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On Brewster's bands—Part I

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Abstract. The paper considers the explanation of Brewster's bands and other allied phenomena from the new and very suggestive standpoint proposed by Schuster (*Philos. Mag.* Oct. 1924). When monochromatic light is reflected by or transmitted through two parallel plates in succession, we have a superposition of the Haidinger ring-systems due to the two plates in the sense that the observed intensity in any given direction is the product of the intensities due to either plate separately. Illustrations showing the effect of such superposition in various cases are reproduced with the paper, differential and summational fringe-systems of various orders being observable. When non-homogeneous light is used, the Haidinger rings disappear and along with them also the superposition pattern, leaving only a uniform illumination in the field, except in the special case of the differential system of the first order for two plates of equal thickness. A simple geometrical explanation is thus forthcoming why Brewster's bands can be observed even in non homogeneous light with thick plates in this case.

1. Introduction

As is well known, coloured interference bands are observed when an extended source of light is viewed in transmission through or by reflection from two glass plates in succession when these are of equal thickness and are held at a suitable small inclination with each other. These bands, first observed by Sir David Brewster, form the basis of the very convenient interference refractometer developed by Jamin, and their explanation is therefore a matter of considerable interest and importance. In a recent and very interesting paper*, Sir Arthur Schuster has proposed a way of regarding the effects observed in this case which is different from that usually given. If we use an extended source of highly monochromatic light, and view it by transmission through or a reflection from a single plate, we should get the well known Haidinger rings, or interference-curves of constant obliquity representing the variation of the transmitting or reflecting power—as the case may be—of the plate for light of a specific wavelength in

*Schuster, *Philos. Mag.* **48** (Oct. 1924) 609–619.

different directions. If two plates are used in combination, Schuster suggests that we may regard each of them as producing its own set of Haidinger rings, but that these are superposed in the sense that the observed brightness in any specific direction is determined by multiplying the fractions of the intensity of the original source transmitted—or reflected, as the case may be—by the two plates in succession. For, if we consider a wave-front travelling out from the source in a specific direction, the two plates act on it in succession, and to find their resultant effect, we have only to multiply their transmitting or reflecting powers for the specific angles of incidence concerned. The joint effect of the two plates is thus merely to superpose their interference-patterns in the sense indicated; when the plates are inclined to each other, the centres of the two ring-systems would of course not be coincident. In his paper, Schuster has endeavoured to trace the geometric character of the complex pattern formed by the two non-concentric sets of rings.

While the treatment outlined by Schuster is of undoubted elegance and simplicity, it must be pointed out that his paper does not actually offer an explanation of Brewster's bands. For, as is well known, it is not necessary to use highly monochromatic light to observe these bands, and they may be seen even with white light and thick plates. Under these conditions, the Haidinger rings due to a parallel plate cannot be observed at all, and it is by no means obvious without explanation why, when each plate separately gives merely uniform illumination, their superposition should give any observable fluctuations of intensity in the field. To furnish an explanation of this and to extend the theory from the new standpoint in other directions is the purpose of the present paper. In part I we shall consider the geometric character of the patterns formed by the superposition of non-concentric ring-systems and explain why Brewster's bands can be observed in non-homogeneous light provided the plates are of equal thickness. In part II we shall consider the explanation of the general (elliptic) form of Brewster's bands observed in monochromatic light with plates inclined at any angle. In part III the character of Brewster's bands as observed with plates of doubly refracting crystals will be discussed.

2. The geometric character of the superposed patterns

To illustrate the effect of multiplying the distribution of light-intensity in two sets of rings when their centres are not coincident, we have prepared the prints reproduced in figures 1 to 6. Of these, figures 1 to 4 represent the superposition of two *identical* ring-systems with gradually increasing distance between their centres. Figures 5 and 6 represent the effect of superposing two *dissimilar* ring-systems, the centres of which are moderately removed from each other in figure 5, and still farther apart in figure 6. The figures were prepared by the very simple expedient of printing twice on bromide paper from negatives on which alternate bright and dark rings were spaced in the manner of a Fresnel zone-plate. The same negative was used twice with a suitable displacement for preparing figures 1, 2, 3 and 4. For preparing figures 5 and 6, two different negatives with ring-systems of unequal size were successively printed on the same sheet of bromide paper.

The law of spacing of the Haidinger rings in a parallel plate of refractive index μ and thickness t is $2\,\mu t \cos r = n\lambda$, where r is the angle of refraction within the plate. If i be the angle of emergence from the plate, the law may be written in the form

$$2t\sqrt{\mu^2 - \sin^2 i} = n\lambda.$$

For moderately small values of i, the law of spacing of the rings is very similar to that on a Fresnel zone-plate. Hence figures 1 to 4 may be regarded as correctly representing the effect of superposing the Haidinger ring-systems of two plates of *equal* thickness moderately inclined to each other, while figures 5 and 6 may be regarded as doing the same for two plates of *unequal* thickness.

