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Some acoustical observations

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Double sources of sound

[Plate I]

Lord Rayleigh has shown (*Scientific Works*, **1**, page 468, and *Theory of Sound*, article 65, *a*) that by rendering an otherwise steady sound intermittent by the periodic interposition of an obstacle, we do not by any means succeed in reproducing the acoustical or dynamical effect of true 'beats'. One essential point of difference is that in true 'beats,' the amplitude of the vibration changes sign every time that it passes through zero, in other words the phase of the vibration is periodically reversed. In the case, however, of an intermittent note, the vibration is resumed after each silence without reversal of phase. A simple analysis shows that as a consequence of this and other differences, we have *three criteria* by which we could or might distinguish in experiment between real 'beats' and an intermittent note, the latter being, after all, only an attempt to simulate the former by purely mechanical methods. Firstly, an intermittent note is resolvable into at least *three* distinct component tones, the 'primary' being always present. In true 'beats' we have only two components, and there is no primary. Secondly, the difference in frequency between the two new secondary tones besides the primary perceivable in the former is *twice* that of the frequency of intermittence, whereas in the latter, the difference of frequency between the components is *equal* to that of the beats. Lastly, the law of variation of amplitude in an intermittent note is arbitrary, and we may have 5 or 7 components instead of 3 as in the simplest case.

These points of difference are very beautifully illustrated in some recent experiments in which I endeavoured to analyse by the aid of resonators, the acoustical effect due to the rotation of 'double' source of sound about a line passing through it perpendicular to its axis. A 'double' source of sound is, in effect, a combination of two equal but opposite sources of identical frequency situated indefinitely near each other, the product of their intensity into their distance being a finite quantity. A 'double' source emits no sound in its equatorial plane, i.e. in any direction at right angles to its axis. It is therefore readily seen that if a double source is rotated with uniform angular velocity about any line passing through it and lying in its equatorial plane, an observer situated at any distance from the source would perceive the effect of 'beats' simulated in a most remarkable manner. At each half-revolution, as the equatorial plane of the source sweeps through his position, there would be an extinction of the sound followed during the intervals by a recrudescence, and the frequency of the 'beats' is therefore twice the number of rotations made per second.

It is not at all a difficult matter to secure in experiment what is very approximately equivalent to a 'double' source of sound. It is known (Rayleigh's *Theory of Sound*, article 325) that the disturbance due to the vibration of a sphere as a rigid body is the same as that corresponding to a double source at the centre whose axis coincides with the line of the sphere's vibration. In practice, the flexural vibrations of a small flat disc or plate would be a very fair substitute, provided that one of the vibrating segments of the plate emits a sufficiently powerful and steady sound which is not sensibly interfered with by any effect of the other segments. With a circular plate, for example, the mode of vibration with one nodal circle would be the most suitable for the purpose, and this would be improved, if the thickness of the disc varied in such manner from the centre outwards that the nodal circle is rather nearer the edge of the disc than the theoretical position for a disc of uniform thickness. I have found that the gong of bell-metal shown in plate I serves most admirably as a double source of sound. When struck with a wooden hammer, its vibrations in the gravest mode are powerfully excited and a clear steady note is emitted which continues to be audible for over a minute, free from any 'beats' or quavers (so long as the plate is held in position) and free from perceptible admixture with upper partials, these latter ceasing to be audible within two or three seconds of the impact of the hammer. This very desirable result appears to be a consequence of the variation

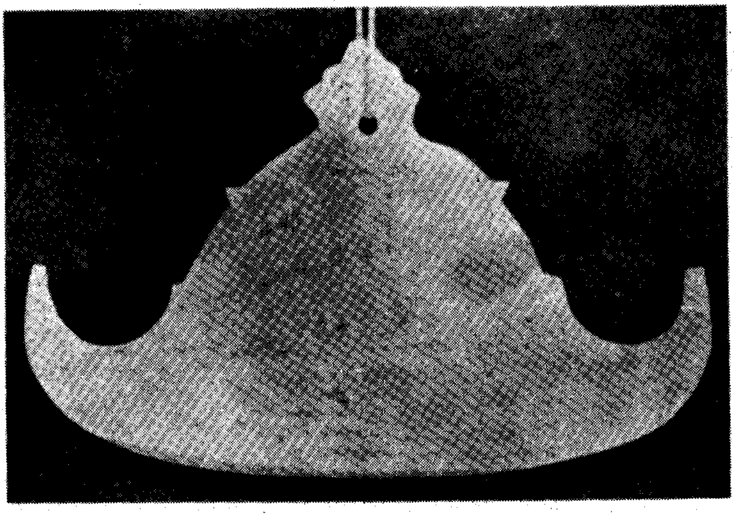
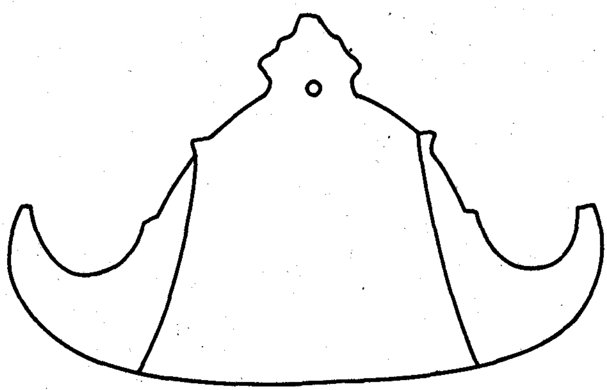


