Haidinger's Rings in Curved Plates

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The paper describes and illustrates the Haidinger interference patterns observed in curved plates of uniform thickness but of arbitrary form, and discusses their theory. Using an illuminated screen as light source and a limited viewing aperture, it is possible to observe such patterns with curved plates, their configuration depending upon the form of the plate as well as the position of the aperture with reference to the centers of curvature of the surface of the plate. It is pointed out that the interference pattern for a curved plate is geometrically similar in form to the image of the standard Haidinger pattern of circular rings as seen viewed by reflection at the appropriate distance from the curved surface of the shell.

1. INTRODUCTION

THE "interference curves of equal inclination" in transparent plates first observed by Haidinger are of great importance in physical optics. For their practical application in the field of spectroscopy, the plane-parallelism of the plate exhibiting these interferences is so essential that great stress is naturally laid on this requirement, and the tendency has therefore been to regard such plane-parallelism as a *sine-qua-non* for the observation of Haidinger's phenomenon. Actually, however, this is far from being the case, and by suitable arrangements the interference curves of equal inclination may be observed in a variety of other circumstances. In particular, these figures are readily to be seen in the transmission or reflection of light at the surface of curved transparent shells of uniform thickness but of arbitrary shape. They then assume a variety of forms depending on the circumstances of observation. From the standpoint of general optics, this enlarged field for the study of Haidinger's phenomenon is not without interest, and it is proposed in this communication to place on record some remarks and observations dealing with it.

2. METHODS OF OBSERVATION

To observe or photograph the Haidinger rings by reflection, a convenient arrangement is to use an opaque diffusing screen which contains a small aperture at its center. This is placed behind the plate under observation and is illuminated on its front by the light of a mercury lamp, while the eye or the camera is directed from the rear through the aperture and the plate towards a dark background. The arrangement permits of the observing aperture being brought very close to the plate, which is of great advantage if the latter is uniform over only restricted areas on its surface. For instance, interferences exhibiting hundreds of concentric rings may easily be observed with this arrangement in suitable areas on any ordinary glass plate such as a cleaned photographic negative. The advantage of restricting the aperture of observation and of bringing it close to the surface is equally evident in the observation of the transmitted system of rings. For the latter purpose, all that is necessary is to view an illuminated screen through the plate held close to the eye. As the plate is moved about, the complete system of rings suddenly springs into view when areas of sufficiently uniform thickness come in front of the eye, while with other parts of the plate nothing is to be seen on the viewing screen.

It is worthy of remark that the Haidinger interferences in a transparent plate may also be exhibited, either in reflection or in transmission, using a point source of light. For this purpose, we merely reverse the roles played by the aperture and the diffusing screen in the arrangements described above. A small aperture in an opaque screen closely backed by a mercury lamp is the origin of a divergent pencil of monochromatic light in the path of which the transparent plate is held. The reflected system of rings is then formed on the rear surface of the screen containing the illuminating aperture. The transmitted system of rings may similarly be observed when the diverging beam after passage through the plate falls on a viewing screen. Though these arrangements are in certain respects less satisfactory than those in which an extended light source is employed, they are nevertheless of interest as showing that such a source is not indispensible for the observation of Haidinger's rings. In what follows, however, it is to be understood that we are referring to the method of observation first described above, unless otherwise explicitly stated.

3. Some Geometrical Considerations

Using an extended light source and a limited aperture of observation, the interference lines of equal inclination may readily be demonstrated with curved plates of any shape. If such plate be of uniform thickness, it is not necessary that the observer's eye should be placed close to its surface. Indeed, a special point of interest in the case is that the configuration of the interference curves varies with the position of the observer's eye. With the eye sufficiently close to the plate, the rings have the same form as for a plane-parallel plate. As the eye recedes from the plate, the curvature of the surface modifies the inclination of the normals to the reflected or transmitted rays which reach the eye; when the eye is sufficiently remote from the plate, its curvature determines such inclinations almost exclusively. The interference curves of equal inclination therefore progressively alter in form and finally reach a limiting configuration which is determined by the shape of the curved shell and which would, in general, be quite different from the familiar arrangement of concentric circular rings.

