THE MAINTENANCE OF VIBRATIONS.

BY C. V. RAMAN.

PART I.

IN my paper on "Some Remarkable Cases of Resonance" published in the PHYSICAL REVIEW for December, 1912, I described a new class of forced vibrations which form apparent exceptions to the general principle of resonance, i. e. in respect of which we find systems exhibiting marked resonance under the action of forces whose periods do not necessarily stand to their own in a relation of approximate equality. Of this class, the first type is the well-known case of double frequency exemplified by the form of Melde's experiment in which the vibrations of a stretched string are sustained by the action of a varying tension imposed by a fork having double the frequency of the string. My experiments showed that the vibrations could be maintained in the following cases:

(1) When the frequency of the fork is 2 times that of the string;
(2) When the frequency of the fork is $\frac{2}{2}$ times that of the string;
(3) When the frequency of the fork is $\frac{2}{3}$ times that of the string;
(4) When the frequency of the fork is $\frac{2}{4}$ times that of the string;
(5) When the frequency of the fork is $\frac{2}{5}$ times that of the string;
And so on.

Photographs illustrating the modes of vibration in each of these cases were published with the paper.

Some very interesting phenomena are noticed when a stroboscopic disk is used in observing these types of maintained motion. A Rayleigh synchronous motor on which is mounted a blackened disk with narrow radial slits cut in it is very suitable for this purpose. One of the disks which I use has thirty slits in it, the armature-wheel of the motor having the same number of teeth. The electric current from the self-interrupter fork which maintains the string in vibration also runs the synchronous motor. In making the observations, the stroboscopic disk is held vertically and the string which is set horizontal and parallel to the disk is viewed through the top row of slits, *i. e.*, those which are vertical or nearly so and move in a direction parallel or practically parallel to the string as the disk revolves. It is advantageous to have the whole length of the string brilliantly illuminated and to let as little stray light as possible fall upon the reverse of the disk at some distance from which the observer takes his stand. A brilliant view is then obtained. Under these circumstances, we see the string in successive cycles of phase along its length, and the peculiar character of the maintained motion in these cases is brought out in a very remarkable way. *The string is seen in the form of a vibration-curve*, which would be identical with those obtained by other methods but for the fact that the amplitude of motion is not the same at all points of the string, being a maximum at the ventral segments and zero at the nodes.

If we use a fork with a frequency of 60 per second, the *free* oscillations of the string should have a frequency of 30 in the case of the first type, 60 in the case of the second, 90 with the third, 120 with the fourth, 150 with the fifth, and so on. With the disk having 30 slits on it we get 60 views per second of any one point on the string, and with the even types of motion, *i. e.*, the second, fourth, etc. the 'vibration-curve' seen through the stroboscopic disk appears single. Figs. 3 and 7 show the curves for the second and fourth types respectively. With the odd types on the other hand, *i. e.*, with the first, third, fifth, etc., *two* vibration-curves are seen, one of which is as nearly as can be seen the mirror-image of the other, intersecting at points which lie or should lie upon the equilibrium position of the string. Figs. 1 and 5 show the curves for the case of the first and third types. The reason why with the odd type we see the vibration-curve double is fairly obvious and forms an excellent illustration of the principles of stroboscopic observation. The double pattern in the case of the third and higher types brings home to the eye in a very vivid and convincing manner the fact that under the action of the variable spring the "amplitude" and "period" of the motion periodically increase and decrease after the manner of "beats."

An interesting variation on the experiment is made by using a disk with 60 slits. We then get 120 views per second and with the even types we get the vibration-curves double, but one of the curves is not the mirror image of the other, the motion not being symmetrical with respect to the position of equilibrium. On the other hand, with the odd types we see the vibration-curves in quadruple pattern, the third and fifth types in particular giving noteworthy effects. Figs 2, 4, 6, and 8 exhibit the patterns obtained in this manner with the first, second, third and fourth types respectively.

