SOME EXACT ANALYTIC MODELS OF TIME DEPENDENT COLLISIONLESS STELLAR SYSTEMS

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DECLARATION

I hereby declare that the work presented in this thesis is entirely original, and has been carried out by me at the Raman Research Institute under the auspices of the Department of Physics, Indian Institute of Science. I further declare that this has not formed the basis for the award of any degree, diploma, membership, associateship or similar title of any University or Institution.

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CONTENTS

CHAPTER 1

SUMMARY

The dynamics of stars as well as dark matter in galaxies is governed purely by mutual gravitational interactions. Over time scales of the order of the age of the universe ($\sim 10^{10}$ years) binary encounters produce a negligible effect on orbits calculated by assuming that the mass distribution of the galaxy is smooth. The galaxy is then well approximated as an incompressible fluid in 6 dimensional phase space that moves under the action of its self gravity. This self consistent dynamics is described by the collisionless Boltzmann equation (CBE). Galaxies are usually modelled as stationary (or stationary in a rotating frame) solutions of the CBE. Stationary solutions are relatively well understood (see eg. Binney and Tremaine 1987) and in fact there is a rich variety of such galaxy models. However, problems concerning the formation of galaxies and interactions between them involve understanding the behaviour of time dependent solutions of the CBE.

The work reported in this thesis concerns some exact oscillating solutions of the CBE. Time dependent solutions have long been conjectured to relax to a steady state. The relaxation is broadly confirmed by numerical experiments (see eg. van Albada 1982) but theoretical understanding has been limited. An early attempt by Lynden-Bell (1967) was based on maximising an entropy function subject to conservation of fine grained phase space density. This led to

novel statistics. More recently, the role of the constraint coming from Liouville's theorem has been emphasised by Tremaine et. al. (1986) - see also Mathur (1988). The statistical arguments of Lynden-Bell require violent and random redistribution of elements in phase space. Even the weaker constraints of Tremaine et. al. were found on closer examination to involve a plausible but nonrigorous assumption of repeated coarse graining (Kandrup 1987, Dejonghe 1987, Sridhar 1987). It is clearly of interest and relevance to clarify the role of exact and approximate invariants in the time dependent behaviour of collisionless self gravitating systems.

Chapter 2 includes a brief sketch of some relevant background material about galaxy dynamics and a discussion of the time dependent behaviour of collisionless stellar systems. The general strategy of using exact invariants (for time dependent harmonic potentials) to construct exact, time dependent, self consistent solutions of the CBE is discussed here. The method is illustrated by constructing the homologous oscillation mode of a homogeneous slab of stars discovered by Kalnajs (1973).

Chapter 3 is a detailed account of uniform density spherical and spheroidal models both of which exhibit undamped oscillations. The equations governing the oscillations are given below without derivation to illustrate the general structure. These equations were written down by

Chandrasekhar & Elbert (1972) and Sam Sunder & Kochhar (1986) for the case of spheres and spheroids respectively based on the tensor virial theorem and an ansatz for the kinetic energy tensor. However the present work goes much further in providing the underlying phase space distribution without which the existence of undamped oscillation could not be regarded as proved.

Spheres. The radius (R) obeys

$$\ddot{R} + \frac{a^2}{R^2} - \frac{1}{R^3} = 0 \qquad (1.1)$$

where a is a constant. So a11 spatially bound solutions are time periodic.

Spheroids. The equations governing the oscillations are

$$\begin{aligned} \ddot{x} + \frac{\beta}{x^2} g - \frac{1}{x^3} &= 0 \\ \ddot{y} + \frac{2\beta}{x^2}(1 - u g) - \frac{\alpha}{y^3} &= 0 \end{aligned} \qquad (1.2)$$

where α and β are constants. y is the axis of symmetry and x is the other axis. u = axis ratio = y/x and $u g(u)$ is the restoring force along the axis of symmetry. It is shown that (1.2) can be put in Hamiltonian form. Surface of section studies reveal periodic, quasiperiodic and chaotic solutions. We conjecture that some of these models (especially the spheres) are stable. This implies that nearby solutions which do not have precisely uniform density remain in the vicinity of the uniform density solutions.

In Appendix A we present some models of periodically oscillating spherical systems with nonuniform density. These are made of particles moving in nonintersecting oscillating shells. An interesting feature is that Jeans' theorem is not explicitly used in building these models. In fact, the constants of motion used are not global in phase space, but are valid only for the specific orbits which are populated.

In Chapter 4 we construct a generalisation of Freeman's (1966) analytic bars. We call these Generalised Freeman Discs (GFDs). In addition to rotation (a property of Freeman's bars) the GFDs are characterised by changes in size and shape. The coupling between rotation and oscillation allows for time varying rotation speeds. A GFD is described by a time dependent 4 x 4 real, positive definite symmetric matrix whose elements are essentially the covariances of the four phase space variables averaged over the whole systems. Self consistency gives a nonlinear matrix equation governing the time evolution of a GFD. Appendix B serves as an extended footnote to Chapter 4, with details that might impede the main argument. In Appendix C we study the general nature of the matrix equation. It is shown there that the equations describe a Hamiltonian dynamical system. Hence, in common with the earlier models, the GFDs also show nonrelaxing behaviour. Appendix D presents a novel numerical scheme for solving the matrix equation. The chief feature of this scheme is that it is symplectic (and hence nondissipative) to machine accuracy.

The only general approach to the problems of galaxy - galaxy interactions appears to be large scale numerical simulation. However, some limiting cases have received analytical study (for references see the review by Alladin and Narasimhan 1982). In Chapter 5 we show that the formalism developed to describe isolated GFDs can be extended to include the effect of tidal forces due to a perturber moving in the plane of the disc. The only approximation made is the lowest order tidal approximation describing the interaction. The matrix equation for the GFD plus the equations of motion for the perturber (4 for a point mass perturber) completely describe the interaction model. So we are able to follow the self consistent response of the GFD without any further approximations (like impulsive or adiabatic). As a specific example, we compute the response of a Kalnajs disc (corotating, nonrotating as well as counter rotating) to a massive point mass perturber moving in the plane of the disc on various hyperbolic trajectories. The fractional energy gained by the disc (which, due to interactions has become a GFD) is plotted as a function of the encounter timescale. Deviations from the predictions of the impulse plus distant encounter approximation are significant. Interesting nonmonotonic behaviour of energy and angular momentum transfer along the sequence of encounters is found. It is suggested that the short time behaviour of these idealised systems which determines energy and angular momentum transfer would also carry over to more realistic cases. Of course the resonant effects would tend to be washed out for galaxy models with a range of orbital periods.