CHAPTER 3

NEW ADDRESSING TECHNIQUES FOR MULTZPLEXING LCDs WITH RMS RESPONSE - ANALYTICAL STUDIES

3. NEW ADDRESSING TECHNIQUES FOR MULTIPLEXING LCDs WITH RMS RESPONSE - ANALYTICAL STUDIES

Various addressing techniques, currently available for multiplexing TNLCDs were reviewed and the need for new addressing techniques was discussed in chapter 2. This chapter is exclusively reserved for the presentation of analytical studies on some new addressing techniques proposed for driving LCDs. The **line-by-line addressing** forms the basis for most of the addressing techniques used for multiplexing a matrix display. A logical question at this stage could be, '**Is line-by-line selection** the only way to address a matrix display?' The Binary Addressing Technique (BAT) put forth first in this chapter is an attempt to answer this question. BAT requires simple addressing waveforms; but, it is suitable only when the number of lines to be multiplexed (N) is small. The selection ratio of BAT is lower than that of the conventional APT and IAPT. However, a lower selection ratio for a small N is not a serious problem, since NLC mixtures suitable for multiplexed displays with N greater than 64 are available at present. The Hybrid Addressing Technique (HAT) discussed next is an extension of BAT to make it suitable for addressing matrix displays with a large N. However, the selection ratio of HAT is also lower as compared to that of APT. This is a major limitation of HAT, considering the fact that N is large here. The Improved Hybrid Addressing Technique (IHAT) presented next achieves the maximum selection ratio possible for any addressing technique, i.e., the same as that of APT. In addition to this, IHAT requires a lower power supply voltage and has a better brightness uniformity of the pixels as compared to IAPT (and APT). These advantages in IHAT are achieved by increasing the complexity of the addressing waveforms of HAT. The trade-off between the selection ratios and the complexity of the addressing waveforms is discussed next. A detailed analysis of these techniques is covered in this chapter along with a comparison of their performance for displaying general patterns in matrix displays based on electro-optic effects with rms response.

The importance of Restricted Pattern Addressing Technique (RPAT) was also covered in the previous chapter. Considering the limitations of the existing techniques, new ones for displaying restricted patterns are also proposed in this chapter. The selection ratio in this case is independent of the matrix size and it just depends on the number of selected pixels in each column.

3.1. BINARY ADDRESSING TECHNIQUE (BAT)

The binary addressing technique follows a totally different approach to multiplex a matrix display as compared to the conventional techniques based on line-by-line addressing. All the lines to be multiplexed are simultaneously selected in BAT instead of one line at a time as in the case of APT. Important aspects of BAT are considered in this section.

3.1.1. Background

The following conditions are satisfied in a matrix display used for numeric and alphanumeric applications:-

- Individual pixels are in two states only, i.e., either ON or OFF, and
- The data to be displayed in a column is an N-bit binary pattern.

Hence, the N rows to be multiplexed in a matrix display can as well be simultaneously selected with voltages corresponding to an N-bit binary pattern. In general, the row-select voltages can be $\pm V_r$ and the column voltages can be $\pm V_c$. The voltages across a pixel, for the various combinations of the row and column voltages are given in Table 3.1. Only the voltage amplitudes are considered here since LCDs respond equally well to both positive and negative voltages. The following conditions are desired in a multiplexed display:-

- All the **OFF** pixels should get as low a voltage as possible across them, and
- All the **ON** pixels should get as large a voltage as possible across them.

Hence a pixel is said to get a favourable voltage when -

- the OFF pixel has a voltage $|V_r - V_c|$ across it; or
- the ON pixel has a voltage $|V_r + V_c|$ across it.

The voltage pattern across the pixels in a column in general corresponds to either -

- the row-select pattern itself, when the sign of the column voltage is the same as that of row-select voltage for logic 0, or
- the complement of the row-select pattern, when the sign of the column voltage is the same as that of row-select voltage for logic 1,

when $|V_r - V_c|$ and $|V_r + V_c|$ are assigned logic 0 and logic 1 respectively.

In general, the data pattern will be different from the row-select pattern or its complement. Hence, some of the pixels in a column will get an unfavourable voltage. The unfavourable voltage or error in this context means, either the column voltage is -

Table 3.1. Pixel voltages for different Row and Column Voltages

Row Voltage	Column Voltage	Resultant Voltage across a pixel
$-V_r$	$-V_c$	$-(V_r - V_c)$
$-V_r$	$+V_c$	$-(V_r + V_c)$
$+V_r$	$-V_c$	$+(V_r + V_c)$
$+V_r$	$+V_c$	$+(V_r - V_c)$

Table 3.2. Column voltages of BAT for all the possible data patterns and the row-select voltages

Row select voltages a_3	a_2	a_1 \ Data patterns								
		d_3	0	0	0	0	1	1	1	1
		d_2	0	0	1	1	0	0	1	1
		d_1	0	1	0	1	0	1	0	1
			Column voltages							
0	0	0	0	0	0	V_s	0	V_s	V_s	V_s
0	0	V_s	0	0	V_s	0	V_s	0	V_s	V_s
0	V_s	0	0	V_s	0	0	V_s	V_s	0	V_s
0	V_s	V_s	V_s	0	0	0	V_s	V_s	V_s	0
V_s	0	0	0	V_s	V_s	V_s	0	0	0	V_s
V_s	0	V_s	V_s	0	V_s	V_s	0	0	V_s	0
V_s	V_s	0	V_s	V_s	0	V_s	0	V_s	0	0
V_s	V_s	V_s	V_s	V_s	V_s	0	V_s	0	0	0

- out-of-phase with the row-select voltage for an **OFF** pixel, resulting in a voltage of $|V_r + V_c|$ instead of $|V_r - V_c|$ across it, or
- in-phase with the row-select voltage for an **ON** pixel, resulting in a voltage of $|V_r - V_c|$ instead of $|V_r + V_c|$ across it.

The presence of an unfavourable voltage across a pixel can be tolerated in a multiplexed display for the following reasons :-

- TNLCDs respond to the rms voltage rather than the instantaneous voltage, due to their slow response time. Hence the presence of an unfavourable voltage across a pixel does not turn **ON** or turn **OFF** the pixel, as long as its duration is smaller than the response time of the display, and
- TNLCDs respond to the external electric field, only above a threshold voltage (V_{th}). Hence, the **OFF** pixels can accommodate a voltage below V_{th} without being turned **ON**.

However, the presence of an error either -

- increases the rms voltage across the **OFF** pixel, or
- decreases the rms voltage across the **ON** pixel.

This leads to a decrease in the selection ratio from the ideal value, i.e., infinite. A similar situation is encountered in the line-by-line addressing due to the presence of the column voltage across the pixels in the unselected rows. Hence the number of errors should be minimum for a given pixel. The number of errors in a column can be 0-N depending on the data and the row-select patterns. However, this can be reduced to be less than or equal to (N/2) by a proper choice of the sign of the column voltage.

This is illustrated in Appendix 6.a. The number of errors in each pixel will be different for the following reasons:-

- the number of mismatches and hence the error depends on both the row-select and the data patterns, and
- the data pattern is different for each column.

The rms voltages across the pixels in the same state should be equal in order to ensure display uniformity. This is possible only when the number of errors is equal for the pixels in the same state. This can be achieved by selecting the rows with all the 2^N binary patterns one after another for an equal duration of time. The number of errors for each pixel (both ON and OFF) in the display is the same and it increases with the value of N. This leads to a decrease in the selection ratio as the number of rows (N) in the matrix increases. This is similar to that of the conventional line-by-line addressing techniques (APT, IAPT, etc.), wherein the decrease in the selection ratio with the increase in N is due to the increase in the duration of the column voltage appearing across the pixels when the corresponding rows are unselected.

3.1.2. Technique

The data to be displayed in a column is an N-bit word and is represented by

$$d_1, d_2, d_3, \ldots\ldots, d_N \ ; \ d_i = 0 \text{ or } 1 \tag{3.1}$$

wherein, logic 0 and logic 1 represent OFF and ON pixels respectively. Similarly the row-select pattern is an N-bit word and is represented by

$$a_1, a_2, a_3, \ldots\ldots, a_N \ ; \ a_i = 0 \text{ or } 1 \tag{3.2}$$

The various steps involved in BAT are given below.

i) An N-bit word is chosen as row-select pattern. The row-select voltages are chosen to be zero for logic 0 and V_s for logic 1 ;

ii) The row-select and the data patterns are compared bit-by-bit using digital comparators, viz., exclusive-OR gates ;

iii) The number of mismatches i between these two patterns is determined by counting the number of exclusive -OR gates with logic 1 output ;

iv) The column voltage V_c is decided by a majority decision. The column voltage is zero (logic 0) if i is less than $N/2$ and is V_s (logic 1) if i is greater than $N/2$.

The steps (ii) - (iv) can be summarized as follows:-

$$i = \sum_{j=1}^{N} a_j \oplus d_j \qquad (3.3)$$

and

$$V_c = \begin{cases} 0 & \text{for} \quad i < (N/2) \\ V_s & \text{for} \quad i > (N/2) \end{cases} \qquad (3.4)$$

The condition $i = (N/2)$ is avoided in this majority decision by choosing N to be odd ;

v) The column voltages for each column in the matrix are determined independently by repeating the steps (ii) - (iv) ;

vi) Both the row-select and the column voltages are applied simultaneously to the matrix display for a time duration T ;

vii) A new row-select pattern is chosen and the column voltages are determined for this by using steps (ii) - (v). The new row and column voltages are applied simultaneously to the matrix display for an equal duration of time at the end of T ;

viii) A cycle is completed when all the 2^N binary patterns are covered as row-select patterns once; and

ix) The display is refreshed by repeating this cycle continuously.

The time duration T should be small as compared to the response time of the display, in order to ensure the rms behavior of the display. The time duration of a cycle (= $2^N . T$) should be low in order to avoid flicker in the display.

The rms voltage across the pixels is independent of the sequence in which the 2^N row select patterns are chosen for addressing the display. This allows some freedom in the choice of the sequence to suit the display characteristics.

The column voltages for all the possible combinations of the row-select and data patterns, for N = 3 are shown in Table 3.2 as an example. From this table, it is evident that the column voltage is -

- complemented when either the row-select pattern or the data pattern is complemented ; or
- the same when both the row-select and the data patterns are complemented.

Typical addressing waveforms of BAT for N = 3 are shown in Fig. 3.1. The row waveforms R0-R2 form the eight binary row-select patterns. The

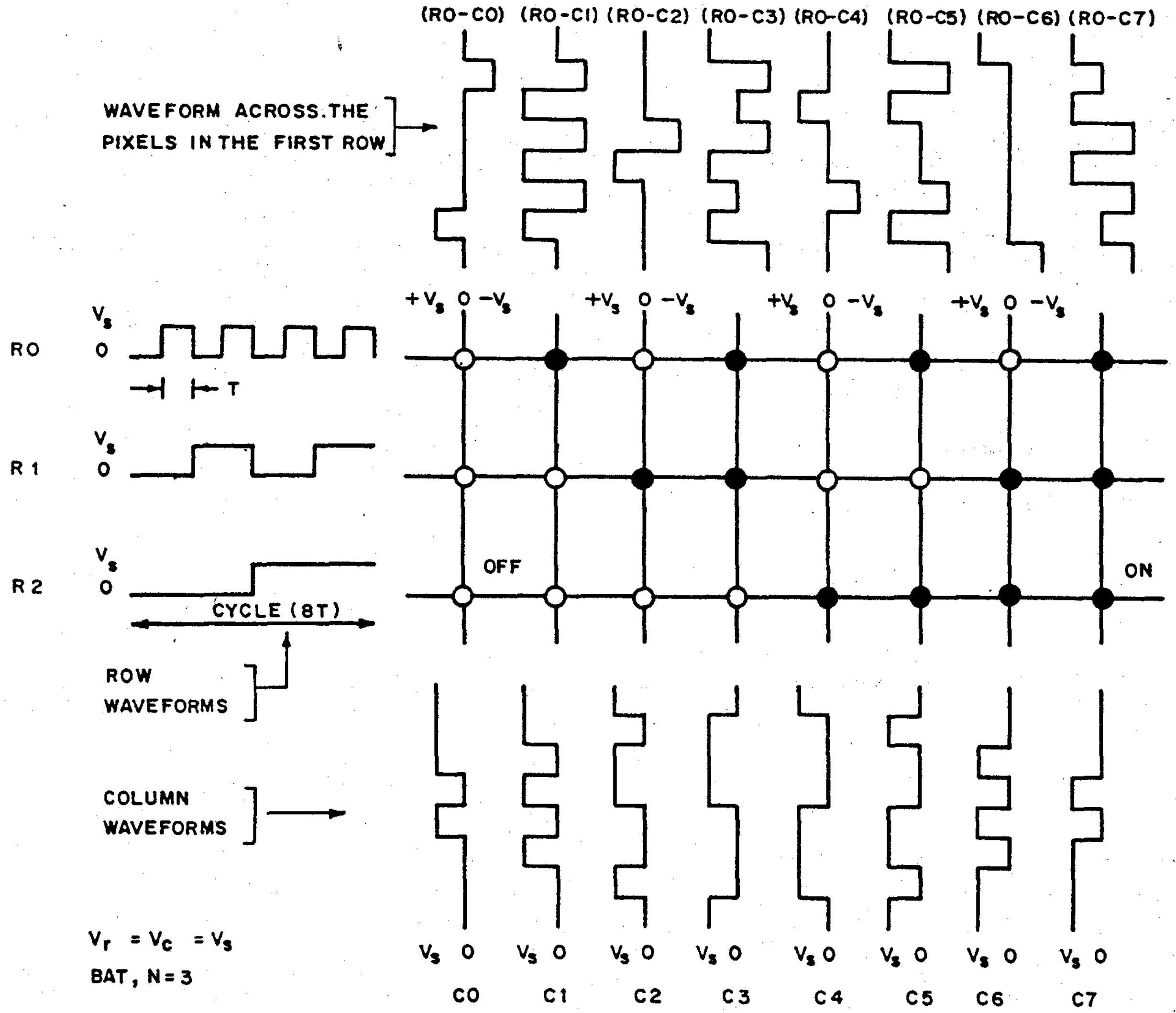

Fig. 3.1. Typical addressing waveform of BAT

column waveforms required to display the eight data patterns (C0 - C7) are shown here. The waveforms across the pixels in the first row are also included in this figure, to illustrate the inherent dc-free operation of BAT.

3.1.3. Analysis

Let the number of rows (N) be odd, and the row-select voltage be $-V_r$ for logic 0 and $+V_r$ for logic 1. Similarly, let the column voltage be $-V_c$ for logic 0 and $+V_c$ for logic 1. Only one column need be considered for the purpose of analysis, since the data is multiplexed through the column line. The number of errors per pixel during a cycle is calculated as follows:-

The number of N-bit row-select patterns which differ from the N-bit data pattern by i bits is given by

$$C_i = \frac{N!}{i!(N-i)!} \tag{3.5}$$

In this context, C_i gives the number of row-select patterns with i mismatches. The total number of mismatches ($= iC_i$) are equally distributed over the N pixels in the column. Hence, the number of mismatches per pixel when there are i mismatches in the column is given by

$$B_i = \frac{iC_i}{N} \tag{3.6}$$

An expression for B_i in terms of N and i is obtained by substituting for C_i using eqn. (3.5). Hence,

$$B_i = \frac{i}{N}\,\frac{N!}{i!(N-i)!} = \frac{i(N-1)!}{i!(N-i)!} \tag{3.7}$$

Since the number of errors is the same as the number of mismatches when i is less than N/2, only those $2^{(N-1)}$ row-select patterns which differ from the data pattern by less than N/2 bits are considered here. The rest of the $2^{(N-1)}$ patterns with the number of mismatches greater than N/2 also give the same number of errors per pixel with the column voltage corresponding to logic 1.