A closer consideration of the geometric problem which here arises indicates that the configuration of the interferences would depend greatly on whether the principal curvatures of the plate are turned towards the observer or away from him, as also upon the situation of the eye with respect to the centers of such curvature. To appreciate the reason for this, we may consider the case of a curved shell in the form of a spherical bowl with its concavity towards the observer. When the eye is exactly at the center of the bowl, all the rays reaching it will be normal to the surface, while if it be situated either nearer or further from the shell, the inclinations of the rays to the normals would increase from the vertex outwards. For this position of the eye, therefore, the circles on the sphere representing varying inclinations of the normal to the light rays would be very widely spaced and they would also shift rapidly as the eye is moved. For the same reason, in a case in which only one of the principal curvatures of the shell faces the observer, both the form and the spacing of the curves of equal inclination would alter rapidly with the position of the eye when the latter is near the center of curvature. On the other hand, if both the principal curvatures are turned away from the observer, no such singularity would be noticed, but the configuration of the interference curves would alter continuously as the eye recedes from the surface of the shell. In general, the form of the interference curves would depart the more widely from circular symmetry the greater the difference is between the principal curvatures at the vertex of the shell opposite to the observer.

4. Observations with Curved Plates

The theoretical considerations set out above are readily capable of experimental test. That a plane-parallel plate is not necessary for the observation of Haidinger's rings may be readily demonstrated with a spherical bulb of glass blown rather thin but uniform and examined by the method already described. The material most suitable, however, for a detailed study of the interference phenomena of curved plates is mica, which was indeed the substance with which Haidinger discovered his rings.[1] Mica is readily bent and held in a cylindrical form, and the various points discussed above may readily be examined with it. When such a cylinder is held with its convexity towards the observer and its axis horizontal and is gradually moved away from the eye, the Haidinger curves progressively change in form from a set of concentric circular rings, to concentric elliptic rings with the major axes horizontal and finally to a set of straight lines parallel to the generating lines of the cylinder.

The accompanying reproductions illustrate the foregoing remarks. Fig. 1 is a picture of the Haidinger's rings in a plane sheet of mica. Fig. 2 is a picture of the rings in a convex cylindrical sheet held fairly near the lens of the camera.

[1] See also Rayleigh, Phil. Mag. **12**, 489 (1906) and Chinmayandandam, Proc. Roy. Soc. **A95**, 176 (1918).

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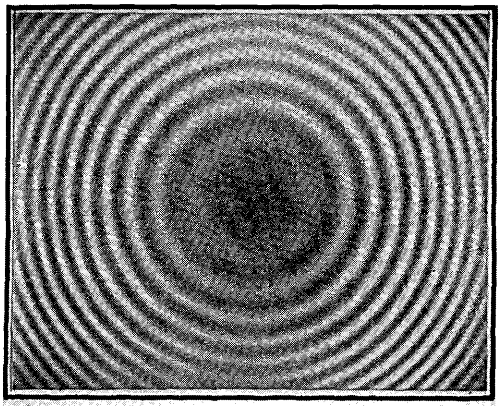

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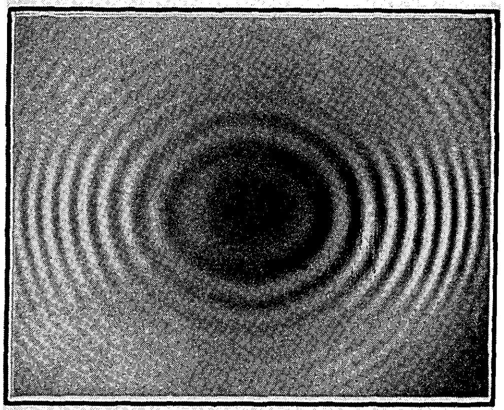

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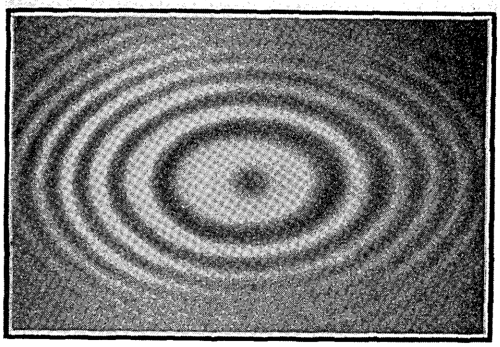

FIGS. 1, 2 AND 3. Haidinger's rings: (1) in a plane sheet of mica; (2) in a convex cylindrical sheet of mica held near the camera lens; (3) same as (2), but with sheet farther from camera lens.

Fig. 3 is a picture taken with the sheet moved further away from the lens, and it will be seen by comparison with Fig. 2 that the ellipticity of the rings is thereby increased. All the three photographs are of the reflected system of rings taken by the method described earlier in this communication.

When the cylinder of mica has its concavity towards the observer, the axis being horizontal, the sequence of changes with increasing distance of the eye from the mica is quite different. We have at first concentric circular rings which change to concentric elliptical rings with the *major axes vertical,* and after passage of the eye through the center of curvature, to circular rings once again, then to elliptical rings with the *major axes horizontal* and finally to straight lines parallel to the generating lines of the cylinder. In the passage through the center of curvature, the configuration of the rings is very varied and depends on the exact form of the mica. When this has the shape of a hyperbolic cylinder, the pattern is observed to consist of two sets of rings joined by oval curves similar to the well-known figures exhibited by a biaxial crystal in the polarization microscope.

