

**Numerical and Field Theoretic
Studies
in Low Dimensional Condensed
Matter Physics**

by

Debnarayan Jana

Thesis submitted to Jawaharlal Nehru University
for the degree of Doctor of Philosophy



**Raman Research Institute
Bangalore 560 080
July, 1997**

DECLARATION

I hereby declare that this thesis is composed independently by me at the Raman Research Institute, Bangalore, under the supervision of Prof. Joseph Samuel. The subject matter presented in this thesis has not previously formed the basis of the award of any degree, diploma, associateship, fellowship or any other similar title in any other University.

Joseph Samuel
29th July 1997

(Prof. Joseph Samuel)

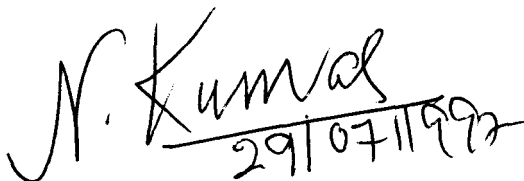
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This is to certify that thesis entitled Numerical and Field Theoretic Studies in Low Dimensional Condensed Matter Physics submitted by Debnarayan Jana, for the award of the degree of DOCTOR OF PHILOSOPHY of Jawaharlal Nehru University is his original work. This has not been published or submitted to any other University for any other Degree or Diploma.

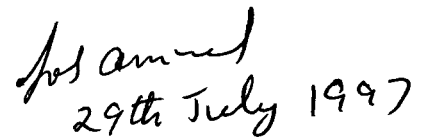

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Acknowledgements

I am deeply indebted to Prof. Joseph Samuel for his valuable guidance in my work. I learnt quite a bit of mathematical physics from him. I also would like to thank him for giving me so much opportunity to work freely.

It is my great pleasure to thank Dr. P. B. Sunil Kumar with whom I did a work which has been presented in this thesis. I thank him for collaboration of this work and many valuable discussions, advices and encouragement during the course of writing this thesis.

I thank Prof. C. S. Sukhre, Prof. Rajaram Nityananda, Prof. N. Kumar and Dr. Yashodhan Hatwalne for many illuminating discussions on the various parts of the work and also giving critical comments on the final version of the thesis.

I thank Prof. V. Radhakrishna, the past Director and Prof. N. Kumar, the present Director for giving me an opportunity to work in RRI.

I also take this opportunity to thank the administrative staff of RRI for various help during the period. Specially, I would like to thank GV and Krishna for help in numerous non-academic subjects.

I thank all the staff members of RRI library. Everyone helped whenever needed in any matter. I cannot think of any other good maintained and up to date library in India.

In the computer section, the help from P. Rajsekhar, P. Ramdurai, Ashima and Devadas are gratefully acknowledged during the period.

I have been benefited from discussions with various people. It is very difficult to put together all the names here. But I would like to mention at least few of them. I would like to thank Prof. Diptiman Sen of CTS, Dr. Madan Rao of IMsc, Dr. Rajesh Parwani, Dr. Supurna Sinha, Dr. Somnath Bharadwaj, Dr. G. Kang, Dr.

V. A. Rangunathan, Prof. A. M. Jayannavar of IOP, Dr. M. Sivakumar of Hyderabad University, Prof. Jayanth Banavar of USA, Prof. P. Grassberger of Germany, Prof. H. E. Stanley of USA, Dr. Gautom Mennon, Dr. Per Elmfors of CERN, Switzerland, Dr. David Persson of Sweden, Dr. Ory Zik of Israel. Prof. Stephane Roux and Prof. D. Sornette of France for various stimulating discussions.

I would like to thank Prof. T. V. Ramakrishnan, the then chairman of the physics dept, Iisc for providing an excellent graduate course work which helped me a lot for better understanding of the subject.

Many thanks to Manju for numerous help in both academic as well as non-academic matter. I will never forget his assistance throughout the period.

Thanks to Alex and Rangan (phys. dept, Iisc) for many illuminating discussions on various topics in physics; Also to Abhik, Somen and Soumya for their help and constant encouragement.

I would like to thank all of my friends in RRI who made my stay very pleasant in Bangalore. More specifically, Sreejith, Sushan, Sarala, Arun. Pramod. Geetha, Madam, Vikram, Gopu, Dipanjan, Ramach, Sobha, Nimisha, Jayadev, Amitabha (and all others) who all directly or indirectly have helped me a lot in various aspects. Special thanks to Sreejith and Andal for going through the earlier version of few chapters in this thesis. In final stage of thesis writing, I would like to thank Kumar, Sandip and Nalini (all from Mauritius) for their witty observations in various matters. Special thanks to Nalini for helping me in understanding a French paper.

Lastly, I would like to thank my parents, aunts, brother, sister, sister-in-law and brother-in-law for their constant support and encouragement in pursuing my research work.