The number of times a pixel gets a favourable voltage when the C_i row-select patterns with i mismatches are considered is given by,

$$A_i = (C_i - B_i) = \left[\frac{N!}{i!(N-i)!} - \frac{i(N-1)!}{i!(N-i)!}\right] \qquad (3.8)$$

which is simplified to

$$A_i = \frac{(N-1)!}{i!(N-i)!}(N-i) = \frac{(N-1)!}{i!(N-i-1)!} \qquad (3.9)$$

The summations of A_i and B_i over i ranging from 0 to $(\frac{N-1}{2})$ give the number of times a pixel gets a favourable voltage and error respectively during the $2^{(N-1)}$ time intervals considered. The maximum value of error is $(\frac{N-1}{2})$ since N is odd here. Hence,

$$A = \sum_{i=0}^{(\frac{N-1}{2})} \frac{(N-1)!}{i!(N-i-1)!} \qquad (3.10)$$

and

$$B = \sum_{i=0}^{(\frac{N-1}{2})} \frac{i.(N-1)!}{i!(N-i)!} \qquad (3.11)$$

Hence, a pixel will get a favourable voltage during 2A time intervals in a cycle. Similarly a pixel will get an unfavourable voltage or error during 2B time intervals in a complete cycle. The expression for the rms voltage across the **ON** and **OFF** pixels are arrived at using the above statistics and the definitions of favourable and unfavourable voltages.

$$V_{ON}(rms) = \left[\frac{2.A.T(V_r + V_c)^2 + 2.B.T\,(V_r - V_c)^2}{2^N\,.\,T}\right]^{1/2} \qquad (3.12)$$

and

$$V_{OFF}(rms) = \left[\frac{2.A.T\,(V_r - V_c)^2 + 2.B.T\,(V_r + V_c)^2}{2^N.T}\right]^{1/2} \qquad (3.13)$$

The selection ratio (R) given by V_{ON}/V_{OFF} should be a maximum, in order to achieve a good discrimination between the **ON** and **OFF** pixels in the display. Here, the selection ratio is a maximum for

$$V_r/V_c = 1 \qquad (3.14)$$

Hence the number of voltage levels in the addressing waveform is just two. A dc-free operation is ensured independent of the polarity of the row and column voltages since every row-select pattern has its complement within the 2^N combinations. Hence unipolar addressing waveforms with two voltage levels, i.e., 0 and V_s are sufficient in BAT.

The maximum selection ratio is

$$R = \frac{V_{ON}(rms)}{V_{OFF}(rms)} = \left[\frac{A}{B}\right]^{1/2} \qquad (3.15)$$

The selection ratio can also be expressed in terms of A and B using the relation given below.

$$(A + B) = 2^{(N-1)} \qquad (3.16)$$

Hence

$$R = \left[\frac{A}{2^{(N-1)} - A}\right]^{1/2} = \left[\frac{2^{(N-1)} - B}{B}\right]^{1/2} \qquad (3.17)$$

The selection ratio of BAT is compared with that of APT (or IAPT) using N_{eq} as follows.

$$R = \left[\frac{A}{B}\right]^{1/2} = \left[\frac{A}{2^{(N-1)} - A}\right]^{1/2} = \left[\frac{N_{eq}^{1/2} + 1}{N_{eq}^{1/2} - 1}\right]^{1/2} \tag{3.18}$$

Hence,

$$N_{eq}(BAT) = \left[\frac{A + B}{A - B}\right]^2 = \left[\frac{2^{(N-1)}}{2A - 2^{(N-1)}}\right]^2 \tag{3.19}$$

The selection ratio of BAT is compared with that of IAPT in Table 3.3 for different values of N. It is evident that the selection ratio of BAT is lower as compared to that of IAPT. The value of N_{eq} for BAT is higher than N and this also reflects the above observation.

The **OFF** pixels are biased close to V_{th} so as to obtain the best contrast ratio in the display. Hence the voltage V_r (= V_c) can be determined in terms of V_{th} as follows.

$$V_{OFF} = V_{th} = \left[\frac{(2^{(N-1)} - A) \cdot 2^2}{2^{(N-1)}}\right]^{1/2} \cdot V_c \tag{3.20}$$

Therefore,

$$V_c = \left[\frac{2^{(N-3)}}{2^{(N-1)} - A}\right]^{1/2} \cdot V_{th} \tag{3.21}$$

The maximum swing in the addressing waveforms is $2.V_c$ since, the column voltages were assumed to be $+V_c$ and $-V_c$. Hence, the supply voltage for this technique is

$$V_{supply} = 2.V_c = \left[\frac{2^{(N-1)}}{2^{(N-1)} - A}\right]^{1/2} \cdot V_{th} \tag{3.22}$$

Table 3.3. BAT vs. IAPT - A comparison

No. of lines multiplexed (N)	Selection ratio (R): BAT $[A/B]^{1/2}$	Selection ratio (R): IAPT $\left[\frac{N^{1/2}+1}{N^{1/2}-1}\right]^{1/2}$	N_{eq} for BAT	Supply voltage normalized to V_{th}: BAT	Supply voltage normalized to V_{th}: IAPT	Supply voltage normalized to V_{th}: IAPT with a reduced selection ratio (IAPT-R)	$\frac{V_{supply}\ (BAT)}{V_{supply}\ (IAPT-R)} \times 100\%$	Duty cycle: BAT	Duty cycle: IAPT
3	1.732	1.932	4	2.000	2.972	2.449	81.65	3/4	1/3
5	1.483	1.618	7.11	1.789	3.078	2.461	72.69	11/16	1/5
7	1.382	1.488	10.24	1.706	3.269	2.558	66.69	42/64	1/7
9	1.324	1.414	13.37	1.659	3.464	2.668	62.17	163/256	1/9
11	1.286	1.365	16.51	1.629	3.652	2.778	58.63	638/1024	1/11

The supply voltage requirement of BAT can also be expressed in terms of B as follows:-

$$V_{supply} = \left[\frac{2^{(N-1)}}{B}\right]^{1/2} \cdot V_{th} \qquad (3.23)$$

The addressing waveforms of BAT can be made unipolar by shifting them by $+V_c$ ($= +V_r$). Hence, the row and column voltages are 0 for logic 0 and V_s for logic 1. However, the supply voltage requirement remains the same here, i.e., $V_s = V_{supply}$.

The supply voltage requirements of BAT and IAPT are also given in Table 3.3. It is evident that BAT requires a lower supply voltage as compared to IAPT. Moreover the supply voltage requirement of BAT decreases with N, while it increases with N in the case of APT and IAPT. However, the selection ratio of IAPT is higher than that of BAT. The selection ratio of IAPT can be reduced to that of BAT in order to properly compare the V_{supply} of these two techniques. Let x be the reduced (V_r/V_c) of IAPT and K be the square of the selection ratio of BAT. The value of x can be determined from the following equation.

$$(R_{BAT})^2 = K = \frac{(x+1)^2 + (N-1)}{(x-1)^2 + (N-1)} \qquad (3.24)$$

Hence,

$$x = \frac{(K+1) - [(K+1)^2 - (K-1)^2 N]^{1/2}}{(K-1)} \qquad (3.25)$$

The supply voltage required for this IAPT with a reduced selection ratio (IAPT - R) is obtained on the same lines as eqn. (2.18)

$$V_{supply}\ (IAPT - R) = (V_r + V_c) = \left[\frac{N}{(x-1)^2 + (N-1)}\right]^{1/2} (x+1).V_{th} \qquad (3.26)$$

The V_{supply} for this IAPT-R leading to the same selection ratio as BAT is also given in Table 3.3 for comparison. It is evident that the supply voltage of BAT is lower even when it is compared with that of the modified IAPT. This is primarily due to the large duty cycle of this technique. The pixels get a favourable voltage during 2A time intervals in a single cycle. Hence, the duty cycle of BAT is

$$\text{Duty cycle} = \frac{2A}{2^N} = \frac{A}{2^{(N-1)}} \tag{3.27}$$

The duty cycle of BAT is also compared with that of IAPT in Table 3.3.

3.1.4. Discussion

The merits and demerits of BAT are listed in Table 3.4. This technique is not suitable for displays with a large number of lines (N), since the number of time intervals to complete a cycle ($= 2^N$) increases rapidly with N. The lower selection ratio of BAT is not a serious problem from the following considerations : -

- This technique is suitable only for a small N, and
- The contrast ratio in the display need not be compromised, since NLC mixtures that are suitable for multiplexing more than 64 lines are available at present.

The BAT can also be used to multiplex a display with even number of rows by choosing the column voltage to correspond to either logic 0 or logic 1, when the number of mismatches is N/2. However, the selection ratio obtained is the same as that of the next odd integer, i.e., (N + 1). A higher selection ratio is possible, if the column voltage is chosen to be mid-way between the voltages corresponding to logic 0 and logic 1.

Table 3.4. Merits and Demerits of BAT

<u>Merits</u>
- Simple addressing waveforms with just two voltage levels ;
- Standard CMOS ICs usable for row and column driving ;
- Natural dc-free operation ;
- Low supply voltage and decreases with increase in N ;
- Large duty cycle
<u>Demerits</u>
- Not suitable for multiplexing displays with large N ;
- Selection ratio lower as compared to IAPT ;
- N to be odd .

However, the number of voltage levels in the column waveform will then be 3 instead of 2. This is not preferred since the main advantage of BAT will be lost. Hence, an additional dummy row can be added (not connected to the display) when N is even. This satisfies the condition that N should be odd.

3.2. HYBRID ADDRESSING TECHNIQUE (HAT)

The hybrid addressing technique presented here is an extension of BAT, in order to make it suitable for multiplexing a large number of lines (N). Although there is no theoretical limit on the value of N in BAT, the duration of a cycle ($= 2^N.T$) increases rapidly with N as discussed in the previous section. Hence, BAT is suitable only when the number of lines to be multiplexed in a display is small. The HAT proposed in this section is a combination of BAT and APT. The number of time intervals to complete a cycle is low in HAT as compared to that of BAT. Important aspects of HAT are analysed here.

3.2.1. Background

The N address lines to be multiplexed in a display can be divided into non-intersecting subgroups, when N is large. Each of these subgroups can be addressed one at a time using the BAT. This is similar to the line-by-line addressing in APT. The number of time intervals to complete a cycle is not high here for the following reasons:-

- Only the subgroups are addressed at a time ; and
- Only a few address lines are present in each subgroup.

The number of address lines in the subgroup (l) should be odd since, BAT is used to address them. Moreover N should be an integral multiple of l for this technique.

3.2.2. Technique

The data to be displayed in the selected subgroup in any column is an l-bit word and is represented by

$$d_{kl+1}, d_{kl+2}, \ldots, d_{kl+l}; \quad d_{kl+j} = 0 \text{ or } 1 \tag{3.28}$$

wherein, logic 0 is for an OFF pixel, logic 1 is for an ON pixel. The value of k ranges from 0 to $[(N/l) - 1]$ corresponding to the selected subgroup. Similarly the row-select pattern is an l-bit word and is represented by

$$a_{kl+1}, a_{kl+2}, \ldots, a_{kl+l}; \quad a_{kl+j} = 0 \text{ or } 1 \tag{3.29}$$

The various steps involved in HAT are given below : -

i) One subgroup at a time is selected for binary addressing;

ii) An l-bit word is chosen as the row-select pattern;

iii) The row-select voltages are chosen to be $-V_r$ for logic 0 and $+V_r$ for logic 1, while the $(N-l)$ unselected rows are grounded;

iv) The row-select and the data patterns in the selected subgroup are compared bit-by-bit using digital comparators, viz., exclusive-OR gates;

v) The number of mismatches i between these two patterns is determined by counting the number of exclusive-OR gates with logic 1 output;

vi) The sign of the column voltage V_c is decided by a majority decision. The column voltage is $-V_c$ if i is less than $l/2$ and is $+V_c$ otherwise. The condition $i = l/2$ is excluded here by choosing l to be odd.

The steps (iv) - (vi) are similar to those in BAT and are summarized as follows : -

$$i = \sum_{j=1}^{l} a_{kl+j} \oplus d_{kl+j} \qquad (3.30)$$

and

$$V_c = \begin{cases} \text{negative for} & i < (l/2) \\ \text{positive for} & i > (l/2) \end{cases} \qquad (3.31)$$

vii) The sign of the column voltage for each column in the matrix is determined independently by repeating the steps (iv) - (vi) ;

viii) Both the row and column voltages are applied simultaneously to the matrix display for a time duration T ;

ix) A new row-select pattern is chosen and the column voltages are determined by using steps (iv) - (vi). The new row and column voltages are applied to the matrix display for an equal duration of time at the end of T ;

x) The binary addressing of a subgroup is completed when all the 2^l binary patterns are covered as row-select patterns once ;

xi) A cycle is completed when all the subgroups (= N/l) are subjected to binary addressing once ;

xii) The display is refreshed by repeating this cycle continuously.

The time duration T should be small as compared to the response time of the display, in order to ensure rms behavior of the display. Apart from the choice in the sequence of row-select pattern as in the case of BAT, the HAT has additional freedom in the manner in which the subgroups are selected as given below : -

- A subgroup can be selected with 2^j row-select patterns consecutively before selecting the subsequent subgroup. Here j can range from 0 to l. However a cycle will be completed only when all the 2^l row-select patterns are covered for each subgroup in the matrix.
- The sequence in which the subgroups are selected can also be altered as long as all the subgroups are addressed with BAT.

Although the rms voltage is the same for all the cases discussed above, the frequency components of the addressing waveform are different for each case. The addressing sequence can be selected to suit the display characteristics.

The column voltages for all the combinations of row-select voltages and data patterns when l is 3 are shown in Table 3.5, as an example.

Typical addressing waveforms of HAT with $l=3$ are given in Fig.3.2. The addressing waveforms of HAT are naturally dc-free since the subgroups are addressed with BAT. Every row-select pattern has its complement within the 2^l row-select patterns and hence the polarity of the row as well as the column voltages are reversed within a cycle. This is evident from the waveforms across the pixel shown in the same figure. The sequences of the row-select pattern are also different for the two subgroups. This is also shown in Fig.3.2. It is evident that the rms voltage across the pixel is independent of the sequence in which the row-select patterns are applied to the subgroups.

Table 3.5. Column voltages of HAT for all the possible data patterns and the row-select voltages

Row-select voltages a_{kl+3}	a_{kl+2}	a_{kl+1}								
		Data patterns d_{kl+3}	0	0	0	0	1	1	1	1
		d_{kl+2}	0	0	1	1	0	0	1	1
		d_{kl+1}	0	1	0	1	0	1	0	1
			Column voltages							
$-V_r$	$-V_r$	$-V_r$	$-V_c$	$-V_c$	$-V_c$	$+V_c$	$-V_c$	$+V_c$	$+V_c$	$+V_c$
$-V_r$	$-V_r$	$+V_r$	$-V_c$	$-V_c$	$+V_c$	$-V_c$	$+V_c$	$-V_c$	$+V_c$	$+V_c$
$-V_r$	$+V_r$	$-V_r$	$-V_c$	$+V_c$	$-V_c$	$-V_c$	$+V_c$	$+V_c$	$-V_c$	$+V_c$
$-V_r$	$+V_r$	$+V_r$	$+V_c$	$-V_c$	$-V_c$	$-V_c$	$+V_c$	$+V_c$	$+V_c$	$-V_c$
$+V_r$	$-V_r$	$-V_r$	$-V_c$	$+V_c$	$+V_c$	$+V_c$	$-V_c$	$-V_c$	$-V_c$	$+V_c$
$+V_r$	$-V_r$	$+V_r$	$+V_c$	$-V_c$	$+V_c$	$+V_c$	$-V_c$	$-V_c$	$+V_c$	$-V_c$
$+V_r$	$+V_r$	$-V_r$	$+V_c$	$+V_c$	$-V_c$	$+V_c$	$-V_c$	$+V_c$	$-V_c$	$-V_c$
$+V_r$	$+V_r$	$+V_r$	$+V_c$	$+V_c$	$+V_c$	$-V_c$	$+V_c$	$-V_c$	$-V_c$	$-V_c$

Table 3.6 N_{eq} of HAT for practical values of l

The number of rows in a subgroup (l)	N_{eq} for HAT
3	$\frac{4N}{3}$
5	$\frac{64N}{45}$
7	$\frac{256N}{175}$