Before leaving this subject, it is worth while to remark that the special method of stroboscopic observation described above can be very effectively applied to the study of the vibrations of stretched strings produced or maintained in any other manner, *e. g.*, by bowing or striking.

The vibrational forms for various points over the entire length of the string can all be observed together simultaneously with the apparatus, and this is often a very great convenience. For example, the widely-differing ratios between the forward and backward velocities at various points on a bowed string can all be seen together and compared in one experiment, and the effect of applying the bow at different positions, of reversing its direction of motion, or of removing it altogether and allowing the motion to die away gradually can be very conveniently studied. It is necessary in such work that the string should be tuned to the correct frequency in the first instance (by observation through the stroboscopic apparatus) so that a stationary figure may be obtained.

PART II.

A stroboscopic disk with radial slits mounted on a synchronous motor can be very effectively used in observing the peculiar properties of the small motion at the nodes of a vibrating string described by me in a previous publication in this REVIEW.[1] In this case an arrangement somewhat different from that described in Part I. of the present paper should be used. An electrically-maintained fork maintains the string in oscillation in any convenient number of loops by imposing a *transverse* obligatory motion at one point on it. The current passing through the fork also drives the synchronous motor on which is mounted a stroboscopic disk having just double as many apertures as the armature-wheel has teeth. The disk therefore gives two views of the fork and of the string maintained by it, which are practically stationary provided the point of observation is fixed and the motor is running satisfactorily. In the present case it is essential that the position taken up by the observer or the camera should be quite close to the stroboscopic disk and should be so chosen that the slits through which the vibration is observed are as nearly as possible parallel to the string and move in a direction at right angles to its length. The most convenient plan is to have the string horizontal and the plane of its vibration vertical. The disk should then be set vertically and the observations made through the region of the disk in which the radial slits are horizontal or practically so. By changing the point of observation, successive phases of the motion and the periodic "travel" of the "nodes" over a large horizontal range seen under the intermittent illumination can all be observed at leisure.

For photographic work, the camera employed is brought up close behind that one of the slits on the disk which is horizontal. The lens is stopped down by a plate which has a rectangular slit cut in it so as to

[1] "The Small Motion at the Nodes of a Vibrating String," PHYS. REV., March, 1911.

correspond with those on the disk. By racking up the lens-front of the camera by successive small distances till it has moved through a length equal to that between contiguous apertures on the disk, a complete set of photographs can be obtained on one plate showing successive stages of the motion of the string. Fig. 9 reproduces a photograph obtained in this manner and showing the cycle of changes in 13 stages. It will be seen that the point of intersection or "node" which is first in the center moves off to one side of the field, first slowly and then more quickly, till after the lapse of a time which can be seen to be exactly half the period of the cycle, it has gone well off the plate and the positions of the string seen in the photograph are sensibly parallel. Direct observation shows that the point has moved off to a great distance, in fact to a distance equal to half the length of a ventral segment. It simultaneously appears at an equal distance on the other side and moves in from that direction first quickly and then more slowly till it reaches the center again and the cycle is complete.

From Fig. 9, it is quite obvious that the phase of the small motion at the node differs by quarter of an oscillation from that of the large motion on either side of it at a distance.

Part III.

A very neat modification of Melde's experiments can be arranged by attaching one extremity of a fine cotton or silk string to a prong of an electrically maintained fork held so that the string lies in a plane perpendicular to the prongs but in a direction inclined to their line of vibration.[1] In view of the ease with which the experiment is performed and the great beauty of the form of oscillation maintained (this must be seen to be fully appreciated), the following brief discussion of the results may be of interest.

