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# Diffraction of light by a transparent lamina

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ABSTRACT

The present paper embodies an attempt to consider the problem of diffraction of light by a *very thin* plate of transparent material, bounded by a straight edge, with greater exactness than is attained in the usual elementary treatment on the Fresnel–Huygen principle. The method adopted, though not completely rigorous, bases itself on the electromagnetic theory of light, and seeks to express the disturbance in the field in the form of functions which are solutions of the equations of wave-propagation. The formulae obtained indicate that the light diffracted by the edge should exhibit colour and polarization effects varying in a remarkable manner with the thickness of the plate and the direction of observation. Effects having the general character of those indicated by the theory have actually been observed in experiment. The theory, however, requires modification in the case of thicker laminae, where further complications arise which are not here taken account of.

## 1. Introduction

The problem of diffraction by a plane transparent lamina arises in considering the theory of such phenomena as the colours exhibited by the "striae" or laminar boundaries in mica,* and the colours of mixed plates.† It has been observed that the edges of thin laminae diffract light through large angles, and that the light thus diffracted exhibits colour and polarization effects which are in some respects analogous to those discovered by Gouy‡ with metallic edges. The elementary treatment of laminar diffraction usually given is thus inadequate. In the present paper an attempt is made to place the theory of diffraction by laminar boundaries on a more satisfactory basis. The case of a thick lamina is too complicated to offer hope of an exact solution. In the case of a thin lamina, however, certain simplifications are possible, as we shall see presently, which enable the problem to be dealt with successfully.

*C V Raman and P N Ghosh, *Nature (London)* **102**, 205 (1918).
†C V Raman and B N Banerji, *Philos. Mag.*, **41**, 338 (1921).
‡G Gouy, *Ann. Chim. Phys.* **8**, 145 (1886).

## 2. Theory

We shall confine ourselves to the case in which the light is incident on the lamina in a plane perpendicular to its edge. The region round the lamina can then be divided into three parts: The first consists of the region in which the light transmitted through the lamina alone appears. The second is the region in which only the incident waves appear. The third is the region in which the incident and reflected waves are superposed.

The middle surface of the lamina may be taken as the plane $\varphi = 0$, and its edge as the $Z$-axis in a system of cylindrical co-ordinates. We may represent the incident wave by the expression

$$\exp(i\cdot 2\pi t/T)\exp[i\kappa\rho\cos(\varphi - \varphi_0)], \tag{1}$$

$\varphi_0$ being the angle between the plane of the lamina and the incident rays. The train of waves reflected from the lamina may be represented by the expression

$$-\exp(i\cdot 2\pi t/T)(A_s + iB_s)\exp[i\kappa\rho\cos(\varphi + \varphi_0)] \tag{2}$$

or by

$$\exp(i\cdot 2\pi t/T)(A_p + iB_p)\exp[i\kappa\rho\cos(\varphi + \varphi_0)] \tag{3}$$

and the train of waves transmitted through the lamina by the expression

$$\exp(i\cdot 2\pi t/T)(C_s + iD_s)\exp[i\kappa\rho\cos(\varphi - \varphi_0)] \tag{4}$$

or by

$$\exp(i\cdot 2\pi t/T)(C_p + iD_p)\exp[i\kappa\rho\cos(\varphi + \varphi_0)] \tag{5}$$

The expressions (2) and (4) refer to the case (which we shall refer to as the $\|^l$ case), in which the planes of incidence and polarization are coincident. The expressions (3) and (5) refer to the case (which we shall refer to as the $\perp^r$ case) in which they are mutually perpendicular. The multiplying factors $(A_s - iB_s)$, etc., are those given by the well known theory of the colours of thin plates,* and may be deduced directly from the electromagnetic theory so as to satisfy the boundary conditions on either face of the lamina.

Now the disturbances represented by expressions (1) to (5) do not extend throughout the whole field, but are confined to the particular regions of the field already indicated. To find the solution of the diffraction problem, we seek a function which satisfies the equations of wave-propagation, and which, while being valid throughout the whole field outside the substance of the lamina, represents a disturbance approaching asymptotically to the values given by those expressions at a sufficient distance from the origin in the respective parts of the field to which they refer.