5. Relation to Geometry of Image Formation

It has already been mentioned that the Haidinger pattern of a plate may be observed with a point source of light and a viewing screen to receive the light transmitted or reflected by the plate. A remarkable fact observed with curved plates in this connection is that *the transmitted and reflected systems of rings behave in a wholly different manner.* The transmitted ring-system seen in this way alters in form with the shape of the plate and its distance from the light-source in the same manner as in the usual method of observation. On the other hand, the reflected interferences remain of invariable form as a system of concentric circular rings, whatever may be the shape of the plate or its position. In both cases, however, the linear dimensions of the pattern seen on the screen increase with its distance from the source of light. The explanation of these facts becomes clear when we trace the formation of the interference curves, by following the rays delineating them from the source to the screen. The curves of equal inclination as traced on the surface of the plate would be of identical form in the two cases. In the case of the reflected system, however, the approach of the curve of a particular order of interference towards the vertex of the shell produced by the curvature of the latter is exactly set off by the recession from the vertex produced by the same curvature in the act of reflection of the ray before it reaches the screen. The effect of the curvature of the shell on the reflected system is thus to alter the distribution of intensity of light on the screen without producing any change in the form or position of the interferences. On the other hand, in the transmitted system, the interference curves seen on the screen are the direct projections of the curves as traced on the surface of the plate and hence exhibit forms depending on the shape of the plate and its distance from the source.

The foregoing considerations indicate that the Haidinger pattern of a curved plate may be regarded as the geometric image of a pattern of the standard type consisting of concentric circular rings as seen by reflection at the curved surface of the plate. The facts as already described then become easier to grasp in the light of the well-known theory of image formation in geometrical optics. That this idea can form the basis for an exact theory of the Haidinger patterns of curved plates can be shown in the following way. A small white circular disk with a central aperture in it is illuminated by monochromatic light and held close to the eye. It is then viewed through the same aperture by reflection at the surface of a curved plate of mica. It is then observed that the configuration of the circular edge of the disk runs exactly parallel to the course of the interference rings seen projected on its surface. Indeed, the changes in the form of the reflected image of the disk in the most varied circumstances describe also the changes in the configuration of the interference rings, thus establishing the thesis stated.

6. Position of Best Focus

An interesting question which is closely related to the topic discussed above relates to the position in space where the Haidinger pattern of a curved plate is seen most clearly in focus. As the visibility of the pattern essentially depends on the limitation of the aperture of observation, and as this aperture is necessarily finite, a certain lack of definition is inevitable. This may however be minimized by suitable accomodation of the observer's eye or focusing of the camera employed to photograph the pattern. The focus would evidently be best when the angle between the normals and the corresponding rays reaching a given point in the image vary as little as possible. When the curvature is small or when the aperture is close to the shell, we should therefore focus for infinity in the same way as for a plane-parallel plate. For greater curvatures of the shell or with increasing distance of the aperture from it, the variation of the angle referred to over the surface of the shell becomes more rapid. We may reduce this variation by altering the focus so as to approach the surface of the shell, but this on the other hand would introduce a variation of the inclination of the rays from the same area of the shell which come to a focus at an image point. The best focus is determined by these opposing considerations which indicate that as the form of the interference curves alters progressively from the standard configuration of concentric circular rings to one determined completely by the form and curvature of the shell itself, the focus of the pattern also shifts steadily from infinity towards the surface of the shell. This general statement must, however, be qualified in certain respects. If the principal curvatures of the shell differ greatly in magnitude or in sign or in both, it is clear that the focusing would show a certain astigmatism, features in the pattern running in one direction being more clearly seen than features running in a perpendicular direction. It is also to be remarked that the focusing of the pattern as a whole would be difficult when the observing aperture is placed at or near a center of curvature of the surface.

From what has already been stated, it is evident that the interference curves of equal inclination for a strongly curved shell at oblique incidences would be best seen when the eye is focused on the surface of the shell itself. A limitation in the area of the light source employed to the extent of practically restricting it to a line-source running parallel to the interferences should then be of marked advantage. For, such a restriction would result in the angle of incidence of the light being better defined and therefore in a better definition of the fringes, since these depend for their visibility on the variation of the angle of incidence over the surface of the shell. In these respects there is a certain similarity with the behavior of the interference curves in plates of varying thickness, though the behavior in other respects is wholly different.