Gravitational Waves from Inspiraling Compact Binaries on Quasi-elliptical Orbits

by

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DECLARATION

I hereby declare that this thesis is composed independently by me at the Raman Research Institute, Bangalore, under the supervision of Professor Bala R. Iyer. The subject matter presented in this thesis has not previously formed the basis of the award of any degree, diploma, associateship, fellowship or any other similar title.

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CERTIFICATE

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for the award of the degree of

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Contents

Preface

Inspiraling compact binaries are the most promising sources of gravitational radiation in the near future for ground-based laser-interferometric detectors. The method of matched filtering will be employed to search for and extract the information from the inspiral waveforms. For this method to be successful, one needs to use search templates (inspiral waveforms *a priori* constructed using the general theory of relativity) that are extremely accurate in their description of the orbital phase, which in turn requires a detailed understanding of how radiation reaction affects the orbital evolution. At present 'ready to use' inspiral waveforms from compact binaries of arbitrary mass ratio, moving in *quasi-circular orbits* are computed to two and half post-Newtonian order. In this thesis, the issues related to the construction of 'ready to use' second post-Newtonian accurate search templates for inspiraling compact binaries in *quasi-elliptical orbits* will be discussed.

The construction of the theoretical search templates for gravitational radiation from inspiraling binaries may be done in two steps, which are generally referred to respectively as the "wave generation problem" and the "radiation reaction problem". The wave generation problem deals with the computation of the gravitational waveforms generated by the binary (at the leading order in $1/R$, where R is the distance of the binary) when the orbital phase and frequency of the binary take some specific values. In other words it deals with the construction of the 'plus' and 'cross' gravitational wave polarizations. The radiation reaction problem consists of determining the evolution of the orbital elements (the orbital phase and parameters like frequency, eccentricity) as a function of time. The parameters describing the orbit vary in a nonlinear manner with respect to time, as the orbit evolves under the action of gravitational radiation reaction forces. In principle, the evolution of the orbital elements should be determined from a knowledge of the radiation reaction forces acting locally on the orbit. However, the radiation reaction forces are at present not known with sufficient accuracy, only the relative first post-Newtonian corrections to the reactive acceleration are known. Therefore, in practice, the evolution of orbital elements is determined assuming energy and angular momentum balance and the far-zone expressions for energy and angular momentum fluxes.

None of the above problems can be solved exactly for compact binaries consisting of two compact objects of comparable masses. They are treated by a combination of approximation methods like Post Minkowskian approximation and Post Newtonian approximation. We employ multi-polar post Minkowskian approach of Blanchet, Damour and Iyer (BDI).

In chapter 2, we apply the second post-Newtonian accurate BDI generation formalism to the case of inspiraling binaries in general orbits. We then compute the post-post-Newtonian (2PN) accurate mass quadrupole moment for compact binaries of arbitrary mass ratio moving in general orbits, in terms of the binary's dynamical variables, using the BDI formalism. Following BDI, we split the 2PN accurate formulae for the gravitational waveform and the far-zone fluxes into "instantaneous" and "tail" parts. The "instantaneous" contribution depends only on the state of the binary at the retarded instant $T_R \equiv (T-R/c)$ while the "tail" contribution is a priori sensitive to the binary's dynamics at all previous instants $T_R - \tau \leq T_R$. The "tails" are caused by the backscatter of outgoing radiation off the background spacetime curvature and appears at $O(\epsilon^{1.5})$ beyond the quadrupole approximation. Note that $\epsilon \sim (v/c)^2 \sim (Gm/rc^2)$, where m , v and r are respectively the total mass, orbital velocity and the separation of the binary system, ϵ is a small parameter in terms of which the orders of post-Newtonian corrections are defined. Using the 2PN accurate mass quadrupole moment and other multipole moments to the required order for binaries in general orbits, we compute the 2PN "instantaneous" corrections to the far-zone energy and angular momentum fluxes for binaries in general orbits. We also compute the 2PN "instantaneous" contributions to the gravitational waveform (the transverse-traceless (TT) part of the radiation-field, representing the deviation of the metric from the flat spacetime) using the STF multipole moments of the BDI formalism. We observe that, though our result for the far-zone energy flux matches precisely with that obtained by Will and Wiseman, our expressions for the waveform differ from the corresponding expressions obtained by them, using the Epstein-Wagoner multipole moments at 1.5PN and 2PN orders. Though the two expressions are totally different looking at these orders, even in the circular limit, we show that they are equivalent. The equivalence is established by showing that the difference between the two expressions, at 1.5PN and 2PN orders has a vanishing transverse-traceless part. We also exhibit various limiting cases of our results. The expressions for the 2PN corrections to the waveform and the far-zone fluxes for binaries in *general* orbits obtained in this chapter will form one of the basic inputs to tackle the "wave generation problem" and the "radiation reaction problem" for the construction of theoretical templates for binaries in *quasi-eccentric* orbits.