*Drude's *Theory of Optics*, English Translation, p. 302.

Such an expression is

$$u = F(\rho, \varphi, \varphi_0) - (A_s + iB_s)F(\rho, \varphi, -\varphi_0) + (C_s + iD_s)F(\rho, \varphi, \varphi_0 - 2\pi) \tag{6}$$

or

$$u = F(\rho, \varphi, \varphi_0) + (A_p - iB_p)F(\rho, \varphi, -\varphi_0) + (C_p - iD_p)F(\rho, \varphi, \varphi_0 - 2\pi) \tag{7}$$

in which $F(\rho, \varphi, \varphi_0)$ is the well known solution of the wave equation due to Sommerfeld.

$$F(\rho, \varphi, \varphi_0) = \exp(i\cdot 2\pi t/T)\left(\frac{i}{\pi}\right)^{1/2} \exp[i\kappa\rho\cos(\varphi - \varphi_0)] \int_{-\infty}^{T} \exp(-i\lambda^2)\mathrm{d}\lambda \tag{8}$$

where

$$T = \sqrt{2k\rho}\cos\tfrac{1}{2}(\varphi - \varphi_0). \tag{9}$$

$F(\rho, \varphi, -\varphi_0)$ and $F(\rho, \varphi, \varphi_0 - 2\pi)$ are obtained by writing $-\varphi_0$ and $(\varphi_0 - 2\pi)$ respectively for $\varphi_0$ in (8) and (9).

In the $\|^l$ case $u$ represents the electric force parallel to the edge of the screen and in the $\perp^r$ case it represents the magnetic force in the same direction. It can readily be verified that the asymptotic values of (6) and (7) are those given by (1), (2), (3), (4) and (5), in the parts of the field to which they refer. When (9) is positive, the asymptotic expansion of (8) is

$$F(\rho, \varphi, \varphi_0) \sim \exp(i\cdot 2\pi t/T)\left[\exp[i\kappa\rho\cos(\varphi - \varphi_0)] + \frac{i^{3/2}\exp(-i\kappa\rho)}{4\pi\sqrt{\rho/\lambda}\cos\frac{1}{2}(\varphi - \varphi_0)}\right] \tag{10}$$

while, if (9) be negative, there is a similar expansion in which the first term is left out. The light diffracted by the edge may accordingly be written thus

$$E_z = \exp(i\cdot 2\pi t/T)\cdot\frac{i^{3/2}\exp(-i\kappa\rho)}{4\pi\sqrt{\rho/\lambda}}\left[\frac{1 - (C_s + iD_s)}{\cos\frac{1}{2}(\varphi - \varphi_0)} - \frac{A_s + iB_s}{\cos\frac{1}{2}(\varphi + \varphi_0)}\right] \tag{11}$$

$$H_z = \exp(i\cdot 2\pi t/T)\cdot\frac{i^{3/2}\exp(-i\kappa\rho)}{4\pi\sqrt{\rho/\lambda}}\left[\frac{1 - (C_p + iD_p)}{\cos\frac{1}{2}(\varphi - \varphi_0)} + \frac{A_p + iB_p}{\cos\frac{1}{2}(\varphi + \varphi_0)}\right]. \tag{12}$$

We may write the expression within the square brackets in (11) and (12) respectively in the form $(F_s + iG_s)$ and $(F_p + iG_p)$. $(F_s^2 + G_s^2)$ and $(F_p^2 + G_p^2)$ are measures of the intensity of the components in the radiation diffracted from the edge, and their ratio indicates its state of polarisation when the incident light is unpolarised. $\delta_s = G_s/F_s$ and $\delta_p = G_p/F_p$ give the phases of the components, and $(\delta_p - \delta_s)/\lambda$ is a measure of the ellipticity of the diffracted radiation when the incident light is polarized in any arbitrary azimuth.