The obligatory motion imposed at one extremity of the string may be resolved into two components, one parallel and the other perpendicular to the string. The two components may be put respectively equal to $\gamma \cos pt \cos \theta$ and $\gamma \cos pt \sin \theta$. The transverse component maintains an oscillation having the same frequency as that of the fork and having an even number of ventral segments when the tension of the string is suitably adjusted. The longitudinal component will then generally be found to maintain an oscillation having half the frequency of that of the fork. The success of the experiment lies in isolating the two vibrations, the frequency of one of which is double that of the other, into perpendicular planes. This is easily secured by a simple little device. The

[1] See Phys. Rev., Vol. XXXII., page 311.

end of the string is attached to a loop of thread which is passed over a prong instead of directly to the prong itself. The result of this mode of attachment is that the frequencies of vibration in the two planes at right angles differs slightly and this has the desired effect of keeping the component vibrations confined to their respective planes, if the tension of the string lies anywhere within a definite range.

If the distance of any point from the fixed end when at rest is x, the transverse components of the maintained motion may be written as under

$$Y = \gamma \sin \frac{\theta R_x}{R_b} \cos (pt + E_x - E_b), \tag{1}$$

vide Lord Rayleigh's Theory of Sound, Art. 134.

$$Z = B \cos \left(\frac{pt}{2} + E\right) \sin \frac{\pi x}{b}, \tag{2}$$

the values of B and E being ascertained from the investigation given in the PHYSICAL REVIEW, Vol. XXXV., page 451. If we exclude any consideration of the motion at points near the nodes of the maintained oscillation, equation (1) may be written in the simple form

$$Y = \gamma \sin \theta \sin \frac{px}{a} \cos (pt + E').$$

If $p/a = 2\pi/b$, (1) and (2) may be written in the form

$$Y = A \sin \frac{2\pi x}{b} \cos (pt + E'), \tag{3}$$

$$Z = B \sin \frac{\pi x}{b} \cos \left(\frac{pt}{2} + E\right). \tag{4}$$

It should be understood that in these equations Y and Z do *not* refer to the coördinates of any point fixed relatively to the string but to the points at which a plane transverse to its equilibrium position cuts the surface generated by the moving string. The distinction is of importance in view of the fact that each point on the string possesses a small longitudinal motion derived from that imposed by the fork and the x coördinate of any point fixed relatively to the string is therefore not itself constant.

In particular cases equations (3) and (4) may be reduced to very simple forms. Thus if $E' = 2E$ and also in the special case when both E and E' are equal to zero

$$\frac{Y}{A} \operatorname{cosec} \frac{2\pi x}{b} = \frac{2Z^2}{B^2} \operatorname{cosec}^2 \frac{\pi x}{b} - 1, \tag{5}$$

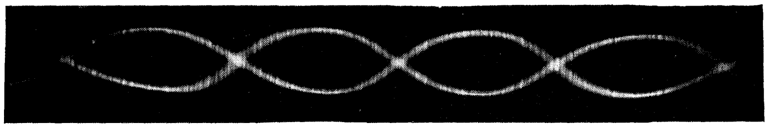

Fig. 1.

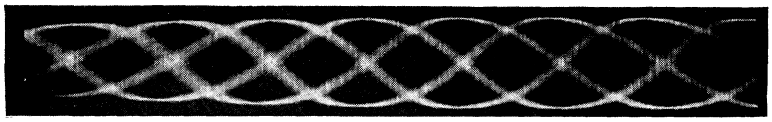

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.

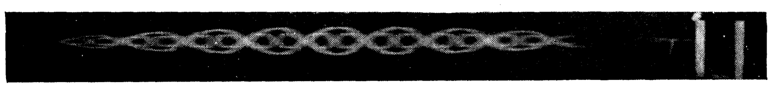

Fig. 6.

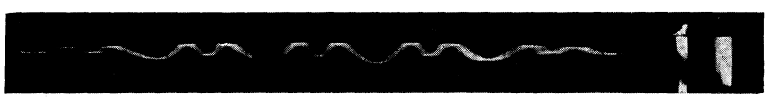

Fig. 7.

Fig. 8.