This thesis is dedicated to my parents and youngest aunt

Contents

1	Magnetism of Charged Particles	1
1.1	Introduction	1
1.2	Classical Approach To Magnetism	2
1.3	Quantum Mechanical Approach To Magnetism	4
1.4	Electric Susceptibility vs Magnetic Susceptibility	5
1.5	Diamagnetic Inequality for Spinless Bose Systems	6
1.6	Brownian Motion and Magnetism	8
1.7	Conclusion and Discussion	10
2	Field Theoretic Treatment of Charged Bosons in a Magnetic Field	13
2.1	Introduction	13
2.2	Computation of Free energy in Free Field Theory	14
2.2.1	Zero Temperature Field Theory	17
2.2.2	Finite Temperature Field Theory	22
2.3	Interacting Field Theory	23
2.4	Conclusion and Perspective	26
3	Numerical Studies on two-dimensional Disordered Systems in a Magnetic Field	30
3.1	Introduction	30
3.2	The Tight-Binding Model	31
3.3	Density of States	32

3.4	Inverse Participation Ratio	32
3.4.1	Variation of IPR with Magnetic Flux	36
3.4.2	GIPR Exponents in Random Flux Model	38
3.5	Multifractality of Eigenstates	39
3.5.1	Relationship of Multi-fractality to Thermodynamics	43
3.5.2	Some General Properties of $\tau(q)$	44
3.6	Conclusion	45
4	Experimental and Numerical Studies on Imbibition	48
4.1	Introduction	48
4.2	Imbibition in Porous Medium	49
4.3	Characterization of the Interface	50
4.4	Review of Previous Work on Imbibition	51
4.5	Experiment	53
4.6	The Cellular Automaton Model	57
4.7	Simulation Results and Discussion	61
4.7.1	Static and Dynamic Exponent	61
4.7.2	Internal Driving Force	65
4.7.3	Finite Size Scaling	67
4.7.4	Distribution of Cluster of Dry Cells	69
4.8	Self-Organized Criticality (SOC)	70
4.9	Multi-affinity	72
4.10	Comparision with KPZ and DPD Models	75
4.11	Conclusion	76
5	Conclusion and Outlook	81
5.1	Introduction	81
5.2	Diamagnetism of Charged Spinless Bosons	81

5.3	2d Disordered Tight-Binding Hamiltonian in a Magnetic Field	83
5.4	Imbibition of Solution through Random Porous Medium	84
5.4.1	On Experimental Aspects	84
5.4.2	On Theoretical Aspects	86
A Diamagnetic behaviour of Dirac electrons in 2d in an external homogeneous magnetic field		91
B Scaling Exponents α and β from Various Theoretical Models and Experiments		95

Numerical and Field Theoretic Studies in Low Dimensional Condensed Matter Physics

Synopsis

In this thesis we study some low dimensional systems (classical as well as quantum) through field theoretic methods and numerical simulations. All the systems considered here are in two spatial dimensions. The thesis consists of three parts. The first part describes an analytical calculation of the response of a charged scalar field in an external magnetic field. We use the finite temperature field theory formalism to compute the partition function of this system in two dimensions. In the second part, we numerically study the properties of the disordered electronic eigenstates of a 2d tight-binding Hamiltonian in the presence of an external magnetic field. The third part deals with experimental and numerical studies of a non-equilibrium classical statistical mechanics problem. The problem we address here is interface formation in a random porous medium in two spatial dimensions.

In chapter one, we give a pedagogical introduction to magnetism of charged particles. We analyse the response of a single particle to an external magnetic field both classically and quantum mechanically. In this connection, we also indicate the difference between the responses of electric and magnetic fields. We review the proof due to B. Simon that N interacting spinless Bosons are diamagnetic. We also review an interesting connection between Brownian motion and magnetism.

In chapter two, we generalize the diamagnetism of N charged spinless Bosons (a system with a finite number of degrees of freedom) to charged scalar field theory (a system with an infinite number of degrees of freedom). For ease of presentation, we restrict ourselves to two spatial dimensions. Because of the infinite number of degrees of freedom present in a field theory the free energy of the system formally diverges. With a suitable regularisation scheme, we compute the difference between

the free energy in the presence of a magnetic field and that in the absence of a magnetic field. This free energy difference is computed exactly and shown to be positive. We also present a non-perturbative proof of diamagnetism in the case of interacting charged scalar fields in an arbitrary dimension.

In chapter three, we present some numerical studies on the electronic eigenstates obtained from the exact diagonalisation of a model tight-binding Hamiltonian in an external magnetic field with on-site disorder potential. We characterize the electronic eigenstates by Generalised Inverse Participation Ratios (GIPR). GIPR is a measure of the spatial extension of the eigenstates and it computes the various disordered averaged moments of the eigenstates. The nature of the eigenstate can be found out from the dependence of the GIPR on the size of the system. We also indicate the dependence of Inverse Participation Ratio (IPR) on the on-site disorder potential and on the flux. We compute GIPR in the case of random magnetic flux. We study the multifractality of these eigenstates (in the case of constant flux) by two methods. Using an existing analogy with thermodynamics, we indicate some interesting features of IPRs and multifractal behaviour of these eigenstates.

In chapter four, we study the interface formed by imbibition of a solution into a random porous medium by experiment and numerical simulation. If one places a filter paper in ink, one finds that the ink rises through the paper. After a while the flow stops and the ink leaves a jagged boundary on the paper. While the precise form of the boundary is different each time one does the experiment, it is known that statistical features of the jagged boundary are reasonably robust and amenable to study. We study the statistical features of this boundary in 'table-top' experiments as well as by numerical simulations using a cellular automaton model. This study focuses in particular on the effect of fluid evaporation on the statistical properties (measured by a roughness exponent) of the boundary. We find that the roughness exponent is not universal and depends on the evaporation rate. This is contrary to earlier claims in the literature about the universality of the roughness exponent.

The numerical results presented from a model simulation on a square lattice are in qualitative agreement with that obtained from experiments. Some aspects of this cellular automaton model have been studied numerically.

In the **final chapter**, we give our conclusions and discuss some possible extensions of the problems addressed in this thesis.