## 3. Normal incidence: Very thin lamina

In the case of normal incidence, we have $\varphi_0 = \pi/2$, and further

$$A_s + iB_s = A_p + iB_p = \frac{i(\mu^2 - 1)\sin p}{i(\mu^2 + 1)\sin p + 2\mu\cos p} \tag{13}$$

$$C_s + iD_s = C_p + iD_p = \frac{2\mu(\cos q + i\sin q)}{i(\mu^2 + 1)\sin p + 2\mu\cos p}, \tag{14}$$

where $p = 2\pi\mu d/\lambda$ and $q = 2\pi d/\lambda$, $d$ being the thickness of the lamina and $\mu$ its refractive index. We have only to substitute (13) and (14) in equations (11) and (12) and evaluate them numerically to find the intensity and state of polarization of the diffracted light in any direction.

Consider first a lamina so thin that we may put $\sin p = p$, $\sin q = q$, $\cos p = \cos q = 1$. Since $p = \mu q$, we find readily on making these substitutions, that

$$1 - (C_s + iD_s) = 1 - (C_p + iD_p) = A_s + iB_s = A_p + iB_p$$

and that

$$F_s + iG_s = \frac{\pi(\mu^2 - 1)d}{\lambda}\left[\frac{1}{\cos\dfrac{\varphi - \varphi_0}{2}} - \frac{1}{\cos\dfrac{\varphi + \varphi_0}{2}}\right] \tag{15}$$

$$F_p + iG_p = \frac{\pi(\mu^2 - 1)d}{\lambda}\left[\frac{1}{\cos\dfrac{\varphi - \varphi_0}{2}} + \frac{1}{\cos\dfrac{\varphi + \varphi_0}{2}}\right]. \tag{16}$$

From (15) and (16), it follows at once that along the surface of the screen ($\varphi = 0$ and $\varphi = 2\pi$), the $\|^l$ component of the diffracted light would be zero, and the $\perp^r$ component would be finite, thus giving complete polarization. In the opposite direction ($\varphi = \pi$), however, the $\|^l$ component of the diffracted light would be finite, and the $\perp^r$ component would be zero, thus again giving complete polarization but in a perpendicular plane. The intensity of the diffracted light would, for such thin laminae, be proportional to the square of the thickness.

## 4. Thicker laminae: Colour and elliptic polarization

If we assume that formulae (11) and (12) remain valid for greater thicknesses of the lamina, at least as an approximation, it is evident from these expressions taken together with (13) and (14) that the intensity and state of polarization of the diffracted light would be functions of the wavelength, the thickness of the lamina, and the angle of diffraction. Hence, in white light, the diffracted radiation from the edge would exhibit colour.

If we assume that the incident light is unpolarized, the diffracted light from the edge would be partially polarized to an extent depending essentially on the contribution proportional to $(A_s + iB_s)$ or $(A_p + iB_p)$ which arises from the reflected wave and appears with a negative sign in (11) and with a positive sign in (12). It follows that the degree of polarization should vary with $\sin p$ in a periodic manner, being zero when the thickness of the lamina is such that the reflected light vanishes and maximum for intermediate thicknesses. The intensity of the diffracted light, however, depends chiefly on the resultant of the contributions from the incident and transmitted waves which are proportional to $(1 - C_s - iD_s)$ or $(1 - C_p - iD_p)$. It is easily shown that this part depends on $\sin(p - q)$ and that its intensity is a maximum when the relative retardation on the two sides of the boundary is half a wavelength, and a minimum when the relative retardation is a complete wavelength, and so on. Two sets of periodic variations in intensity thus occur giving rise to corresponding colour effects in the diffracted light. One set of variations affects both components of the vibration in the same way, while the other set affects the two components in opposite senses. Hence, the colour of the diffracted light would, to some extent, depend on the azimuth of polarization, and the two images of the diffracting edge seen through a double-image prism may in favourable cases actually exhibit complementary colours.