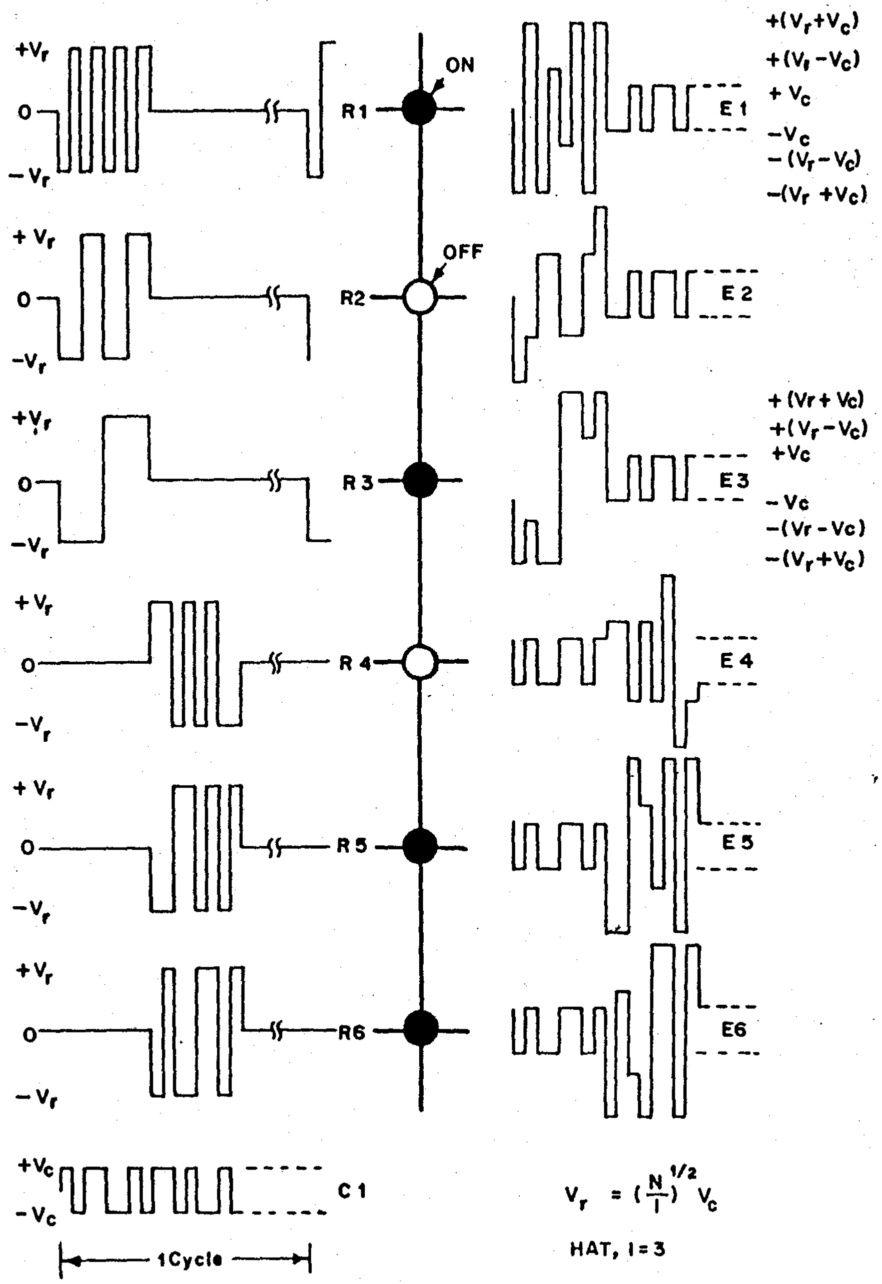

Fig. 3.2. Typical addressing waveforms of HAT, when l =3.

3.2.3. Analysis

Let the row-select voltage be $-V_r$ for logic 0 and $+V_r$ for logic 1. The $(N-l)$ rows in the unselected subgroups are grounded. Similarly the column voltages are $-V_c$ and $+V_c$ corresponding to logic 0 and logic 1 respectively. The instantaneous voltage across a pixel is either $|V_r+V_c|$ or $|V_r-V_c|$ as shown in Table 3.1. Here again, only a single column is considered for the purpose of analysis, since the data is multiplexed through the column line. The statistics of the occurrence of favourable and unfavourable voltages across a pixel during a cycle can be arrived at in a manner similar to that in the case of BAT.

The number of times a pixel gets a favourable voltage during a cycle is obtained as follows, based on eqn. (3.10) : -

$$2A = 2\sum_{i=0}^{(\frac{l-1}{2})} A_i = 2\sum_{i=0}^{(\frac{l-1}{2})} \frac{(l-1)!}{i!(l-i-1)!} \qquad (3.32)$$

Similarly the number of times a pixel gets an error is obtained by rewriting eqn. (3.11) in terms of l as follows : -

$$2B = 2\sum_{i=0}^{(\frac{l-1}{2})} B_i = 2\sum_{i=0}^{(\frac{l-1}{2})} \frac{i(l-1)!}{i!(l-i)!} \qquad (3.33)$$

The pixels get a voltage $\pm V_c$ during rest of the $2^l(\frac{N}{l}-1)$ time intervals, when the corresponding row is unselected. Hence, the rms voltage across the **ON** and **OFF** pixels are as follows : -

$$V_{ON}(rms) = \left[\frac{2.A.T.(V_r+V_c)^2 + 2.B.T\,(V_r-V_c)^2 + (\frac{N}{l}-1)2^l.T.V_c^2}{2^l\frac{N}{l}\,.\,T}\right]^{1/2}$$

(3.34)

and

$$V_{OFF}(rms) = \left[\frac{2.A.T.\,(V_r - V_c)^2 + 2.B.T\,(V_r + V_c)^2 + (\frac{N}{l} - 1).2^l.T.V_c^2}{2^l\,\frac{N}{l}\,.\,T}\right]^{1/2}$$

....(3.35)

The selection ratio (R) given by V_{ON}/V_{OFF} should be high in order to get a good contrast ratio in the display. R is a maximum for the condition

$$\frac{V_r}{V_c} = \left[\frac{N}{l}\right]^{1/2} \qquad (3.36)$$

and the maximum selection ratio is

$$R = \frac{V_{ON}\,(rms)}{V_{OFF}\,(rms)} = \left[\frac{2^l(\frac{N}{l})^{1/2} + 2(A - B)}{2^l(\frac{N}{l})^{1/2} - 2(A - B)}\right]^{1/2} \qquad (3.37)$$

The selection ratio can also be expressed as follows by using the relation given below.

$$(A + B) = 2^{(l-1)} \qquad (3.38)$$

Hence,

$$R = \left[\frac{2^l.(\frac{N}{l})^{1/2} + (4A - 2^l)}{2^l.(\frac{N}{l})^{1/2} - (4A - 2^l)}\right]^{1/2} = \left[\frac{2^l.(\frac{N}{l})^{1/2} + (2^l - 4B)}{2^l.(\frac{N}{l})^{1/2} - (2^l - 4B)}\right]^{1/2} \qquad (3.39)$$

The selection ratios of HAT for two values of l, i.e., $l = 3, 5$, are compared with those of APT (same as IAPT) as a function of N in Fig. 3.3. It is clear from this that the selection ratio of HAT is lower as compared to that of APT or IAPT. The N_{eq} of HAT is also evaluated as before to get an idea about the performance of this technique in relation to the standard APT or IAPT.

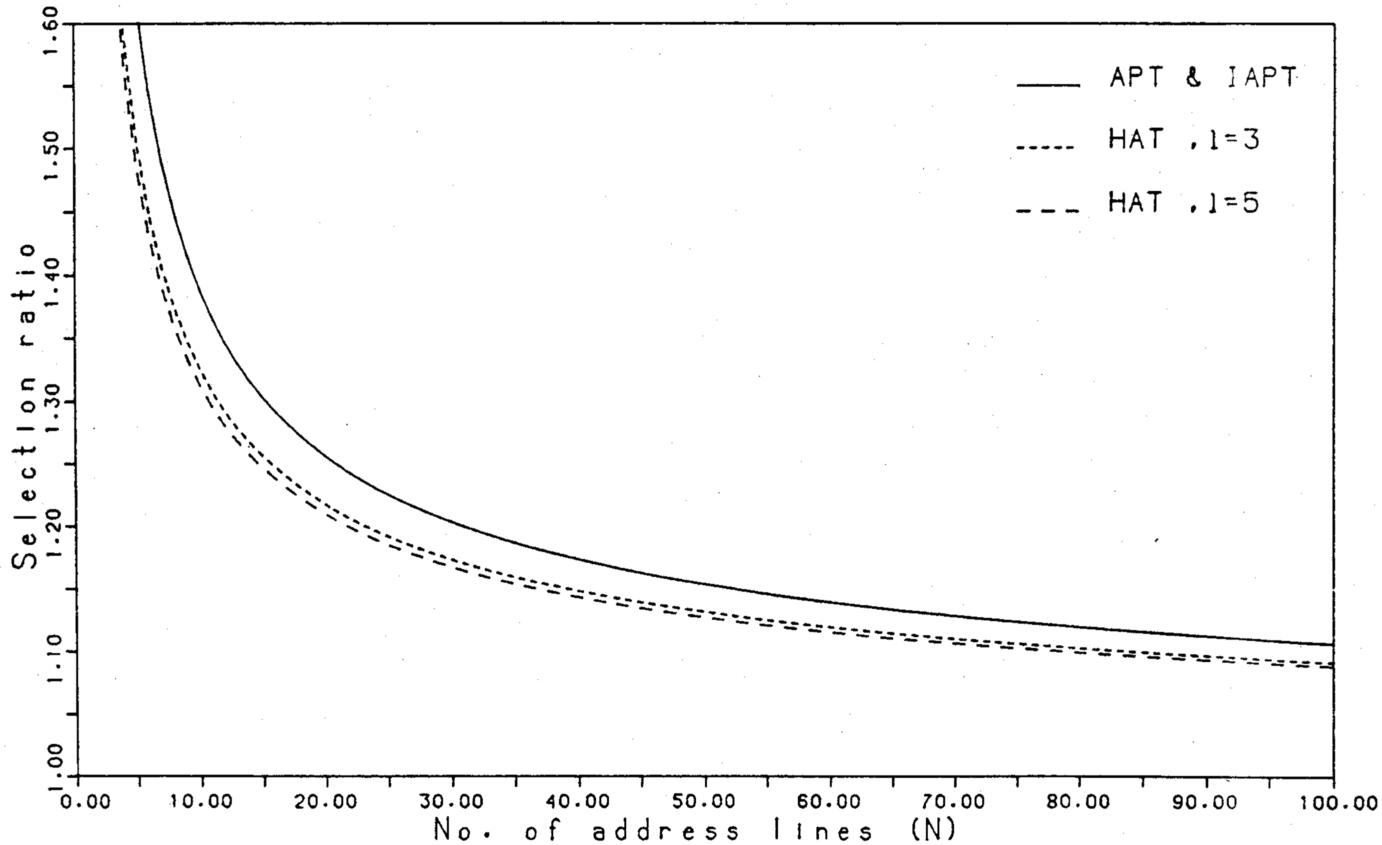

Fig. 3.3. Selection ratio of HAT and IAPT - A comparison

$$R = \left[\frac{2^{\ell}(\frac{N}{\ell})^{1/2} + 2(A - B)}{2^{\ell}(\frac{N}{\ell})^{1/2} - 2(A - B)}\right]^{1/2} = \left[\frac{(A+B)(\frac{N}{\ell})^{1/2} + (A - B)}{(A+B)(\frac{N}{\ell})^{1/2} - (A - B)}\right]^{1/2} = \left[\frac{N_{eq}^{1/2} + 1}{N_{eq}^{1/2} - 1}\right]^{1/2} \quad ....(3.40)$$

Therefore,

$$N_{eq} = \left[\frac{A + B}{A - B}\right]^{2} \left(\frac{N}{\ell}\right) \quad (3.41)$$

The term within the square brackets in the above equation is the same as the N_{eq} for BAT. This appears here since the subgroups are subjected to binary addressing in HAT. The values of N_{eq} of HAT for various values of ℓ are given in Table 3.6. It is observed here that the value of N_{eq} is higher than N and this confirms the lower selection ratio of HAT as compared to APT (or IAPT).

The expressions for the rms voltages for a maximum selection ratio is obtained by substituting the condition of eqn. (3.36) in eqns. (3.34) and (3.35). Thus,

$$V_{ON}\,(rms) = \left[\frac{2^{\ell}\,(\frac{N}{\ell}) + (4A - 2^{\ell})\,(\frac{N}{\ell})^{1/2}}{2^{\ell-1}\,(\frac{N}{\ell})}\right]^{1/2} \cdot V_c \quad (3.42)$$

and

$$V_{OFF}\,(rms) = \left[\frac{2^{\ell}\,(\frac{N}{\ell}) - (4A - 2^{\ell})\,(\frac{N}{\ell})^{1/2}}{2^{\ell-1}\,(\frac{N}{\ell})}\right]^{1/2} \cdot V_c \quad (3.43)$$

The magnitude of the column voltage is determined by equating V_{OFF} to V_{th}, since the OFF pixels are biased near V_{th}. Hence,

$$V_c = \left[\frac{2^{\ell-1}\,(\frac{N}{\ell})}{2^{\ell}\,(\frac{N}{\ell}) - (4A - 2^{\ell})\,(\frac{N}{\ell})^{1/2}}\right]^{1/2} \cdot V_{th} \quad (3.44)$$

The magnitude of V_r can be obtained by using eqn.(3.36) as

$$V_r = \left[\frac{N}{l}\right]^{1/2} \cdot V_c \qquad (3.45)$$

The maximum voltage swing in the addressing waveforms of HAT, as evident from Fig.3.2 is $2V_r$. Hence the supply voltage requirement of HAT is determined using eqns. (3.44) and (3.45) as below:-

$$V_{supply} = 2V_r = \left[\frac{2(\frac{N}{l})}{1 - [(4A - 2^l)/(2^l(\frac{N}{l})^{1/2})]}\right]^{1/2} V_{th} \qquad (3.46)$$

The supply voltage of HAT is compared with that of APT as well as IAPT in Table 3.7. It is clear that HAT requires a lower supply voltage as compared to APT and IAPT. This is mainly due to the large duty cycle of HAT $(= \frac{2Al}{2^l N})$ as compared to that of APT or IAPT $(= \frac{1}{N})$. The large duty cycle leads to a low V_r/V_c and hence a low supply voltage. However, the selection ratio is lower as compared to that of IAPT. In order to have a fair comparison, these two techniques are compared for the same selection ratio. Let R^2 of HAT be K, the supply voltage of the IAPT-R having the same selection ratio as HAT is calculated as before. The eqns. (3.26) is reproduced here for the sake of convenience:-

$$V_{supply}\,(IAPT\text{-}R) = \left[\frac{N}{(x-1)^2 + (N-1)}\right]^{1/2} (x+1)\, V_{th} \qquad (3.47)$$

where

$$x = \frac{(K+1) - [(K+1)^2 - (K-1)^2 N]^{1/2}}{(K-1)} \qquad (3.48)$$

The supply voltage requirement of HAT is compared with that of IAPT-R for various values of N in Table 3.7. This indicates that the supply voltage of HAT is lower for limited values of N, when the comparison is made for the same selection ratio.

Table 3.7. HAT Vs. IAPT - A comparison

N	l	Selection Ratio (R)		Supply voltage of HAT (normalized to V_{th})	Reduced V_r/V_c of IAPT-R	$\frac{V_{supply}\ HAT}{V_{supply}\ (IAPT-R)} \times 100\%$
		HAT	IAPT			
3	3	1.732	1.932	2.000	1.000	81.65
6	3	1.447	1.543	2.487	1.415	95.62
5	5	1.483	1.618	1.789	1.214	72.69
10	5	1.312	1.387	2.333	1.718	83.73
15	5	1.246	1.302	2.767	2.105	89.77
20	5	1.209	1.255	3.138	2.432	93.80
25	5	1.184	1.225	3.466	2.714	96.87
30	5	1.167	1.203	3.764	2.981	99.05
7	7	1.382	1.488	1.706	1.400	56.69
14	7	1.252	1.315	2.266	1.977	76.00
21	7	1.200	1.248	2.706	2.422	80.97
28	7	1.171	1.211	3.079	2.794	84.42
35	7	1.151	1.186	3.409	3.132	86.60

3.2.4. Discussion

The merits and demerits of HAT are given in Table 3.8. The HAT extends the BAT for higher values of N and can be considered as an intermediate step between the BAT and the IHAT, discussed in the next section.

3.3. IMPROVED HYBRID ADDRESSING TECHNIQUE (IHAT)

The IHAT proposed in this section is similar to the HAT discussed earlier, except for the choice of the column voltages. This technique has the same selection ratio as that of the conventional APT or IAPT, which is higher than that of HAT. A considerable reduction in the supply voltage requirement is possible in IHAT, as compared to IAPT or APT. In APT, the supply voltage required increases with the number of address lines (N). The IHAT presented here, requires a lower supply voltage as compared to IAPT even when the value of N is large. Important aspects of IHAT are considered in this section.

3.3.1. Background

The number of voltage levels in the column waveform is restricted to just two in BAT and HAT. However, the column voltage can be chosen depending on the number of mismatches (i) between the row-select and the data patterns. The IHAT discussed here, has multiple voltage levels in the column waveform and the choice of the column voltage depends on the value of i. Hence, both the amplitude and the sign of the column voltage depend on the value of i in IHAT, while only the sign of the column voltage is chosen according to the value of i in HAT. This increased freedom results in the following improvements in IHAT as compared to HAT:-

Table 3.8. Merits and Demerits of HAT

Merits - Extension of BAT for high values of N ; - Good pixel brightness uniformity ; - Natural dc-free operation ; - Lower supply voltage requirement as compared to IAPT for limited values of N ; - High duty cycle as compared to IAPT . **Demerits** - Selection ratio lower than that of IAPT ; - Number of time intervals to complete a cycle higher than IAPT .