In chapter **3** we investigate the "radiation reaction problem" for eccentric binaries. Here the aim is to obtain the 2PN "instantaneous" corrections to the evolution of the orbital elements like the orbital phase and parameters like frequency and eccentricity. As mentioned earlier, these computations are done *assuming* energy and angular momentum balance and the far-zone expressions for the energy and angular momentum fluxes, averaged over an orbit. This naturally requires, for the elliptical binaries, a convenient solution to the 2PN accurate equations of motion. A very elegant 2PN accurate generalized quasi-Keplerian parameterization for elliptical orbits in the Arnowit, Deser and Misner (ADM) coordinates has been implemented by Damour, Schafer, and Wex. This representation is thus the most natural and best suited for our purpose to parametrize the dynamical variables that enter the expressions for the far-zone fluxes. We first obtain the 2PN corrections to the far-zone fluxes averaged over an orbit extending computations performed at 1PN and

1.5PN orders, taking due care of a new complication at this order that the far-zone fluxes are computed in the harmonic or De-Donder coordinates, while the orbital representation is available in the ADM coordinates. To obtain the evolution of the orbital elements, we start from the 2PN accurate expressions for the orbital elements in terms of the conserved energy and angular momentum available in the literature and compute the time variation of these orbital elements. One ends up with a result, in terms of the time variation of the 'conserved' energy and angular momentum. By a heuristic argument, one replaces these by the corresponding average far-zone fluxes obtained previously. In the limit of $\eta \rightarrow 0$ our results reduce to the test particle results of Tagoshi to 2PN accuracy. These results along with the 'tail' contributions to the evolution of orbital elements, computed by Blanchet and Schafer and Schafer and Reith form the basic set of equations from where one can numerically evaluate the evolution of the orbital phase and other orbital elements as a function of time under the effects of gravitational radiation reaction forces.

In chapter 4 we address the "wave generation problem" which deals with the computation of the gravitational wave polarizations h_+ and $h_\times$, at the leading order in $1/R$, when the orbital phase and other parameters of the binary orbit take some specific values. In this chapter we compute all the 'instantaneous' 2PN contributions to h_+ and $h_\times$ for two compact objects of arbitrary mass ratio moving in elliptical orbits, using the 2PN corrections to the gravitational waveform for the general orbits obtained in chapter 2 and the generalized quasi-Keplerian representation for the 2PN motion. The expressions for h_+ and $h_\times$ obtained here represent gravitational radiation from an elliptical binary during that stage of inspiral when orbital parameters are essentially the same over a few orbital periods, in other words when the gravitational radiation reaction is negligible. We investigate the effect of eccentricity and orbital inclination on the amplitude of the Newtonian part of h_+ and $h_\times$. We observe that orbital inclination changes the magnitudes of $|h_+|^2$ and

$|h_\times|^2$ appreciably. The reduction in $|h_+|^2$ and $|h_\times|^2$ for small and medium eccentricities, is small compared to that for higher eccentricities, when the inclination angle is varied from 0 to $\pi/2$. We compute $\left(\frac{h_\times}{h_+}\right)^2$ at the Newtonian order and conclude that this ratio may be used as a good indicator for the orbital inclination, for very small to very high values of eccentricity. The modulation of h_+ and $h_\times$ due to the precession of the periastron, which occurs at the 1PN order is also explicitly shown. We recover in the circular limit the earlier results modulo the tail terms.

As mentioned earlier, the evolution of the orbital elements should be determined from the knowledge of the radiation reaction forces acting locally on the orbit. However, in practice this is determined *assuming* energy and angular momentum balance and the far-zone expressions for energy and angular momentum fluxes. The complete determination of the radiation reaction terms in the equations of motion requires a full iteration of the Einstein's field equations in the near-zone. In the absence of this complete result an interesting question to pose is the following. To what extent do the expressions of energy and angular momentum fluxes and the *assumption* of energy and angular momentum balance constrain the equations of motion? In the next chapter we address this question using the 'refined balance procedure' proposed by Iyer and Will (IW)

In chapter 5, we deduce the gravitational radiation reaction to the second post-Newtonian order beyond the quadrupole approximation – 4.5PN terms in the equations of motion – using the refined balance method of IW. We critically explore the features of their construction and illustrate them by contrast with other possible variants. We observe that in terms of the number of arbitrary parameters and the corresponding gauge transformations, the IW scheme exhibits remarkable stability for a variety of choices for the ambiguity in energy and angular momentum. We also show that the far-zone formulae and the balance equations by themselves do not constrain the reactive acceleration to be a power series in the binary's individual masses m_1 and m_2 as assumed by Iyer and Will, but are consistent with a more general form of the reactive acceleration. The equations of motion are valid for general binary orbits and for a class of coordinate gauges. The limiting cases of circular orbits and radial infall are also investigated.

In chapter 6, we summarize the work presented in this thesis and specify the theoretical issues which need to be addressed in the near future before one can explicitly provide the 'ready to use' 2PN accurate search templates for compact binaries in *quasi-elliptical orbits.*

These investigations are done in collaboration with the thesis advisor Prof. Bala R. Iyer and partly with Dr. Sai Iyer. These investigations are presented in the following papers.

1. *Second post-Newtonian gravitational radiation reaction for two-body systems: Nonspinning bodies,* A. Gopakumar, Bala R. Iyer, and Sai Iyer, Physical Review D, *55*, (1997) pp 6030 - 6054.

2. *Gravitational waves from inspiraling compact binaries: Angular momentum flux, evolution of the orbital elements and the wave form to the second post-Newtonian order,* A. Gopakumar and Bala R. Iyer, Physical Review D, *56*, (1998) pp 7708 - 7731.

3. *Second post-Newtonian gravitational wave polarizations for inspiraling compact binaries in elliptical orbits,* A. Gopakumar and Bala R. Iyer, Physical Review D (In preparation).