Plate I. Vibrating plate used as a double source of sound.

of the thickness of the plate, which is least at the centre of the horizontal edge of the plate and gradually increases in all directions upwards. The 'wings' at the end of the base of the plate are particularly massive and appear to be both ornamental and useful, as can be seen from the position of the nodal lines shown in the figure. The emission of sound is principally due to the central segment of the plate, the two wings having very little effect as could be shown by covering up the former when the bulk of the sound falls off.



When suspended by a fairly long thin cord and struck on one of the wings, the gong is excited and can at the same time be set in rotation at any moderate speed. The effect of 'beats' is then reproduced in a most remarkable manner. The analysis of these beats was effected in the following manner by the aid of a tuning-fork mounted on a resonance box. The frequency of the fork was 512, and was a little greater than that of the vibrations of the plate. By loading the prongs of the fork suitably with wax, it was found possible to get the two to be of identical frequency or to have the frequency of the fork slightly in excess or defect as desired. For the analysis of the beats due to the rotation, the fork mounted upon its box was itself used as a resonator.

Experiment 1
The plate was excited and held in position. The fork was adjusted for exact unison, and held a few feet off. On stopping the vibration of the plate the fork was found to be sounding loudly.

Experiment 2
As above, but with the plate in rotation. There was no resonance.

Experiments 3 *and* 4
The fork was thrown out of unison in one direction or the other by putting on or

removing a little wax, so that when sounded it made approximately *one* beat per second with the gong. On repeating experiment 1, there was no resonance.

Experiments 5 *and* 6

As in experiments 3 and 4 but the plate in rotation at such a speed that *two* beats per second were heard. There was marked resonance.

These observations are of interest as illustrating the mechanical production and analysis of 'true' beats. At each half-revolution of the plate we get an extinction of the sound, followed by a revival in which the phase of the motion is reversed. This can readily be understood from the fact that the motion on the two sides of the plate is in any instant in opposite phases, a compression on one side and a rarefaction on the other and vice versa.

Some reciprocal effects

On adjusting the fork and the vibrating plate used in the preceding experiments to *exact* unison and sounding the former, very marked resonance was exhibited by the plate. The arrangement was found to be very sensitive, a deviation from unison giving only one beat per second practically abolishing the resonance. This observation suggested the following experiments which appear to be of some interest as illustrations of the dynamical theory of resonance.

Experiment 7

Fork and plate in exact unison. Holding the fork mounted on its box a few feet from and *facing* the plate, it was found on exciting the fork that the plate showed marked resonance.

Experiment 8

As above, but with the fork and resonance box in *the plane* of the plate. Resonance extremely feeble.

Experiment 9

As in experiment 7, but with the plate in rotation. No resonance.

Experiments 10 *and* 11

As in the experiment 7 with the plate held in position, but the fork thrown slightly out of unison in one direction or the other (to the extent of one beat per second) by putting on or removing a little wax. No resonance.

Experiments 12 *and* 12(*a*)
As in experiments 10 and 11, but with the plate in rotation at a speed of one revolution per second in either case. Marked resonance was noticed.

Mathematical note

The equivalence of the effect at any point due to a rotating double source and that due to the beats of two simple tones of differing frequencies can easily be shown theoretically. Thus:

The velocity-potential of a double source is proportional to the real part in

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{\exp(ik(at-r))}{r}\right]=\mu\frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{\exp(ik(at-r))}{r}\right]=-ik\mu\frac{\exp(ik(at-r))}{r}\left[1+\frac{1}{ikr}\right]$$

where x is the axis and μ is the cosine of the angle between x and r. Putting $\mu=\cos\theta=\cos\omega t$ where ω is the angular velocity of rotation of the double source; the expression given above may be written as

$$-ik\exp(-ikr)\frac{\exp(i(ka+\omega)t)+\exp(i(ka-\omega)t)}{2r}\left[1+\frac{1}{ikr}\right].$$

From this, the inference is obvious.