- A higher selection ratio ;
- A considerable reduction in the supply voltage requirement as compared to IAPT for all values of N by a proper choice in the value of l, and
- The value of l can be odd or even as against odd only in the case of HAT.

3.3.2. Technique

The N rows to be multiplexed in a display are divided into N/l non-intersecting subgroups, each consisting of l address lines. Here again, the data to be displayed in the selected subgroup in any one of the columns is an l-bit word represented by ,

$$d_{kl+1},\ d_{kl+2},\ \ldots,\ d_{kl+l}\ ;\quad d_{kl+j} = 0 \text{ or } 1 \qquad (3.49)$$

wherein, logic 0 and logic 1 represent the OFF and ON pixels respectively. The row-select pattern is again an l-bit word represented by,

$$a_{kl+1},\ a_{kl+2},\ \ldots,\ a_{kl+l}\ ;\ a_{kl+j} = 0 \text{ or } 1 \qquad (3.50)$$

The value of k ranges from 0 to $[(N/l)-1]$ corresponding to the selected subgroups.

The IHAT is similar to HAT except for the choice of the column voltage. The various steps involved in IHAT are given below:-

i) One subgroup is selected at a time for addressing ;
ii) An l-bit word is chosen as the row-select pattern ;
iii) The row-select voltages are $-V_r$ for logic 0 and $+V_r$ for logic 1, while the $(N-l)$ unselected rows are grounded ;

iv) The row-select and the data patterns in the selected subgroup are compared bit-by-bit using digital comparators, viz., exclusive-OR gates ;

v) The number of mismatches i between these two patterns is determined by counting the number of exclusive-OR gates with logic 1 output ;

The steps (iv) and (v) can be summarized as follows:-

$$i = \sum_{j=0}^{l} a_{kl+j} \oplus d_{kl+j} \qquad (3.51)$$

vi) The column voltage is chosen to be V_i, if the number of mismatches is i ;

vii) The column voltage for each column in the matrix is determined independently by repeating the steps (iv) - (vi) ;

viii) Both the row and column voltages are applied simultaneously to the matrix display for a time duration T ;

ix) A new row-select pattern is chosen and the column voltages are determined using steps (iv) - (vi). The new row and column voltages are applied to the display for an equal duration of time at the end of T ;

x) A cycle is completed when all the subgroups (= N/l) are selected with all the 2^l row-select patterns once ;

xi) The display is refreshed by repeating this cycle continuously.

The time duration T should be small as compared to the response time of the display, in order to ensure the rms behavior of the display. The

IHAT has the same freedom as in the case of HAT in the choice of the row-select sequence and the subgroup selection as given below:-

- The sequence in which the 2^l row-select patterns are applied to the subgroup can be changed as in the case of BAT.
- A subgroup can be selected with 2^j row-select patterns consecutively before selecting the next subgroup. Here, j can range from 0 to l.
- The order in which the subgroups are selected can also be changed as long as all the subgroups are selected with all the 2^l row-select patterns.

The rms voltages across similar pixels are equal in all the cases discussed above, but the frequency components are different in each case. One of these combinations can be chosen as the addressing sequence to suit the display characteristics.

The column voltage V_i for the various values l and i are given in Table 3.9. The column voltages here are normalized to V_0. Adressing waveforms of IHAT with $l = 2$ and 3 are shown in Fig.3.4 and 3.5 respectively, as typical examples. The natural dc-free operation of IHAT is evident from the waveforms across the pixels illustrated in Fig. 3.4.

3.3.3. Analysis

Let $-V_r$ and $+V_r$ be the row-select voltages of the l-bit word. The $(N-l)$ rows in the unselected subgroups are grounded. Let the column voltage be V_i, corresponding to the i mismatches between the row-select and column voltages. Let the sign of the column voltage be in-phase with the row-select

Table 3.9. Column voltages of IHAT for various values of l and i

l	Column voltages corresponding to number of mismatches i =							
	0	1	2	3	4	5	6	7
2	-1	0	+1	-	-	-	-	-
3	-1	$-\frac{1}{3}$	$+\frac{1}{3}$	+1	-	-	-	-
4	-1	$-\frac{1}{2}$	0	$+\frac{1}{2}$	+1	-	-	-
5	-1	$-\frac{3}{5}$	$-\frac{1}{5}$	$+\frac{1}{5}$	$+\frac{3}{5}$	+1	-	-
6	-1	$-\frac{4}{6}$	$-\frac{2}{6}$	0	$+\frac{2}{6}$	$+\frac{4}{6}$	+1	-
7	-1	$-\frac{5}{7}$	$-\frac{3}{7}$	$-\frac{1}{7}$	$+\frac{1}{7}$	$+\frac{3}{7}$	$+\frac{5}{7}$	+1

Table 3.10. Supply voltage requirements of IHAT Vs. APT for $N > l^2$

No. of address lines in a subgroup	$\frac{V_{supply}(IHAT)}{V_{supply}(APT)} \times 100\%$
2	70.71
3	57.74
4	50.00
5	44.72
6	40.82
7	37.80

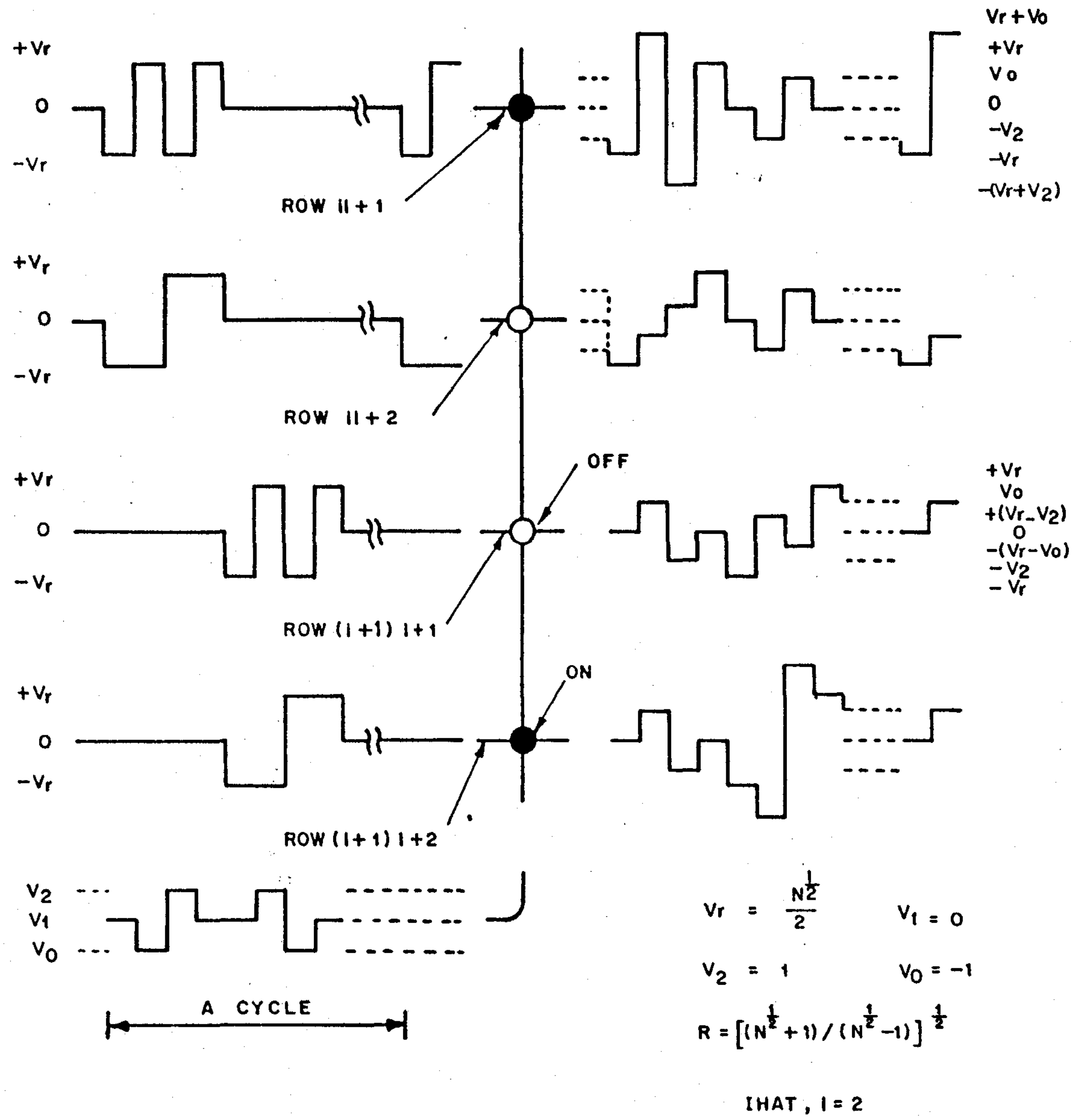

Fig. 3.4. Typical addressing waveforms of IHAT, when $l = 2$.

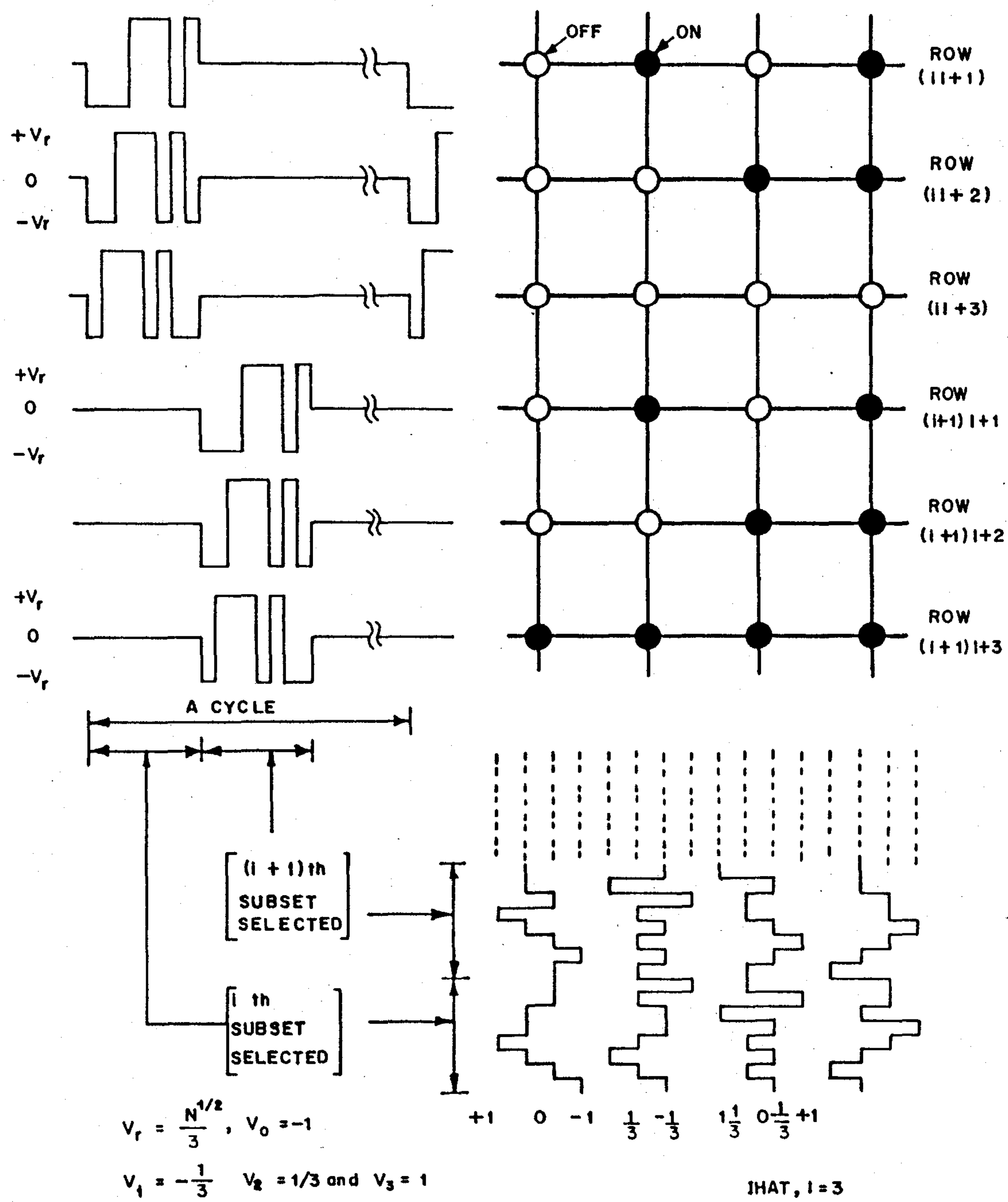

Fig. 3.5. Typical addressing waveforms of IHAT, when $l = 3$.

voltage corresponding to logic 0. The instantaneous voltage across the pixels are either $|V_r + V_i|$ or $|V_r - V_i|$.

The number of times the l-bit row-select and data patterns differ by i bits is based on eqn.(3.5). Thus,

$$C_i = \frac{l!}{i!(l-i)!} \tag{3.52}$$

The number of times a pixel gets a favourable voltage, when the number of mismatches in the column is i is based on eqn. (3.9) as given below:-

$$A_i = \frac{(l-1)!}{i!(l-i-1)!} \tag{3.53}$$

Similarly, the number of times a pixel gets an unfavourable voltage or error, within the C_i row-select patterns considered is given below:-

$$B_i = \frac{i(l-1)!}{i!(l-i)!} \tag{3.54}$$

The **ON** pixels get a voltage of $|V_r + V_i|$ during A_i time intervals and a voltage $|V_r - V_i|$ during B_i time intervals out of the total C_i $(= A_i + B_i)$ time intervals considered, when the corresponding row is selected. These pixels also get a voltage $|V_i|$ during $(\frac{N}{l}-1)C_i$ time intervals, when the corresponding row is unselected. The rms voltage across the **ON** pixel is determined by summing these voltages for all possible value of mismatches, i.e., summation over i from 0 to l, as follows:-

$$V_{ON}^2\,(rms) = \left(\sum_{i=0}^{l} A_i(V_r+V_i)^2 + \sum_{i=0}^{l} B_i(V_r-V_i)^2 + \sum_{i=0}^{l} (\frac{N}{l} - 1)(A_i+B_i)\,V_i^2\right) \Big/ (2^l N/l) \tag{3.55}$$

The rms voltage across an **OFF** pixel can be arrived at in a similar way as follows:-

$$V^2_{OFF}\,(rms) = \left(\sum_{i=0}^{l} A_i(V_r - V_i)^2 + \sum_{i=0}^{l} B_i(V_r + V_i)^2 + \sum_{i=0}^{l} \left(\frac{N}{l} - 1\right)(A_i + B_i)V_i^2\right) \Big/ (2^l N/l) \qquad (3.56)$$

The ratio V_{ON}/V_{OFF} should be a maximum in order to achieve a good contrast in the display.

$$\left[\frac{V_{ON}(rms)}{V_{OFF}(rms)}\right]^2 = \frac{\sum_{i=0}^{l} (A_i+B_i)V_r^2 + \frac{N}{l}\sum_{i=0}^{l} (A_i+B_i)V_i^2 + 2\sum_{i=0}^{l} (A_i - B_i)V_r V_i}{\sum_{i=0}^{l} (A_i+B_i)V_r^2 + \frac{N}{l}\sum_{i=0}^{l} (A_i+B_i)V_i^2 - 2\sum_{i=0}^{l} (A_i - B_i)V_r V_i} \qquad (3.57)$$

This ratio is of the form

$$\left[\frac{V_{ON}}{V_{OFF}}\right]^2 = \frac{f_1 + f_2}{f_1 - f_2} \qquad (3.58)$$

where,

$$f_1 = V_r^2 \sum_{i=0}^{l} (A_i + B_i) + \left(\frac{N}{l}\right) \sum_{i=0}^{l} (A_i + B_i)V_i^2 \qquad (3.59)$$

and

$$f_2 = 2V_r \sum_{i=0}^{l} (A_i - B_i)V_i \qquad (3.60)$$

The following equations must be satisfied for obtaining an optimum selection ratio.

