Numerical and Field Theoretic Studies in Low Dimensional Condensed Matter Physics

by

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for the degree of Doctor of Philosophy

Raman Research Institute
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DECLARATION

I hereby declare that this thesis is composed independently by me at the Raman Research Institute, Bangalore, under the supervision of Prof. Joseph Samuel. The subject matter presented in this thesis has not previously formed the basis of the award of any degree, diploma, associateship, fellowship or any other similar title in any other University.

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CERTIFICATE

This is to certify that thesis entitled **Numerical and Field Theoretic Studies in Low Dimensional Condensed Matter Physics** submitted by **Debnarayan Jana**, for the award of the degree of DOCTOR OF PHILOSOPHY of Jawaharlal Nehru University is his original work. This has not been published or submitted to any other University for any other Degree or Diploma.

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Numerical and Field Theoretic Studies in Low Dimensional Condensed Matter Physics

Synopsis

In this thesis we study some low dimensional systems (classical as well as quantum) through field theoretic methods and numerical simulations. All tlic systems considered here are in two spatial dimensions. The thesis consists of three parts. The first part describes an analytical calculation of the response of a charged scalar field in an external magnetic field. We use the finite temperature field theory formalism to compute the partition function of this system in two dimensions. In the second part, we numerically study the properties of the disordered electronic eigenstates of a 2d tight-binding Hamiltonian in the presence of an external magnetic field. The third part deals with experimental and numerical studies of a non-equilibrium classical statistical mechanics problem. The problem we address here is interface formation in a random porous medium in two spatial dimensions.

In chapter one, we give a pedagogical introduction to magnetism of charged particles. We analyse the response of a single particle to an external magnetic field both classically and quantum mechanically. In this connection. we also indicate the difference between the responses of electric and magnetic fields. We review tlie proof due to B. Simon that N interacting spinless Bosons are diamagnetic. We also review an interesting connection between Brownian motion and magnetism.

In chapter two, we generalize the diamagnetism of N charged spinless Bosons (a system with a finite number of degrees of freedom) to charged scalar field theory (a system with an infinite number of degrees of freedom). For ease of presentation, we restrict ourselves to two spatial dimensions. Because of the infinite number of degrees of freedom present in a field theory the free energy of the system formally diverges. With a suitable regularisation scheme, we compute the difference between the free energy in the presence of a magnetic field and that in the absence of a magnetic field. This free energy difference is computed exactly and shown to be positive. We also present a non-perturbative proof of diamagnetism in the case of interacting charged scalar fields in an arbitrary dimension.

In chapter three, we present some numerical studies on the electronic eigenstates obtained from the exact diagonalisation of a model tight-binding Hamiltonian in an external magnetic field with on-site disorder potential. We characterize the electronic eigenstates by Generalised Inverse Participation Ratios (GIPR). GIPR is a measure of the spatial extension of the eigenstates and it computes the various disordered averaged moments of the eigenstates. The nature of the eigenstate can be found out from the dependence of the GIPR on the size of the system. We also indicate the dependence of Inverse Participation Ratio (IPR) on the on-site disorder potential and on the flux. We compute GIPR in the case of random magnetic flux. We study the multifractality of these eigenstates (in the case of constant flux) by two methods. Using an existing analogy with thermodynamics, we indicate some interesting features of IPRs and multifractal behaviour of these eigenstates.

In chapter four, we study the interface formed by imbibition of a solution into a random porous medium by experiment and numerical simulation. If one places a filter paper in ink, one finds that the ink rises through the paper. After a while the flow stops and the ink leaves a jagged boundary on the paper. While the precise form of the boundary is different each time one does the experiment, it is known that statistical features of the jagged boundary are reasonably robust and amenable to study. We study the statistical features of this boundary in 'table-top' experiments as well as by numerical simulations using a cellular automaton model. This study focuses in particular on the effect of fluid evaporation on the statistical properties (measured by a roughness exponent) of the boundary. We find that the roughness exponent is not universal and depends on the evaporation rate. This is contrary to earlier claims in the literature about the universality of the roughness exponent.

The numerical results presented from a model simulation on a square lattice are in qualitative agreement with that obtained from experiments. Some aspects of this cellular automaton model have been studied numerically.

In the **final chapter,** we give our conclusions and discuss some possible extensions of the problems addressed in this thesis.