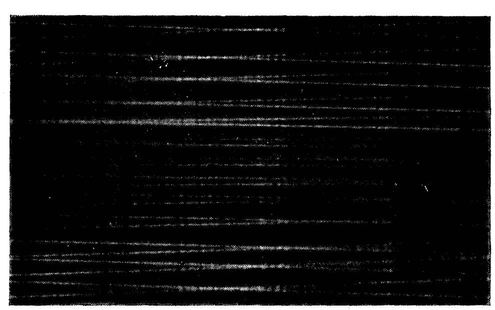

Fig. 9.

which is the equation of the surface generated by the moving string, the sections of which by planes perpendicular to the axis of x are parabolic arcs. The curvatures of the arcs are in opposite direction in the two halves of the string. When $E' = 2E + \pi$, a parabolic type of motion should also be obtained. This case can be realized approximately by having the tension of the string as low as possible consistent with the maintenance of the motion. In the intermediate case when $E' = 2E + \pi/2$, the sections of the surface described by the moving string elsewhere than at its center are 8 curves. At the center, the curve is a very flat parabolic arc, as the phase of the small motion at the node differs by quarter of an oscillation from that of the rest of the string.

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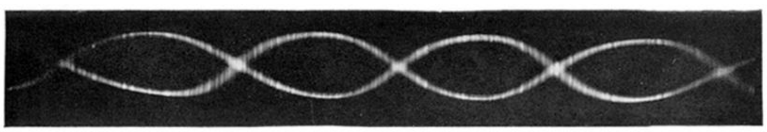


FIG. 1.

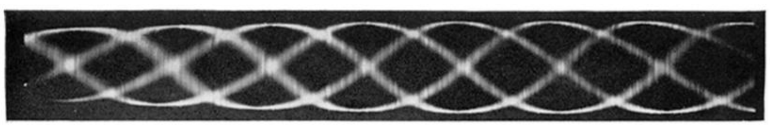


FIG. 2.

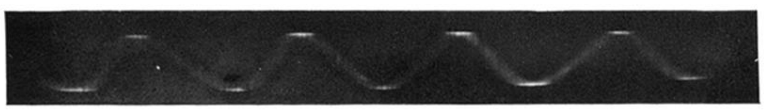
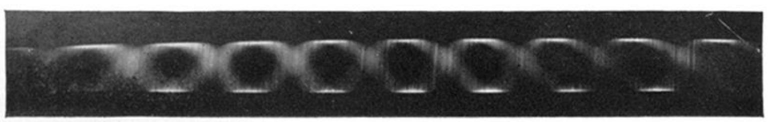


Fig. 3.



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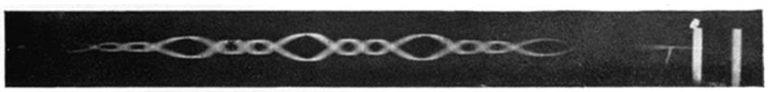

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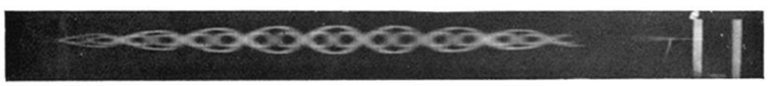

FIG. 6.

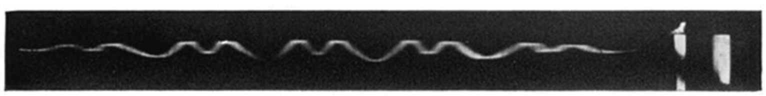

Fig. 7.

FIG. 8.

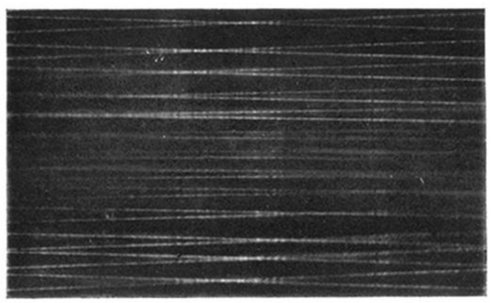

FIG. 9.