$$f_1 \cdot f'_{2V_r} = f'_{1V_r} \cdot f_2 \qquad (3.61)$$

and

$$f_1 \cdot f'_{2V_i} = f'_{1V_i} \cdot f_2 \qquad \text{for} \quad i = 0 \text{ to } l \qquad (3.62)$$

where,

$$f'_{1V_r} = 2V_r \sum_{i=0}^{l} (A_i + B_i) \qquad (3.63)$$

$$f'_{1V_i} = 2\left(\frac{N}{l}\right).(A_i + B_i) \cdot V_i \qquad (3.64)$$

$$\delta'_{2V_r} = 2 \sum_{i=0}^{l} (A_i - B_i) V_i \tag{3.65}$$

and

$$\delta'_{2V_i} = 2V_r(A_i - B_i) \tag{3.66}$$

The following relation is obtained by substituting for δ_2, δ'_{1V_r} and δ'_{2V_r} in eqn.(3.61)

$$\delta_1 . 2 \sum_{i=0}^{l} (A_i - B_i)V_i = \left[2V_r \sum_{i=0}^{l} (A_i + B_i)\right] 2V_r \sum_{i=0}^{l} (A_i - B_i)V_i \tag{3.67}$$

or

$$\delta_1 = 2V_r^2 \sum_{i=0}^{l} (A_i + B_i) \tag{3.68}$$

The following relation is well known from the properties of binomial coefficients:-

$$\sum_{i=0}^{l} (A_i + B_i) = \sum_{i=0}^{l} C_i = \sum_{i=0}^{l} \frac{l!}{i!(l-i)!} = 2^l \tag{3.69}$$

Hence,

$$\delta_1 = 2^{(l+1)} V_r^2 \tag{3.70}$$

The following equations are obtained by substituting for δ'_{1V_i} and δ'_{2V_i} in eqns. (3.62)

$$\delta_1 2V_r(A_i - B_i) = 2\frac{N}{l}(A_i + B_i)V_i \delta_2 \qquad \text{for all } i\text{'s} \tag{3.71}$$

Hence,

$$\frac{(A_i - B_i)}{(A_i + B_i)V_i} = \frac{2N\delta_2}{2lV_r\delta_1} = \text{constant for all } i\text{'s} \tag{3.72}$$

A relation between the column voltages for i and j mismatches can be obtained, since the term on the right hand side is a constant in the above equation. Hence,

$$\frac{(A_i - B_i)}{(A_i + B_i)V_i} = \frac{(A_j - B_j)}{(A_j + B_j)V_j} \qquad (3.73)$$

or

$$\frac{V_i}{V_j} = \frac{(A_i - B_i)\,(A_j + B_j)}{(A_i + B_i)\,(A_j - B_j)} \qquad (3.74)$$

The column voltages can be normalized with any one of the column voltages. However, it is convenient to normalize them to V_0, i.e., the column voltage corresponding to zero mismatches. Substituting for A_0 (=1) and B_0 (=0) in eqn. (3.74), the column voltage for i mismatches is,

$$V_i = \frac{(A_i - B_i)}{(A_i + B_i)} V_0 \qquad (3.75)$$

This can be further simplified by substituting for A_i and B_i from eqns. (3.53) and (3.54). Hence,

$$V_i = \frac{(l - 2i)}{l} V_0 \qquad (3.76)$$

It can be shown (Appendix 3.a) that

$$V_{(l-i)} = -V_i \qquad (3.77)$$

Hence, the column voltages for i and (l-i) mismatches have the same amplitude and they differ only in phase (sign). The number of errors in a column is made to be the same for i and (l-i) mismatches in both BAT and HAT by a proper choice of the polarity of the column voltage. A similar condition is imposed here by eqn. (3.77).

The equation (3.68) can be rewritten as follows by substituting for f_1 from eqn. (3.59):-

$$V_r^2 \sum_{i=0}^{l} (A_i + B_i) = \frac{N}{l} \sum_{i=0}^{l} (A_i + B_i) V_i^2 \qquad (3.78)$$

However, V_i^2 is obtained from eqn. (3.75) as follows:-

$$V_i^2 = \frac{(A_i - B_i)^2}{(A_i + B_i)^2} V_o^2 \qquad (3.79)$$

Hence, the eqn. (3.78) becomes

$$V_r^2 \sum_{i=0}^{l} (A_i + B_i) = \frac{N}{l} \sum_{i=0}^{l} \frac{(A_i - B_i)^2}{(A_i + B_i)} V_o^2 \qquad (3.80)$$

The following relation is obtained by substituting for A_i and B_i from eqns. (3.53) and (3.54) in the above equation.

$$V_r^2 \sum_{i=0}^{l} \frac{l!}{i!(l-i)!} = \frac{N}{l^2} V_o^2 \sum_{i=0}^{l} \frac{(l-1)!\,(l-2i)^2}{i!\,(l-i)!} \qquad (3.81)$$

It can be shown (Appendix 3b) that

$$\sum_{i=0}^{l} \frac{l!}{i!(l-i)!} = \sum_{i=0}^{l} \frac{(l-1)!\,(l-2i)^2}{i!\,(l-i)!} = 2^l \qquad (3.82)$$

Hence,

$$V_r^2 = \frac{N}{l^2} V_o^2 \qquad (3.83)$$

The selection ratio is a maximum for the following conditions:-

$$V_r = + \frac{N^{1/2}}{l} V_o \qquad (3.84)$$

and

$$V_i = \left(\frac{l - 2i}{l}\right) V_o \qquad (3.85)$$

The function f_2 from eqn.(3.60) can be modified to the following form by substituting for V_i in terms of A_i and B_i using eqn. (3.75). Thus,

$$f_2 = 2V_r \sum_{i=0}^{l} (A_i - B_i)V_i = 2V_r V_o \sum_{i=0}^{l} \frac{(A_i - B_i)^2}{(A_i + B_i)} \quad (3.86)$$

It can be shown (Appendix 3.c) that

$$\sum_{i=0}^{l} \frac{(A_i - B_i)^2}{(A_i + B_i)} = \frac{2^l}{l} \quad (3.87)$$

Hence,

$$f_2 = 2V_r V_o (\frac{2^l}{l}) \quad (3.88)$$

Substituting for V_r using eqn.(3.84),

$$f_2 = 2^{l+1} \frac{N^{1/2}}{l^2} V_o^2 \quad (3.89)$$

The selection ratio is obtained by substituting for f_1 and f_2 using eqns. (3.70) and (3.89) in eqn.(3.58)

$$\left[\frac{V_{ON}\ (rms)}{V_{OFF}\ (rms)}\right]^2 = \left[\frac{2^{l+1} \frac{N}{l^2} V_o^2 + 2^{l+1} \frac{N^{1/2}}{l^2} V_o^2}{2^{l+1} \frac{N}{l^2} V_o^2 - 2^{l+1} \frac{N^{1/2}}{l} V_o^2}\right] \quad (3.90)$$

or

$$R = \frac{V_{ON}\ (rms)}{V_{OFF}\ (rms)} = \left[\frac{N^{1/2} + 1}{N^{1/2} - 1}\right]^{1/2} \quad (3.91)$$

The selection ratio here, is the same as that of the APT or IAPT. This is the maximum value possible for any addressing technique [69]. However, the complexity of the column waveform is more in IHAT, since the number of column voltages is $(l+1)$ instead of just 2 in HAT. The sign of the column voltage V_i is assumed to be in-phase with the row-select voltage corresponding to logic 0. The column voltages are normalized to V_o, where $-V_r$ is for logic 0 and $+V_r$ is for logic 1 in the l-bit row-select pattern. This is given in Table 3.9.

The rms voltages across the **ON** and **OFF** pixels, when the selection ratio is a maximum are as follows:-

$$V_{ON}\,(rms) = \left[\frac{2^{l+1}\,V_o^2(N + N^{1/2})\,l}{l^2\,2^l\,N}\right]^{1/2} = \left[\frac{2(N + N^{1/2})}{Nl}\right]^{1/2} V_o \qquad (3.92)$$

and

$$V_{OFF}\,(rms) = \left[\frac{2^{l+1}\,V_o^2(N - N^{1/2})\,l}{l^2\,2^l\,N}\right]^{1/2} = \left[\frac{2(N - N^{1/2})}{Nl}\right]^{1/2} V_o \qquad (3.93)$$

The **OFF** pixels in the display are biased near V_{th} in order to obtain a good contrast ratio. Hence,

$$V_{OFF} = V_{th} \qquad (3.94)$$

or

$$V_o = \left[\frac{N.l}{2(N - N^{1/2})}\right]^{1/2} V_{th} \qquad (3.95)$$

The supply voltage requirement is determined by the maximum swing in the addressing waveforms. The amplitude of the row select voltage (V_r) is lower than V_o for $N < l^2$, is the same as V_o for $N = l^2$ and is greater than V_o for $N > l^2$. The column voltages for $i \geq 0$ are however lower or equal to V_o (Table 3.9). Hence, the supply voltage of 1HAT is calculated for two ranges of N as given below:-

$$V_{supply}\,(1HAT) = 2V_o \qquad \text{for } N \leq l^2 \qquad (3.96)$$

and

$$V_{supply}\,(IHAT) = 2V_r = 2\frac{N^{1/2}}{l}V_o \quad for \quad N \geq l^2 \tag{3.97}$$

Hence,

$$V_{supply}\,(IHAT) = \left[\frac{4l}{2(1 - N^{-1/2})}\right]^{1/2} V_{th} \quad for\ N \leq l^2 \tag{3.98}$$

and

$$V_{supply}\,(IHAT) = \left[\frac{4(N/l)}{2(1 - N^{-1/2})}\right]^{1/2} V_{th} \quad for \quad N \geq l^2 \tag{3.99}$$

The supply voltage of APT for a comparison is obtained from eqn. (2.17) as follows:-

$$V_{supply}\,(APT) = \left[\frac{2N^{1/2}}{[2(1 - N^{-1/2})]^{1/2}}\right] = \left[\frac{4N}{2(1 - N^{-1/2})}\right]^{1/2} \tag{3.100}$$

The supply voltage requirement of IHAT is compared with that of APT using the following ratio:-

$$\frac{V_{supply}\,(IHAT)}{V_{supply}\,(APT)} = \begin{cases} (\frac{l}{N})^{1/2} & for \quad N \leq l^2 \\ (\frac{1}{l})^{1/2} & for \quad N \geq l^2 \end{cases} \tag{3.101}$$

It is evident that this ratio is independent of N, when $N \geq l^2$. This condition can be met with a proper choice of l for a given N. The supply voltage requirements of IHAT for various values of l ($N \geq l^2$) are compared with that of APT in Table 3.10. From this table, it is clear that IHAT requires a lower supply voltage as compared to APT.

The IAPT is popular at present, due to its lower supply voltage requirement as compared to the APT. Hence, the supply voltage of IHAT is also compared with that of IAPT.

The supply voltage of IAPT is recalled from equation (2.18)

$$V_{supply}\ (IAPT) = \frac{(N^{1/2} + 1)}{[2(1 - N^{-1/2})]^{1/2}} \tag{3.102}$$

Hence,

$$\frac{V_{supply}\ [IHAT)}{V_{supply}\ (IAPT)} = \begin{cases} \dfrac{2\ell^{1/2}}{(N^{1/2} + 1)} & for\ \ N \leq \ell^2 \\ \dfrac{2(N/\ell)^{1/2}}{(N^{1/2} + 1)} & for\ \ N \geq \ell^2 \end{cases} \tag{3.103}$$

The supply voltage requirement of IHAT is compared with that of IAPT for various values of N and ℓ in Figs. 3.6 and 3.7. The following points are evident from these figures :-

- The reduction in the supply voltage is maximum when $N = \ell^2$, and
- It is possible to achieve a considerable reduction in the power supply voltage as compared to IAPT by a proper choice of the value of ℓ.

Table 3.11 gives the value of ℓ leading to a good reduction in the supply voltage for a wide range of values of N. From this table it is clear that IHAT with $\ell = 7$ is quite adequate for a wide range, especially for high values of N ($N > 49$). The supply voltage increases with N in general. It is important to note that IHAT requires a considerably lower supply voltage as compared to IAPT, even when a few thousand lines are multiplexed.

3.3.4. Discussion

The merits and demerits of IHAT as compared to IAPT are given in Table 3.12. IHAT requires a lower supply voltage as compared to IAPT,

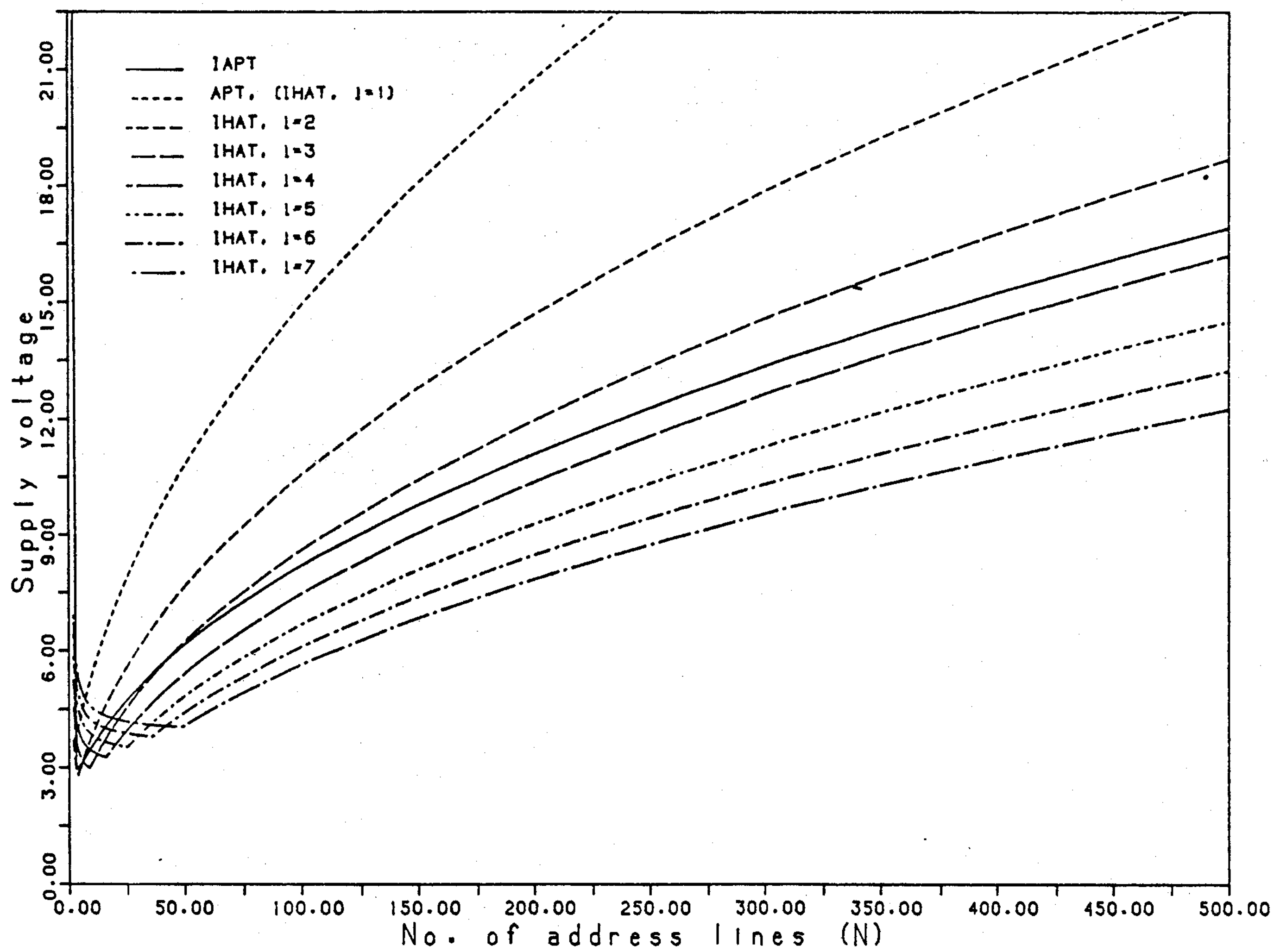

Fig. 3.6. Supply voltage (normalized to V_{th}) vs. N for both IHAT and IAPT.