Figure 1 represents the values of $F_s^2 + G_s^2$ and $F_p^2 + G_p^2$ corresponding to an angle of diffraction of 90° ($\varphi = 180°$) for different thicknesses of the lamina, the refractive index being assumed to be 1·5. The periodic variations of the $\|^l$ and $\perp^r$ components of the intensity of the diffracted light are clearly seen. The

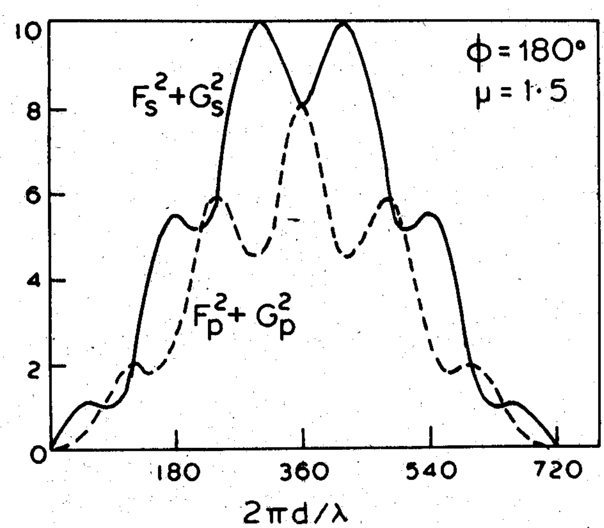


**Figure 1.** Intensities of components of diffracted light as functions of thickness.

polarization, which is given by the ratio of the two components, is almost complete for small thicknesses of the lamina; it is zero for thicknesses given by $2\pi d/\lambda = 78°$, $120°$, $218°$, $240°$, and $360°$. For intermediate thicknesses, the polarization fluctuates.

From (11), (12), (13) and (14) it is obvious that, when the incident light is plane-polarized in any azimuth, the diffracted light would in general be elliptically polarized. The phases of the $\perp^r$ and $\|^l$ components as calculated from the values of $\tan^{-1}(G_p/F_p)$ and $\tan^{-1}(G_s/F_s)$ respectively are shown in figure 2 for the same cases as those considered in figure 1. It is evident from the figure that, in certain cases, particularly for small thicknesses of the lamina, the phase differences are large and should easily be observed.

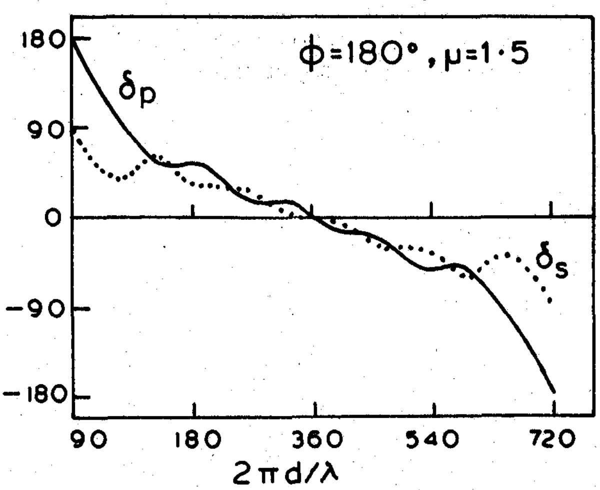


**Figure 2.** Phases of components of diffracted light as functions of thickness.

## 5. Supplementary remarks

The theory set out above is an approximation to the truth which is strictly applicable only to the thinnest laminae. It succeeds in explaining the polarization effects which would otherwise be unintelligible. With laminae of greater thickness, however, complications arise which are not here taken account of. In reality, we have not one edge, but two edges to deal with, namely, those relating to the front and rear surfaces of the lamina, at which the waves passing on either side of the boundary are diffracted and, diverging with a difference of path, interfere with each other.

It is readily seen that the path-difference between the interfering rays from the two edges would alter continually with increasing deviation of the diffracted ray, but in different ways in the two portions of the field to the right and the left of the direction of the incident pencil. Since the conditions on the two sides of the boundary between the incident and transmitted waves are thus dissimilar, the diffraction effects observed should be asymmetrical in intensity with reference to this boundary. Experimentally this is actually observed to be the case. Further, owing to the interference of the diffracted waves from the front and rear edges occurring under varying difference of path in different directions, the colour and intensity of the diffracted light should vary periodically with increasing deviation of the diffracted ray. This again is actually observed in experiment. It is not unlikely that a modified treatment which takes account of the complications referred to here may be successfully worked out. To enter into them more fully, however, or to make detailed comparisons between theory and observation would lie beyond the scope of the present paper.