successive efforts. It will also be conceded that 80 or 90 vibration-curves of successive points 'stringed' together in one photograph will convey to the eye far more of the finer effects of the gradation of the motion from point to point, than 5, 10 or even 15 separate photographs could. This result I have succeeded in securing and 16 records are presented in plate III.

The experimental procedure is very simple indeed. A fine violin string is stretched on low bridges mounted on a hollow-box. The top of the box between the two bridges is open and fitted with grooves inside which a long strip of wood about 2 inches wide slides very smoothly parallel to the string. When the apparatus is not at work, the strip completely covers the opening, so that the box is light-tight. An aperture in the strip is fitted with a fine slit transverse to the string. A small brilliant arc placed at some distance in front of the box provides the source of illumination. An arrangement is made by which a length of sensitive film or bromide paper is held up just behind the sliding cover. The string is bowed steadily, and the illuminated slit with the shadow of the string across it is caused to slide with uniform velocity from end to end of the string, and thus to record the motion of the latter on the sensitive surface.

The necessary uniform movement of the slide carrying the slit is secured by the aid of a small fly-wheel and a string wrapped round its axle. A little string is left slack at first while the fly-wheel is set into rotation, and the string tightens and moves the slide forward with uniform velocity. When the slit has passed over the whole of the string, the slide comes up against a stop and the connecting string snaps off.

The place of bowing is indicated in the photographs by the shadow of the bow crossing the string. The vibration-curves come right up to this short length on either side, and the nature of the motion at the bowed point itself is thus sufficiently indicated. It will be seen that in plate III, each record consists of 90 to 100 vibration-curves or 'waves' continuously stringed together. The length of each of these waves is the distance through which the slit travelled in one complete period of oscillation of the string, and they represent with perfect fidelity the time-displacement diagrams of the points on the string at which they respectively appear. For, the length of the 'wave' is a very small fraction of the length of the string, in fact much smaller than the length of the ventral segments of all harmonics which have sensible amplitudes, i.e. say up to the 10th or 11th; and further, discontinuous changes of velocity which involve harmonics of very high orders are also reproduced in the curve as discontinuous changes in direction.

In other words, these curves are for all practical purposes precisely similar to those obtained for individual points on the string by the method of a stationary slit and moving photographic plate. Each 'wave' differs from those on either side of it by almost imperceptible gradations (see for instance a section reproduced as figure 4, plate I), and thus, an accurate picture of the vibration of all points on the string is presented to the eye.

This method, of course, is suitable for the photographic record of all cases of the vibrations of strings or thin rods, provided these oscillations are maintained steadily during the interval the illuminated slit takes to pass over from end to end of the string or thin rod, or over any desired section of it. In the case of damped oscillations, e.g. those produced by striking or plucking the method is not so effective. Even here, however, some use can be made of the method, and to illustrate this, I have included the case of a plucked string (Record no. 17).

In the case of bowed strings, when the bowing has been fairly steady, the whole picture, in addition to conveying the vibration-form for individual points on the string, also indicates the configuration of the string as a whole, i.e. shows the relative amplitudes of the motion at various sections, and also the deflexion of the string due to the friction of the bow, which I have referred to previously. Further, the curves when held nearly in a line with the eye, show very well the division of the string into alternately brighter and darker strips parallel to its length over various sections of it, which are so readily observed by the unassisted eye when a bowed string is seen under good illumination. The 'bright' strips are seen over the regions where, for a short distance, points on the string reverse their direction of motion and retrace their paths, and the 'dark' strips where the points move down with increased velocities.

The projection and photography of vibration-microscope figures

Helmholtz, as is well known, deduced the vibration-forms of bowed strings in some of the simple cases by the very beautiful, if a trifle complicated, method of the vibration-microscope. The figures seen in this instrument possess great fascination and interest to the experimenter on account, partly, of the variety and the beauty of the forms which they may assume, and also on account of the peculiar, almost stereoscopic effect produced by the slow rotation of the figures under slightly imperfect tuning. In the form devised by Helmholtz, however, the instrument can only be used by one observer at a time, and as a monocular microscope is employed, there is some discomfort in its use which renders the observation of these figures less attractive than it otherwise might be. In fact, the number of those who have seen even the simplest of these curves for themselves, i.e. elsewhere than in text-book diagrams, is probably not very large, and the interest of these figures does not appear to be very generally realized.

There are some difficulties in obtaining satisfactory projections of these figures on the screen, which I have successfully overcome. For use with an ordinary lantern, even in one in which the electric arc is employed, Helmholtz's original method is obviously quite unsuitable, as the amount of light available is altogether insufficient. The following arrangement is a very suitable one. The condenser of the lantern is covered by a cap carrying an adjustable slit with vertical jaws. A steel wire about 80 cm long is employed as the 'string' to be bowed, and this is stretched horizontally close to the cap covering the condenser of the lantern, so as to bisect the slit. An electrically-maintained interruptor-fork of frequency, say 60, is used. This is held vertically, and upon one of the prongs of this, a small achromatic lens of short focal length, say about 7·5 cm, is fixed on with wax, and the other prong is loaded to balance the masses. This lens forms an enlarged image of the slit with the shadow of the string across it, on a distant screen, or on the ground-glass of a camera (with the lens removed) placed in front of it. The principal difficulty which remains to be overcome is that with the low frequency of the fork employed, the diameter of the wire used cannot be made very small, and the magnified image of its shadow across the slit is excessively broad. This cannot be conveniently avoided by the use of fine wires and a high frequency fork, as the amplitudes of vibration then become too small for successful projection. The device finally adopted by me is to flatten a very short length of the wire just opposite the illuminated slit, with a hammer, so as to have the wire at this point practically a very thin flat ribbon seen edgewise and therefore giving a very sharp and narrow shadow on the screen. As the linear density of the wire is not even locally altered by this treatment, the vibrational modes are quite unaffected, and it becomes possible to obtain satisfactory curves on the screen by bowing the string and setting the fork in vibration.

An extraordinary and most interesting variety of curves can be demonstrated to a large audience in this way, and the gradual rotation of the curve due to slight imperfections in the tuning has a very striking and almost stereoscopic effect.

To obtain photographic records of these curves, the arrangement described above is adopted. The string is so tuned that when it is bowed, the curve seen on the ground-glass of the camera is practically steady, and a picture is secured with an exposure of only $\frac{1}{25}$th of a second. With such an exposure, the effect of any imperfection in the tuning on the sharpness of the figure photographed is quite negligible. The available light is so great that, even with this short exposure, it is generally found necessary to use a neutral-tinted screen in front of the shutter of the camera to prevent over-exposure of the plate.

Plate II shows nine pictures secured in the preliminary work. In each case, it will be seen that the frequency of the fundamental vibration of the string employed was double that of the fork employed, and some of the curves are evidently identical with those observed and drawn by Helmholtz. I hope later to find time to secure and publish a more extensive series of curves, grouped according to the places of bowing and observation and with various ratios of frequency between the vibrations of the bowed string and that of the interruptor-fork.

In concluding this Bulletin, I have great pleasure in acknowledging the valuable assitance I have received from Mr Asutosh Dey, the senior demonstrator of the Association Laboratory, in planning and carrying out the experiments described above. I must also express my gratitude to Dr Amrita Lal Sircar who, as Honorary Secretary, put the resources of the Laboratory unreservedly at my disposal during hours at which few institutions, if any, would remain open for work, and who was also unfailing with his personal encouragement and advice.