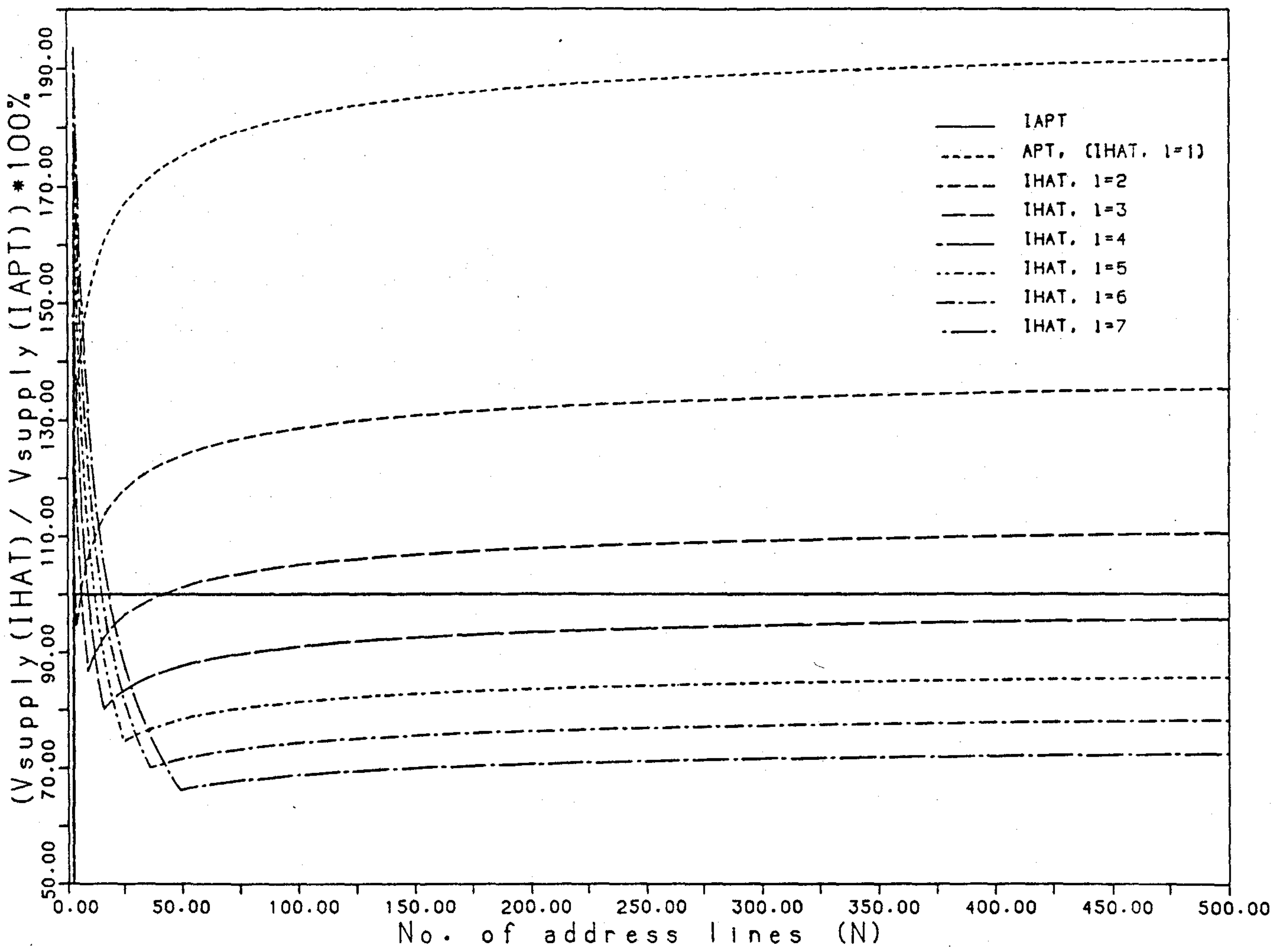

Fig. 3.7. A comparison of supply voltage requirement of IHAT with that of IAPT.

Table 3.11. Possible values of l, for different N leading to a good reduction in the supply voltage requirement

N	l	$\frac{V_{supply}\ (IHAT)}{V_{supply}\ (IAPT)} \times 100\%$, V_{th} constant	No. of voltage levels in the addressing waveforms	No. of time intervals in a cycle of (IHAT)/(IAPT)
4	2	94.28	3	1
9	3	86.60	5	4/3
12	3,4	89.60	7,8	4/3, 8/4
16	4	80.00	5	8/4
20	4,5	81.73	8,9	8/4, 16/5
25	5	74.54	7	16/5
30	5,6	75.63	9,10	16/5, 32/6
36	6	69.99	7	32/6
42	6,7	70.74	10,11	32/6, 64/7
49	7	66.14	9	64/7
70	7	67.52	11	64/7
140	7	69.70	11	64/7
280	7	71.33	11	64/7
560	7	72.59	11	64/7
1120	7	73.40	11	64/7
2240	7	74.03	11	64/7
4480	7	74.48	11	64/7

Table 3.12. Merits and Demerits of IHAT as compared to IAPT

Merits

- Good reduction in supply voltage possible by proper choice in the value of l ;
- Better pixel brightness uniformity by choosing scanning sequence to match display characteristics ;
- Natural dc-free operation ;
- Higher duty cycle .

Demerits

- Number of voltage levels in the column waveforms is $(l+1)$ against four in IAPT ;
- Number of time intervals to complete a cycle higher by a factor $[2^{(l-1)}/l]$.

especially for a high value of N. However, the hardware complexity of IHAT increases with l, since $(l+1)$ switches are required in each column driver to generate the column waveforms.

3.4. IHAT - SPECIAL CASES (IHAT-S)

The IHAT presented in the previous section is well suited for multiplexing TNLCDs, wherein the selection ratio is an important parameter. However, displays based on SBE exhibit steep electro-optic characteristics. Hence, a selection ratio lower than that of IHAT or IAPT is acceptable here. This facilitates a reduction in the hardware as the number of voltage levels in the column waveform can be restricted as in the case of HAT. The trade-off between the hardware complexity and the resulting selection ratio as well as the supply voltage requirement is analyzed in this section.

3.4.1. Background

The number of voltage levels in the column waveform (n_c) can be restricted by grouping the number of mismatches (i) and assigning a column voltage for each group. The value of n_c can be restricted to 3 in the case of even l and 4 in the case of odd l instead of $(l+1)$ in the case of IHAT. This leads to a lower selection ratio as compared to that of IHAT. An optimum grouping of mismatches with a minimum degradation in the selection ratio, can be arrived at by an exhaustive search of all the possible groupings for each l.

3.4.2. Techniques

The technique for even values of l will be referred to as IHAT-S3, since the number of column voltages is restricted to 3 here. This technique is similar to 1HAT except for the choice of a column voltage (V_c) as given below:-

$$V_c = \begin{cases} -V_m & \text{for } 0 \leq i \leq m \\ 0 & \text{for } m < i < (l-m) \\ +V_m & \text{for } (l-m) \leq i \leq l \end{cases} \qquad (3.104)$$

The number of column voltages is restricted to 4 in the case of odd values of l and will be referred to as 1HAT-S4. Here again, the technique is similar to 1HAT except for the choice of a column voltage (V_c) as given below:-

$$V_c = \begin{cases} -V_{m1} & \text{for } 0 \leq i \leq m1 \\ -V_{m2} & \text{for } m1 < i < (l/2) \\ +V_{m2} & \text{for } (l/2) < i < (l-m1) \\ +V_{m1} & \text{for } (l-m1) \leq i \leq l \end{cases} \qquad (3.105)$$

3.4.3. Analysis

Case 1: 1HAT-S3

Three groups, covering the range 0 to l are formed when the value of l is even and are represented as given below:-

$$(0,1,\ldots,m) \; ; \; (m+1,\ldots\ldots,l-m-1) \; ; \; (l-m \; ; \ldots\ldots, l) \qquad (3.106)$$

The number of entries in the first and the last group are chosen to be equal here since the number of errors can be minimized by a proper choice of the polarity of the column voltage. This is similar to the other techniques discussed earlier.

Let the column voltages be V_{g1}, V_{g2} and V_{g3} respectively for the three groups. The following constraints are imposed on these voltages based on the analysis of IHAT :

$$V_{g1} = -V_{g3} ; \quad \text{Let } |V_{g1}| = |V_{g3}| = |V_m| \tag{3.107}$$

and

$$V_{g2} = 0 \tag{3.108}$$

Only $2^{(l-1)}$ row-select patterns leading to $i \leq (l/2)$ are considered here, for the purpose of analysis. Let the row-select voltage be $-V_r$ for logic 0 and $+V_r$ for logic 1 in the row-select pattern. Here again, the instantaneous voltages across the pixels are classified as favourable or unfavourable as given below:-

- The voltage $|V_r + V_m|$ is favourable for an **ON** pixel and unfavourable for an **OFF** pixel ; and
- The voltage $|V_r - V_m|$ is favourable for an **OFF** pixel and unfavourable for an **ON** pixel.

A pixel in the selected row gets a favourable voltage during A_i time intervals and gets an unfavourable voltage during B_i time intervals, when C_i row-select patterns with i mismatches are considered. The expressions for A_i and B_i are reproduced below from the eqns. (3.53) and (3.54) for the sake of convenience.

$$A_i = \frac{(l-1)!}{i!(l-i-1)!} \tag{3.109}$$

and

$$B_i = \frac{i(l-1)!}{i!(l-i)!} \tag{3.110}$$

A pixel in the unselected row gets a voltage $|V_m|$ for the time intervals given below:-

$$\left(\frac{N}{\ell} - 1\right) \sum_{i=0}^{m} (A_i + B_i) \tag{3.111}$$

Hence, the voltages across the **ON** and **OFF** pixels are as given below:-

$$V_{ON}\,(rms) = \left[\frac{S_1 + S_2 + S_3 + S_4}{2^{(\ell-1)}\,\frac{N}{\ell}}\right]^{1/2} \tag{3.112}$$

and

$$V_{OFF}\,(rms) = \left[\frac{S_5 + S_6 + S_3 + S_4}{2^{(\ell-1)}\,\frac{N}{\ell}}\right]^{1/2} \tag{3.113}$$

where

$$S_1 = \sum_{i=0}^{m} A_i(V_r + V_m)^2 \tag{3.114}$$

$$S_2 = \sum_{i=0}^{m} B_i(V_r - V_m)^2 \tag{3.115}$$

$$S_3 = \left[2^{(\ell-1)} - \sum_{i=0}^{m} (A_i + B_i)\right] V_r^2 \tag{3.116}$$

$$S_4 = \left(\frac{N}{\ell} - 1\right) \sum_{i=0}^{m} (A_i + B_i)\, V_m^2 \tag{3.117}$$

$$S_5 = \sum_{i=0}^{m} A_i(V_r - V_m)^2 \tag{3.118}$$

and

$$S_6 = \sum_{i=0}^{m} B_i(V_r + V_m)^2 \tag{3.119}$$

The ratio (V_{ON}/V_{OFF}) should be a maximum in order to obtain a good discrimination between the **ON** and **OFF** pixels. The procedure is similar to that of IHAT and is given in Appendix 4 . The selection ratio is a maximum when the following condition is satisfied :

$$\frac{V_r}{V_m} = \left[\frac{C^*_m N}{2^{(l-1)} l} \right]^{1/2} \tag{3.120}$$

where

$$C^*_m = \sum_{i=0}^{m} (A_i + B_i) = \sum_{i=0}^{m} C_i \tag{3.121}$$

The maximum selection ratio is given by the following expression:-

$$R = \frac{V_{ON}\,(rms)}{V_{OFF}\,(rms)} = \left[\frac{\left(\frac{C^*_m N}{l}\right)^{1/2} + \left(\frac{A^2_m}{2^{(l-1)}}\right)^{1/2}}{\left(\frac{C^*_m N}{l}\right)^{1/2} - \left(\frac{A^2_m}{2^{(l-1)}}\right)^{1/2}} \right]^{1/2} \tag{3.122}$$

where,

$$A_m = \frac{(l-1)!}{m!\,(l-m)!} \tag{3.123}$$

The selection ratio of this technique can be compared with that of APT, IAPT and IHAT using N_{eq} as follows:-

$$R = \left[\frac{\left(\frac{C^*_m N}{l}\right)^{1/2} + \left(\frac{A^2_m}{2^{(l-1)}}\right)^{1/2}}{\left(\frac{C^*_m N}{l}\right)^{1/2} - \left(\frac{A^2_m}{2^{(l-1)}}\right)^{1/2}} \right]^{1/2} = \left[\frac{N^{1/2}_{eq}+1}{N^{1/2}_{eq}-1} \right]^{1/2} \tag{3.124}$$

or

$$N_{eq} = \frac{2^{(l-1)}\, C^*_m N}{A^2_m\, l} \tag{3.125}$$

The values of N_{eq} for various possible groupings when $l=4$ and 6 are given in Table 3.13, as typical examples. The values of N_{eq} is greater than N as shown in this table. This indicates that the selection ratio is lower than that of IHAT, when the value of n_c is restricted to 3. The best possible grouping for each l is indicated by an asterisk in Table 3.13. The selection ratios of IHAT–S3 with these best possible groupings are compared with those of

Table 3.13. Various possible groupings of mismatches in IHAT-S3 - A comparison

l	Grouping of mismatches (i) for deciding column voltage	Parameters		Row-select voltage normalized to V_m	N_{eq}	Best grouping for each l
		A_m	C^*_m			
4	(0,1) (2) (3,4)	3	5	$(\frac{5N}{32})^{1/2}$	$\frac{10N}{9}$	*
4	(0) (1,2,3) (4)	1	1	$(\frac{N}{32})^{1/2}$	$2N$	
6	(0,1,2)(3)(4,5,6)	10	22	$(\frac{11N}{96})^{1/2}$	$\frac{88N}{75}$	*
6	(0,1)(2,3,4)(5,6)	5	7	$(\frac{7N}{192})^{1/2}$	$\frac{112N}{75}$	
6	(0)(1,2,3,4,5)(6)	1	1	$(\frac{N}{192})^{1/2}$	$\frac{32N}{6}$	

IAPT, for l = 4 and 6 in Tables 3.14 and 3.15 respectively. The rms voltage across the ON and OFF pixels when the selection ratio is a maximum, are given below:-

$$V_{ON}\ (rms) = \left[\frac{2\left(\frac{C^*_m N}{l}\right)^{1/2} \left(\left(\frac{C^*_m N}{l}\right)^{1/2} + \left(\frac{A_m^2}{2^{(l-1)}}\right)^{1/2} \right)}{2^{(l-1)} \frac{N}{l}} \right]^{1/2} V_m \qquad (3.126)$$

and ,

$$V_{OFF}\ (rms) = \left[\frac{2\left(\frac{C^*_m N}{l}\right)^{1/2} \left(\left(\frac{C^*_m N}{l}\right)^{1/2} - \left(\frac{A_m^2}{2^{(l-1)}}\right)^{1/2} \right)}{2^{(l-1)} \frac{N}{l}} \right]^{1/2} V_m \qquad (3.127)$$

The OFF pixels are usually biased near V_{th}. Hence, V_m is determined in terms of V_{th} as follows:-

$$V_{OFF} = V_{th} \qquad (3.128)$$

Thus,

$$V_m = \left[\frac{2^{(l-2)} \left(\frac{N}{l}\right)}{\left(\frac{C^*_m N}{l}\right)^{1/2} \left(\left(\frac{C^*_m N}{l}\right)^{1/2} - \left(\frac{A_m^2}{2^{(l-1)}}\right)^{1/2} \right)} \right]^{1/2} \qquad (3.129)$$

The supply voltage is determined by the maximum voltage swing in the addressing waveform

$$V_{supply}\ (IHAT\text{-}S3) = 2V_m \qquad \text{for } N \leq \left(\frac{2^{(l-1)} \cdot l}{C^*_m}\right) \qquad (3.130)$$

$$V_{supply}\ (IHAT\text{-}S3) = 2V_r \qquad \text{for } N \geq \left(\frac{2^{(l-1)} \cdot l}{C^*_m}\right) \qquad (3.131)$$

Hence ,

Table 3.14. IHAT-S3 (with $l=4$) vs. IAPT - A comparison

N	Selection ratio of		N_{eq} of IHAT-S3	V_{supply} of IHAT-S3	Reduced V_r/V_c for IAPT-R	$\frac{V_{supply}(IHAT\text{-}S3)}{V_{supply}(IAPT\text{-}R)} \times 100\%$
	IHAT-S3 with $l=4$	IAPT				
4	1.675	1.732	4.444	2.467	1.442	90.32
8	1.418	1.447	8.889	2.453	2.039	81.13
12	1.324	1.346	13.333	2.875	2.497	86.35
16	1.274	1.291	17.778	3.238	2.883	89.79
20	1.240	1.255	22.222	3.563	3.223	92.30
24	1.217	1.230	26.667	3.858	3.531	94.24
28	1.199	1.211	31.111	4.130	3.814	95.81
32	1.184	1.196	35.556	4.385	4.077	97.09
36	1.173	1.183	40.000	4.624	4.325	98.22
40	1.163	1.173	44.444	4.851	4.558	99.18