The characteristic feature shown in figures 1 to 3 is the system of rectilinear equidistant bands running transverse to the line joining the centres of the ring-systems. These bands are very broad in figure 1, corresponding to a small inclination of the plates, and become closer and closer together with increasing inclination of the plates until, as in figure 4, they are so close together as to be lost to the eye in the general structure of the field. At the same time, other features appear in the superposition pattern, viz., the two external circular fringe-systems shown in figure 3, and the internal circular fringe-system shown in figure 4. The significance of these other features we shall consider presently. A noteworthy feature of the rectilinear bands is that the central bright one passing through the point midway between the centres of the two systems is always fixed in position and orientation.

We have now to consider the manner in which the superposition patterns are formed. Taking first the case in which the two sets of rings are identical in size, we may, following Schuster, take the centre of each system to be a point of maximum intensity; the radii of the rings having maximum intensity are (approximately) in the ratio of the square roots of successive integral numbers. Referred to a system of co-ordinates in which the line joining the two centres is the axis of x and the origin is a point midway between the centres, the rings are represented by the equations

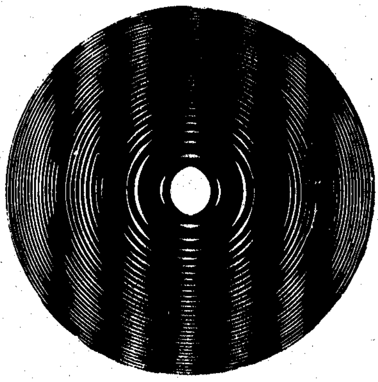
$$y^2 + (x - a)^2 = R^2 m, \quad y^2 + (x + a)^2 = R^2 n,$$

where m and n are integers and R is a linear constant. From these equations, we readily obtain by addition and subtraction

$$4ax = R^2(n - m), \tag{1}$$

$$y^2 + x^2 + a^2 = \tfrac{1}{2}R^2(m + n), \tag{2}$$

$$y^2 + x^2 + 6ax + a^2 = R^2(2n - m), \tag{3}$$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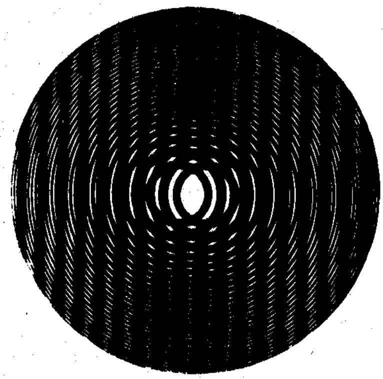
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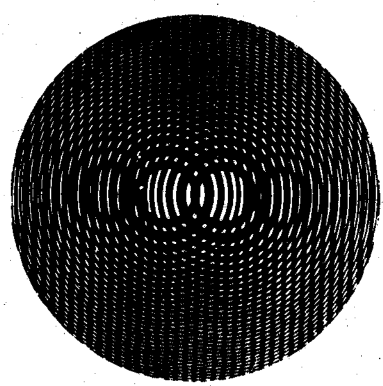
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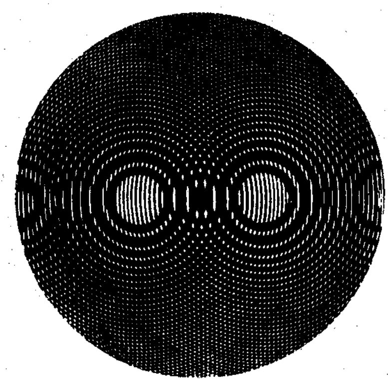
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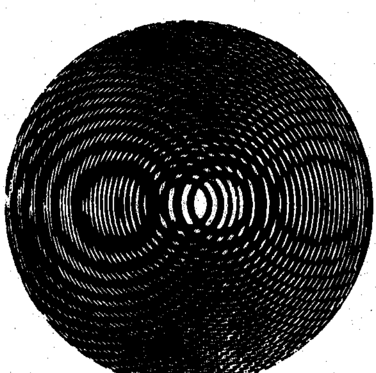
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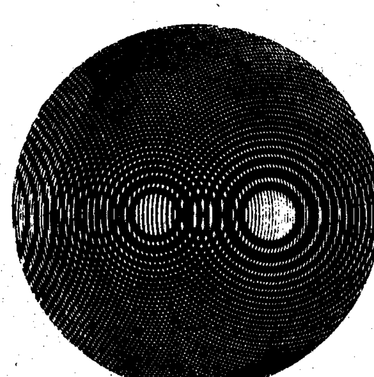
(6)