Table 3.15. IHAT-S3 with ($l = 6$) vs. IAPT - A comparison

N	Selection ratio (R) of IHAT-S3	Selection ratio (R) of IAPT	N_{eq} of IHAT-S3	V_{supply} of IHAT-S3	Reduced V_r/V_c of IAPT-R	$\frac{V_{supply}\text{(IHAT-S3)}}{V_{supply}\text{(IAPT-R)}} \times 100\%$
6	1.487	1.543	7.04	2.161	1.633	77.85
12	1.314	1.346	14.08	2.335	2.310	72.62
18	1.247	1.272	21.12	2.769	2.829	76.89
24	1.210	1.230	28.16	3.140	3.267	79.67
30	1.1855	1.203	35.20	3.468	3.653	81.69
36	1.168	1.183	42.24	3.766	4.001	83.25
42	1.154	1.168	49.28	4.040	4.322	84.50
48	1.143	1.156	56.32	4.296	4.620	85.55
54	1.135	1.147	63.36	4.537	4.900	86.43
60	1.127	1.139	70.40	4.765	5.166	87.18
90	1.103	1.112	105.60	5.765	6.326	89.89
120	1.088	1.096	140.80	6.609	7.305	91.53
150	1.078	1.085	176.00	7.354	8.167	92.71
300	1.055	1.060	352.00	10.278	11.550	95.77

$$V_{supply}\,(1HAT\text{-}S3) = \begin{cases} 2V_m & \text{for } N \leq \left(\dfrac{2^{(l-1)}.l}{C^*_m}\right) \\ 2\left[\dfrac{C^*_m\, N}{2^{(l-1)}.l}\right]^{1/2}.V_m & \text{for } N \geq \left(\dfrac{2^{(l-1)}.l}{C^*_m}\right) \end{cases} \qquad (3.132)$$

Since the selection ratio is lower than that of 1APT the supply voltage requirement of the present technique is compared for an equal selection ratio. This is possible by reducing the (V_r/V_c) from the optimum value of $(N)^{1/2}$ of 1APT as discussed in section 3.1.3. The expression for this is reproduced from eqns. (3.26) and (3.25) here, for ready reference.

$$V_{supply}\,(1APT\text{-}R) = \left[\frac{N}{(x-1)^2 + (N-1)}\right]^{1/2} . (x+1).V_{th} \qquad (3.133)$$

where

$$x = \frac{(K+1) - [(K+1)^2 - (K-1)^2\, N]^{1/2}}{(K-1)} \qquad (3.134)$$

and

$$K = R^2 \qquad (3.135)$$

i.e., the square of the selection ratio of the technique being compared with 1APT.

The supply voltage requirements of 1HAT-S3 are compared in Tables 3.14 and 3.15 with that of 1APT (for equal value R), when l = 4 and 6 respectively. The grouping here is for the best possible selection ratio as indicated in Table 3.13. The number of voltage levels in the column wave-forms is reduced to 3 from 7 of 1HAT when l = 6. It is evident from Table 3.15 that the supply voltage is lower than that of 1APT for a wide range of N, even when (V_r/V_c) of 1APT is reduced to make the selection ratio equal.

Case 2: 1HAT-S4

The four groups covering the entire range of mismatches, i.e., 0 to l, when the value of N is odd are represented as given below:-

$$(0,1, \ldots, m1); \; (m1+1, \ldots, \frac{l-1}{2}); \; (\frac{l+1}{2}, \ldots, l-m1-1); \; (l-m1, \ldots, l) \qquad (3.136)$$

The number of entries in the first and the last group are the same. Similarly, the number of entries in the middle two groups are the same for the following reason: The number of unfavourable voltages in a column can be made the same for i and $(l-i)$ mismatches by a proper choice of the polarity of the column voltage. Let the column voltages corresponding to these groups be V_{g1}, V_{g2}, V_{g3} and V_{g4} respectively. The following constraints are imposed due to the above grouping as in the case of 1HAT:-

$$V_{g1} = - V_{g4} ; \quad \text{Let } |V_{g1}| = |V_{g4}| = |V_{m1}| \qquad (3.137)$$

and

$$V_{g2} = - V_{g3} ; \quad \text{Let } |V_{g2}| = |V_{g3}| = |V_{m2}| \qquad (3.138)$$

Only $2^{(l-1)}$ row-select patterns with $i \leq (l/2)$ are considered here for the purpose of analysis, since the error can be made less than $(l/2)$ by a proper choice of the polarity of the column voltage.

Let the row voltage be $-V_r$ for logic 0 and $+V_r$ for logic 1, in the row-select pattern.

The rms voltages across the ON and OFF pixels can be arrived at from the statistics of the instantaneous voltage across the pixels as before.

$$V_{ON}\,(rms) = \left[\frac{s_1 + s_2 + s_3}{2^{(l-1)}\,\frac{N}{l}}\right]^{1/2} \qquad (3.139)$$

and

$$V_{OFF}\ (rms) = \left[\frac{s_4 + s_5 + s_3}{2^{(\ell-1)}\frac{N}{\ell}}\right]^{1/2} \tag{3.140}$$

where,

$$s_1 = \sum_{i=0}^{m1} A_i(V_r+V_{m1})^2 + \sum_{i=(m1+1)}^{(\frac{\ell-1}{2})} A_i(V_r+V_{m2})^2 \tag{3.141}$$

$$s_2 = \sum_{i=0}^{m1} B_i(V_r-V_{m1})^2 + \sum_{i=(m1+1)}^{(\frac{\ell-1}{2})} B_i(V_r-V_{m2})^2 \tag{3.142}$$

$$s_3 = (\frac{N}{\ell} - 1)\left[\sum_{i=0}^{m1} (A_i+B_i)V^2_{m1} + \sum_{i=(m1+1)}^{(\frac{\ell-1}{2})} (A_i+B_i)V^2_{m2}\right] \tag{3.143}$$

$$s_4 = \sum_{i=0}^{m1} A_i(V_r-V_{m1})^2 + \sum_{i=(m1+1)}^{(\frac{\ell-1}{2})} A_i(V_r-V_{m2})^2 \tag{3.144}$$

and

$$s_5 = \sum_{i=0}^{m1} B_i(V_r+V_{m1})^2 + \sum_{i=(m1+1)}^{(\frac{\ell-1}{2})} B_i(V_r+V_{m2})^2 \tag{3.145}$$

The selection ratio given by (V_{ON}/V_{OFF}) should be a maximum in order to obtain a good contrast ratio in the display. The steps involved in the maximization of this ratio are similar to those of IHAT and are given in Appendix 5 . The column voltages V_{m1} and V_{m2} are inter-related as given below, for a maximum selection ratio:-

$$V_{m2} = \frac{DG}{EF} V_{m1} \tag{3.146}$$

where

$$D = \sum_{i=0}^{m1} (A_i + B_i) = \sum_{i=0}^{m1} C_i \qquad (3.147)$$

$$E = \sum_{i=(m1+1)}^{(\frac{l-1}{2})} (A_i + B_i) = \sum_{i=(m1+1)}^{(\frac{l-1}{2})} C_i \qquad (3.148)$$

$$F = \sum_{i=0}^{m1} (A_i - B_i) \qquad (3.149)$$

and

$$G = \sum_{i=(m1+1)}^{(\frac{l-1}{2})} (A_i - B_i) \qquad (3.150)$$

The expression for (V_r/V_{m1}) corresponding to the maximum selection ratio is as given below:-

$$\frac{V_r}{V_{m1}} = \left[\frac{DN}{2^{(l-1)}\,Fl}\right]^{1/2} \times \left[\frac{EF^2 + DG^2}{EF}\right]^{1/2} \qquad (3.151)$$

The maximum selection ratio under these conditions is as follows:-

$$R = \frac{V_{ON}\,(rms)}{V_{OFF}\,(rms)} = \left[\frac{\left(\frac{DN}{Fl}\right)^{1/2} + \left(\frac{EF^2 + DG^2}{2^{(l-1)}\,EF}\right)^{1/2}}{\left(\frac{DN}{Fl}\right)^{1/2} - \left(\frac{EF^2 + DG^2}{2^{(l-1)}\,EF}\right)^{1/2}}\right]^{1/2} \qquad (3.152)$$

An expression for N_{eq} for this technique is obtained as before by equating R to the standard form of selection ratio for APT, IAPT and IHAT. Thus,

$$R = \left[\frac{N_{eq}^{1/2} + 1}{N_{eq}^{1/2} - 1}\right]^{1/2} \qquad (3.153)$$

or

$$N_{eq} = \frac{2^{(l-1)}\,DE\,N}{(EF^2 + DG^2)\,l} \qquad (3.154)$$

The N_{eq} for various groupings when $l=5$ and 7 are given in Table 3.16 as typical examples. The selection ratios of the present technique are compared with those of IAPT for $l=5$ and 7 in Tables 3.17 and 3.18 respectively. It is evident from this table that the selection ratio is lower than that of IAPT since the value of N_{eq} is higher than N. The best groupings leading to a minimum degradation in the selection ratio are indicated by an asterisk in this table.

The rms voltages across the ON and OFF pixels for a maximum selection ratio are given below:-

$$V_{ON}\ (rms) = \left[\frac{2\left(\frac{EF^2 + DG^2}{EF}\right)\left(\frac{DN}{Fl}\right)^{1/2}\left(\left(\frac{DN}{Fl}\right)^{1/2} + \left(\frac{EF^2 + DG^2}{2^{(l-1)}\ EF}\right)^{1/2}\right)}{2^{(l-1)}\ \frac{N}{l}} \right]^{1/2} V_{m1} \quad \ldots\ (3.155)$$

and

$$V_{OFF}\ (rms) = \left[\frac{2\left(\frac{EF^2 + DG^2}{EF}\right)\left(\frac{DN}{Fl}\right)^{1/2}\left(\left(\frac{DN}{Fl}\right)^{1/2} - \left(\frac{EF^2 + DG^2}{2^{(l-1)}\ EF}\right)^{1/2}\right)}{2^{(l-1)}\ \frac{N}{l}} \right]^{1/2} V_{m1} \quad \ldots\ (3.156)$$

The OFF pixels are usually biased near V_{th}, in order to obtain a good contrast ratio in the display. Hence, V_{m1} is determined in terms of V_{th} by equating V_{OFF} to V_{th}. Thus,

$$V_{m1} = \left[\frac{2^{(l-1)}\ \frac{N}{l}}{2\left(\frac{EF^2 + DG^2}{EF}\right)\left(\frac{DN}{Fl}\right)^{1/2}\left(\left(\frac{DN}{Fl}\right)^{1/2} - \left(\frac{EF^2 + DG^2}{2^{(l-1)}\ EF}\right)^{1/2}\right)} \right]^{1/2} V_{th} \quad \ldots\ (3.157)$$

The supply voltage is determined by the maximum voltage swing in the addressing waveforms. The supply voltage requirement of the present technique

Table 3.16. Various possible groupings of mismatches in IHAT-S4 - A comparison

l	Grouping of mismatches (i) for deciding the column voltage	Parameters				Row-select voltage $\lvert V_r \rvert$	Column voltages				N_{eq}	Best grouping for each l
		D	E	F	G		V_{g1}	V_{g2}	V_{g3}	V_{g4}		
5	(0,1)(2)(3)(4,5)	6	10	4	2	$(\frac{69N}{800})^{1/2}$	-1	$-\frac{3}{10}$	$+\frac{3}{10}$	$+1$	$\frac{24N}{23}$	*
5	(0)(1,2)(3,4)(5)	1	15	1	5	$(\frac{N}{30})^{1/2}$	-1	$-\frac{1}{3}$	$+\frac{1}{3}$	$+1$	$\frac{6N}{5}$	
7	(0,1,2)(3)(4)(5,6,7)	29	35	15	5	$(\frac{1247N}{17640})^{1/2}$	-1	$-\frac{29}{105}$	$+\frac{29}{105}$	$+1$	$\frac{232N}{215}$	*
7	(0,1)(2,3)(4,5)(6,7)	8	56	6	14	$(\frac{2N}{63})^{1/2}$	-1	$-\frac{1}{3}$	$+\frac{1}{3}$	$+1$	$\frac{8N}{7}$	
7	(0)(1,2,3)(4,5,6)(7)	1	63	1	19	$(\frac{53N}{3528})^{1/2}$	-1	$-\frac{19}{63}$	$+\frac{19}{63}$	$+1$	$\frac{72N}{53}$	

Table 3.17. IHAT-S4 (with $l=5$) vs. IAPT - A comparison

N	Selection ratio of IHAT-S4	Selection ratio of IAPT	N_{eq} of IHAT-S4	V_{supply} of IHAT-S4	Reduced V_r/V_c of IAPT-R	$\frac{V_{supply}(IHAT\text{-}S4)}{V_{supply}(IAPT\text{-}R)} \times 100\%$
5	1.599	1.618	5.217	2.872	1.818	98.49
10	1.377	1.387	10.435	2.592	2.571	77.72
15	1.295	1.302	15.652	2.834	3.001	77.59
20	1.249	1.255	20.870	3.200	3.636	78.63
25	1.219	1.225	26.087	3.526	4.065	80.46
30	1.198	1.203	31.304	3.826	4.453	81.87
35	1.182	1.186	36.522	4.096	4.935	83.00
40	1.169	1.173	41.739	4.351	5.142	83.93
45	1.158	1.162	46.957	4.591	5.454	84.72
50	1.150	1.153	52.174	4.818	5.749	85.40
60	1.135	1.139	62.609	5.241	6.297	86.52
70	1.125	1.128	73.043	5.631	6.802	87.41
80	1.116	1.119	83.478	5.994	7.272	88.14
90	1.109	1.112	93.913	6.336	7.713	88.75
100	1.103	1.106	104.348	6.659	8.130	89.28
200	1.072	1.073	208.696	9.271	11.497	92.24
300	1.058	1.060	313.043	11.278	14.081	93.61
400	1.050	1.051	417.391	12.971	16.260	94.45
500	1.0448	1.0458	521.739	14.462	18.179	95.03

Table 3.18. IHAT-S4 (with $l=7$) vs. IAPT - A comparison

N	Selection ratio of IHAT-S4	Selection ratio of IAPT	N_{eq} of IHAT-S4	V_{supply} of IHAT-S4	Reduced V_r/V_c of IAPT-R	$\frac{V_{supply}(IHAT\text{-}S4)}{V_{supply}(IAPT\text{-}R)} \times 100\%$
7	1.464	1.488	7.553	2.521	2.004	83.95
14	1.3010	1.315	15.107	2.333	2.835	65.77
21	1.238	1.248	22.660	2.756	3.472	68.72
28	1.202	1.211	30.214	3.127	4.009	70.84
35	1.178	1.186	37.767	3.456	4.482	72.37
42	1.161	1.168	45.321	3.754	4.910	73.54
49	1.148	1.155	52.874	4.029	5.303	74.47
56	1.138	1.144	60.428	4.285	5.669	75.24
63	1.130	1.135	67.981	4.526	6.013	75.90
70	1.123	1.128	75.535	4.754	6.338	76.45
140	1.0850	1.0884	151.07	6.599	8.964	79.63
210	1.0688	1.0716	226.60	8.017	10.978	81.13
280	1.0593	1.0617	302.14	9.213	12.677	82.04
350	1.0529	1.0550	377.67	10.268	14.173	82.68
420	1.0481	1.0500	453.21	11.221	15.526	83.16
490	1.0445	1.0462	528.74	12.092	16.770	83.54
560	1.0415	1.0432	604.28	12.915	17.928	83.84
630	1.0391	1.0407	679.81	13.681	19.015	84.10
700	1.0371	1.0385	755.35	14.407	20.044	84.31
1050	1.0302	1.0314	1133.02	17.584	24.548	85.05
1400	1.0261	1.0271	1510.70	20.262	28.345	85.50

is as given below:-

$$V_{supply}\,(1HAT\text{-}S4) = \begin{cases} 2V_{m1} & \text{for } V_r \leq V_{m1} \\ 2V_r & \text{for } V_r \geq V_{m1} \end{cases} \quad (3.158)$$

The voltage V_r is greater than V_{m1} when the following condition is satisfied:-

$$N > \left[\frac{2^{(l-1)}\, l\, EF^2}{D(EF^2 + DG^2)}\right] = N_t \quad (3.159)$$

Hence,

$$V_{supply}\,(1HAT\text{-}S4) = \begin{cases} 2V_{m1} & \text{for } N \leq N_t \\ 2\left[\frac{D(EF^2 + DG^2)N}{2^{(l-1)}\; EF^2 \,.\, l}\right]^{1/2} V_{m1} & \text{for } N \geq N_t \end{cases} \quad (3.160)$$

The selection ratio of 1APT is higher than the present technique. The supply voltage requirements of 1HAT-S4 is compared with 1APT-R, having a reduced (V_r/V_c) and a selection ratio equal to the present technique using eqns. (3.133)-(3.135).