Figures 1–6

$$y^2 + x^2 - 6ax + a^2 = R^2(2m - n), \tag{4}$$

and so on. Equation (1) represents a set of straight lines parallel to the x-axis. Equation (2) represents a set of circles with their centres at the mid-point between the centres of the ring-systems. Equations (3) and (4) represent sets of circles with their centres at $x = \pm 3a$, and so on. These loci contain points where the maxima of intensity of one set of rings coincide with the maxima of the other set and the minima of one set with the minima of the other. Alternating between these, we have other loci of similar shape where the maxima of one set coincide with the minima of the other set, and vice versa. Since the intensity in the superposed patterns is the product of the intensity in the patterns due to the two plates separately, the loci mentioned represent lines along which there is alternately a general strengthening and weakening of the ring-pattern. This is the feature exhibited in figures 1 to 4. The systems represented by equations (2), (3), and (4) become prominent in the superposition pattern only when the rectilinear fringes given by (1) are so narrow as not to obscure the structure of the field.

The case of the superposition of two ring-systems of unequal size may be similarly discussed. We have as the equations of the ring-systems

$$y^2 + (x - a)^2 = R_1^2 m, \quad y^2 + (x + a)^2 = R_2^2 n,$$

where R_1 and R_2 are now different linear constants. Adding and subtracting we have

$$(R_1^2 - R_2^2)(x^2 + y^2 + a^2) + 2(R_1^2 + R_2^2)ax = R_1^2 R_2^2 (n - m), \tag{5}$$

$$(R_1^2 + R_2^2)(x^2 + y^2 + a^2) + 2(R_1^2 - R_2^2)ax = R_1^2 R_2^2 (m + n), \tag{6}$$

$$(2R_1^2 - R_2^2)(x^2 + y^2 + a^2) + 2(2R_1^2 + R_2^2)ax = R_1^2 R_2^2 (2n - m), \tag{7}$$

$$(2R_2^2 - R_1^2)(x^2 + y^2 + a^2) - 2(2R_2^2 + R_1^2)ax = R_1^2 R_2^2 (2m - n), \tag{8}$$

and so on. These equations also represent sets of concentric circles, the centres lying on the x-axis.

3. Phenomena in non-homogeneous light

We are now in a position to discuss the effect of using non-homogeneous light on the appearance of the superposed patterns. An alteration of wavelength changes the linear constant of the Haidinger ring-systems, which consequently expand or contract, as the case may be. If we were considering only each plate separately, the fluctuations of intensity in the field would be wiped out completely, and we would get only uniform illumination. With two plates, as the wavelength is altered, a radial movement of each ring-system occurs and the superposition patterns also alter. The *circular* fringe-systems formed by superposition, whose equations are given in (2), (3), (4), (5), (6), (7), and (8), move radially from their respective centres, and when any appreciable range of wavelengths is used, they are completely wiped out and disappear. But the rectilinear fringes given in equation (1), which correspond to the differential-pattern for two plates of *equal thickness*, behave otherwise. For the central bright fringe of this system cuts the circular rings orthogonally and therefore remains fixed in position, and the same is also true *approximately* of the other fringes running parallel to it on either side. From equation (1) it can be seen that an alteration of R simply means a change of the width of the rectilinear fringes. Thus in white light the system would remain visible, the central fringes being achromatic and the outer ones coloured. It is only in highly monochromatic light that we can expect to see the circular fringe-systems corresponding to the summationals and differentials of higher order. As a matter of fact, using two plane-parallel glass plates 1 mm thick and sodium light, the Haidinger rings and also the superposition pattern consisting of the rectilinear fringes corresponding to the differential system, and the circular fringes corresponding to the summationals and the differentials of higher order, can be readily observed. We have studied these features and will describe them more fully in part II.

The first differential system for two plates of equal thickness appears in equation (1) as a system of straight lines. This is only the case when the Haidinger rings are taken to be spaced exactly according to the law of the square roots of the natural numbers. Actually, as is well known, the Haidinger rings in a glass plate are spaced somewhat differently, the successive rings becoming closer and closer together only up to about an angle of emergence of 45° and then widening out again. As a consequence of this, the differential pattern is not really a system of straight fringes throughout the field, and actually consists of a system of closed ellipses. The modification of the theory necessary on this account and the special phenomena observed in crystalline plates will be considered more fully in parts II and III of the paper*.

We have to thank Mr Asutosh Dey for preparing the illustrations appearing with this paper.

*Not published.