Tables 3.17 and 3.18 also compare the supply voltage requirements of 1HAT-S4 when $l = 5$ and 7 respectively. This comparison is done for an equal selection ratio as discussed above. It is evident from these tables that the present technique requires a lower supply voltage as compared to 1APT for a wide range of values of N. The number of voltage levels when $l = 7$ has been reduced to 4 as compared to 8 in 1HAT. It is evident from Table 3.18 that the selection ratio is close to that of 1APT or 1HAT, although there is a 50% reduction in the complexity of the column drivers.

3.4.4. Discussion

1HAT-S can also be used to multiplex TNLCDs if a selection ratio lower but close to that of IAPT is adequate for a given application. The lower supply voltage requirement of IHAT-S as compared to IAPT makes it attractive for such applications.

3.5 A COMPARISON

The IAPT has become a standard technique for multiplexing LCDs at present, due to its low supply voltage requirement as compared to APT. Hence the new addressing techniques are compared with IAPT in Table 3.19.

The BAT is suitable for multiplexing displays with a single row of characters due to its simple addressing waveforms. The HAT serves as a link between BAT and IHAT. The IHAT is especially suited for multiplexing displays with large N, due to its low supply voltage requirement as compared to IAPT. However, the hardware complexity increases with l. The hardware complexity of IHAT is reduced in IHAT-S3 and IHAT-S4. The supply voltages of these techniques are also lower as compared to that of IAPT.

HATs proposed in this thesis require N to be an integral multiple of l. This condition is imposed to reduce the number of time intervals to complete a cycle. However displays which do not meet this condition can also be addressed with HATs by adopting any one of the following approaches :-

- Additional dummy rows (not connected to the display) can be introduced to satisfy the addressing requirement.
- Subgroups can be overlapped; after subjecting l-rows to binary addressing a new sub-group is formed by dropping one row and adding a new row to the sub-group. Here the number of time intervals to complete a cycle increases by a factor of l as compared to the case with non-intersecting sub-groups.

Table 3.19. New Addressing Techniques vs. IAPT - A Comparison

Parameters	TECHNIQUES					
	BAT	HAT	IHAT	IHAT-S3	IHAT-S4	IAPT
Selection Ratio (R)	Low	Low	High	Intermediate		High
N_{eq}	$\geqslant \frac{4N}{3}$	$\geqslant \frac{4N}{3}$	$= N$	$>N$ (Values close to N are possible)		$= N$
Supply voltage requirement as compared to IAPT	Low	Low for limited values of N	Low for a wide range of N especially high values of N	Low for wide range of N		-
Duty cycle	$\left[\frac{A}{2^{l-1}}\right]$	$\left[\frac{A}{2^{l-1}}\right]\frac{l}{N}$	$\left[\frac{A}{2^{l-1}}\right]\frac{l}{N}$	$\left[\frac{\sum_{i=0}^{m} A_i}{2^{l}-1}\right]\frac{l}{N}$	$\left[\frac{A}{2^{l-1}}\right]\frac{l}{N}$	$\left[\frac{1}{N}\right]$
No. of voltage levels in the row waveforms	2	3	3	← 3 →		4
No. of voltage levels in the column waveforms	2	2	$(l+1)$	3	4	4
No. of time intervals in a cycle (dc-free operation)	2^N (High)	← $\left[\frac{2^l N}{l}\right]$ (Intermediate) →				2N (low)
Salient feature	Simple addressing waveforms	Extends BAT for higher values of N	Good reduction in supply voltage with proper l	Hardware complexity is less as compared to IHAT.		Lower supply voltage as compared to APT
Comments	Suitable for low values of N only	Link between the BAT and IHAT	Hardware complexity increases with l	Performance close to IHAT possible in spite of the hardware reduction		The standard technique

3.6 RESTRICTED PATTERN ADDRESSING TECHNIQUES (RPAT)

Oscilloscope and logic analyzer displays are primarily used for displaying waveforms. The patterns to be displayed in these applications are restricted since the waveforms are mostly single valued functions of time. Two addressing techniques for displaying multiple waveforms are proposed in this section. The selection ratios of these techniques are independent of the matrix size and are higher as compared to the conventional addressing techniques.

3.6.1 Background

Special addressing techniques for displaying waveforms were reviewed in Chapter 2. The theoretical limits of the value of the selection ratio for the addressing techniques were also reviewed in Chapter 2. The non-multiplexed addressing reviewed in Chapter 2, can be used for displaying restricted patterns. However, basics of simple new addressing techniques are developed here.

The Oscilloscope displays require a large matrix size, in order to reproduce the waveforms with a high resolution. The number of rows (N) should be at least 256 in order to achieve an eight bit accuracy for a single waveform. The value of N should be even higher for displaying multiple waveforms. The waveforms to be displayed are mostly single valued functions. The following conditions are satisfied when W such waveforms are to be displayed:-

- The number of selected pixels in each column is W; and
- The number of background pixels (N-W) is far greater than W.

Hence, the column voltage corresponding to the background pixels can be chosen to be zero (i.e., same as that of unselected rows). This suppresses the voltage across the pixels in the unselected rows to zero during (N-W) time intervals out of the total N time intervals in a cycle. In contrast, this cannot be done in the case of general patterns since the number of selected pixels in a column is not known.

The column voltage for a selected pixel can be chosen as follows:-

- In-phase with the row-select voltage ; or
- Out-of-phase with the row-select voltage.

This leads to two different addressing techniques as given below:-

- The selected pixels, i.e., the points on the waveform, get a lower rms voltage as compared to the background pixels, when the column voltage (V_c) is in-phase with the row-select voltage (V_r). This results in a negative contrast mode in TNLCDs (polarizers perpendicular to each other), with the waveform appearing bright against a dark background. A positive contrast mode is possible however, if a Guest-Host display is used instead of TNLCDs. This technique is referred to as RPAT-NC.
- The selected pixels get a higher rms voltage as compared to the background pixels, when V_c is out-of-phase with V_r. This results in a positive contrast mode in TNLCDs, with the waveforms appearing dark against a bright background. This technique is referred to as RPAT-PC.

3.6.2 Techniques

The RPAT techniques are based on the line-by-line addressing scheme and are similar to APT except for the choice of the column voltage. The RPAT-PC and RPAT-NC themselves differ only in the phase of the column voltage as discussed earlier. The various steps involved in these techniques are given below:-

i) The N rows to be multiplexed are selected sequentially with a row-select voltage $\pm V_r$ one after the other ;

ii) The unselected rows are grounded. There will be (N-1) unselected rows at any given instant of time ;

iii) The column voltages are decided, depending on the data to be displayed in the selected row ;

iv) The column voltage is zero for a background pixel in the selected row ;

v) The column voltage is $\pm V_c$, i.e., in-phase with the row-select voltage for a selected pixel, in the case of the RPAT-NC. The column voltage is $\mp V_c$, i.e., out-of-phase with the row-select voltage for a selected pixel, in the case of RPAT-PC ;

vi) Both the row and column voltages are simultaneously applied to the matrix display for a time duration T ;

vii) The column voltages for the subsequent row are decided using steps (iv) and (v). This row is selected for an equal duration of time at the end of T ;

viii) A cycle is completed, when all the rows (N) in the matrix are selected once ;

ix) The display is refreshed by repeating this cycle continuously ;

x) The polarity of the addressing waveforms is reversed periodically in order to achieve a dc-free operation. This can be reversed either within a row-select period (T) or after completion of every cycle, i.e., period NT. The duration T should be small as compared to the response time of the display in order to ensure the rms behavior of TNLCDs. The duration of a cycle (=NT) should be sufficiently low in order to avoid flicker, in the display.

Typical addressing waveforms of both the RPAT-NC and RPAT-PC are given in Figures 3.8 and 3.9 respectively. The maximum voltage swing in the addressing waveform is $2V_r$, due to the polarity reversal in them for a dc-free operation. However, it is possible to reduce the voltage swing to V_r in the case of RPAT-NC and to $(V_r + V_c)$ in the case of RPAT-PC by modifying the addressing waveforms. The technique employed here is similar to that of IAPT. Typical addressing waveforms leading to a reduction in the supply voltage requirement are shown in Figs. 3.10 and 3.11 for the RPAT-NC and RPAT-PC respectively.

3.6.3 Analysis

Let the number of waveforms to be displayed be W. The background pixels get a voltage $|V_r|$ for a time duration T, when the corresponding row is selected. This pixel gets a voltage $|V_c|$ during W time intervals corresponding to the W selected pixels in that column. The voltage across the pixel is zero during the rest of the (N-W-1) time intervals. Hence,

$$V_{Background}\,(rms) = \left[\frac{T \cdot V_r^2 + W \cdot T \cdot V_c^2}{NT}\right]^{1/2} \qquad (3.161)$$

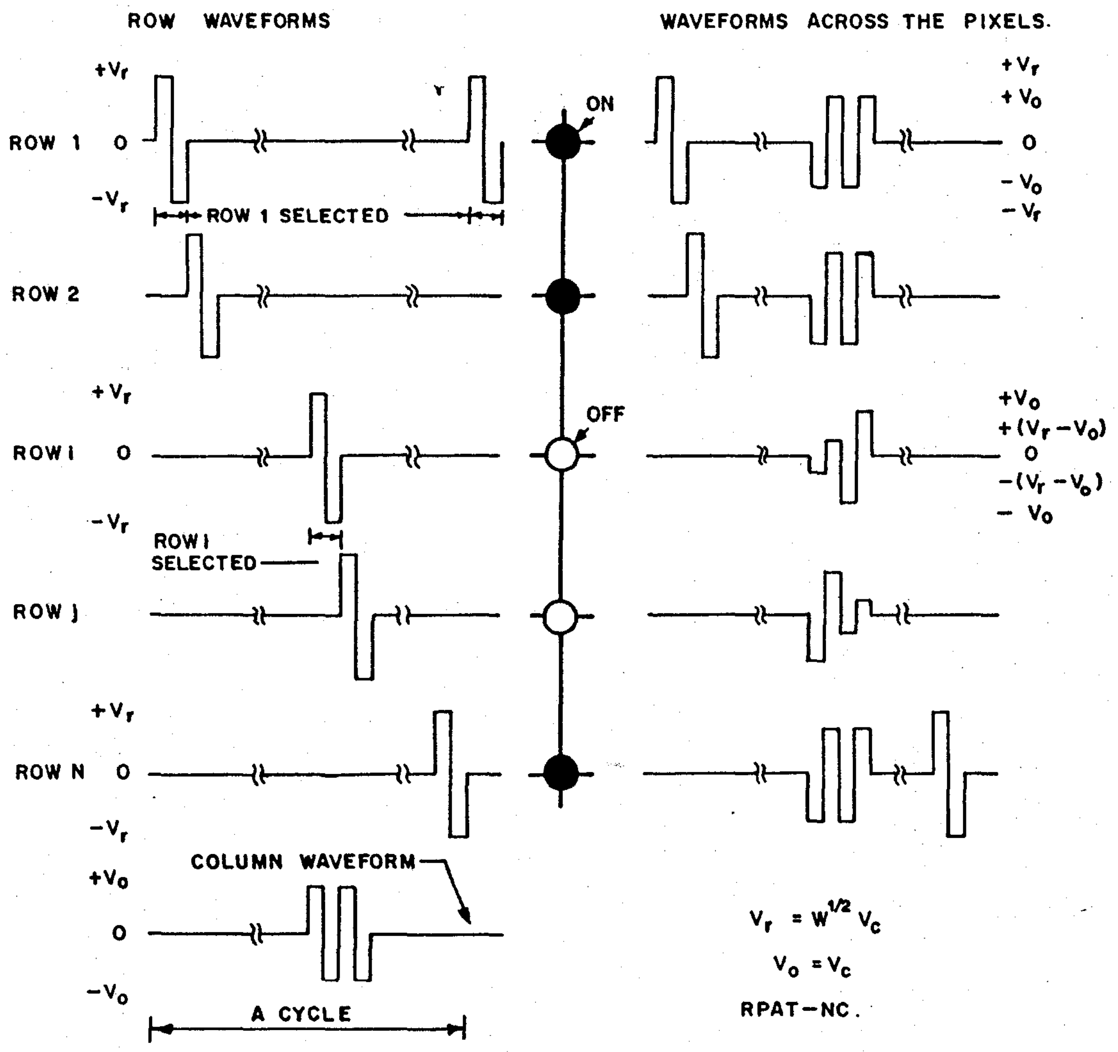

Fig. 3.8. Typical addressing waveforms of RPAT-NC.

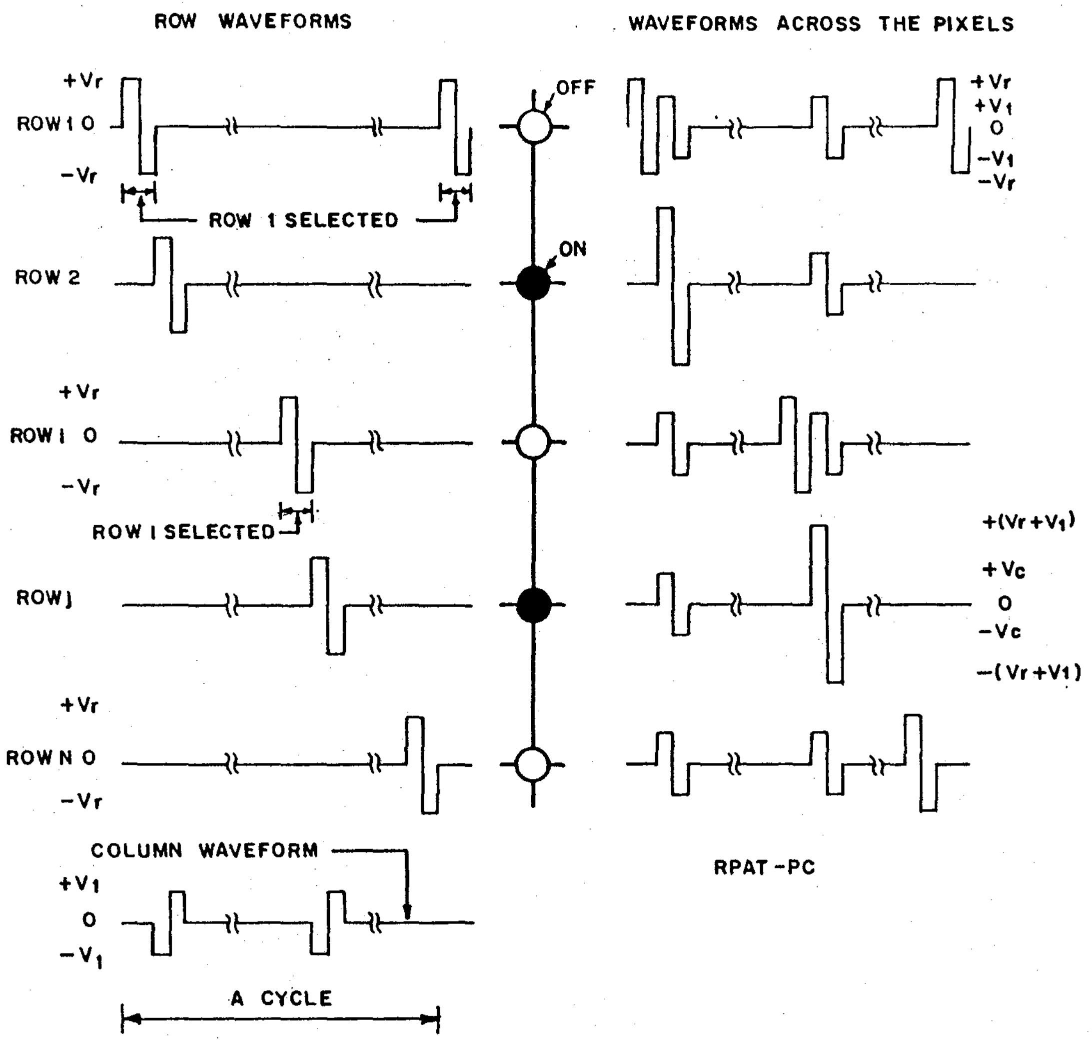

Fig. 3.9. Typical addressing waveforms of RPAT-PC.

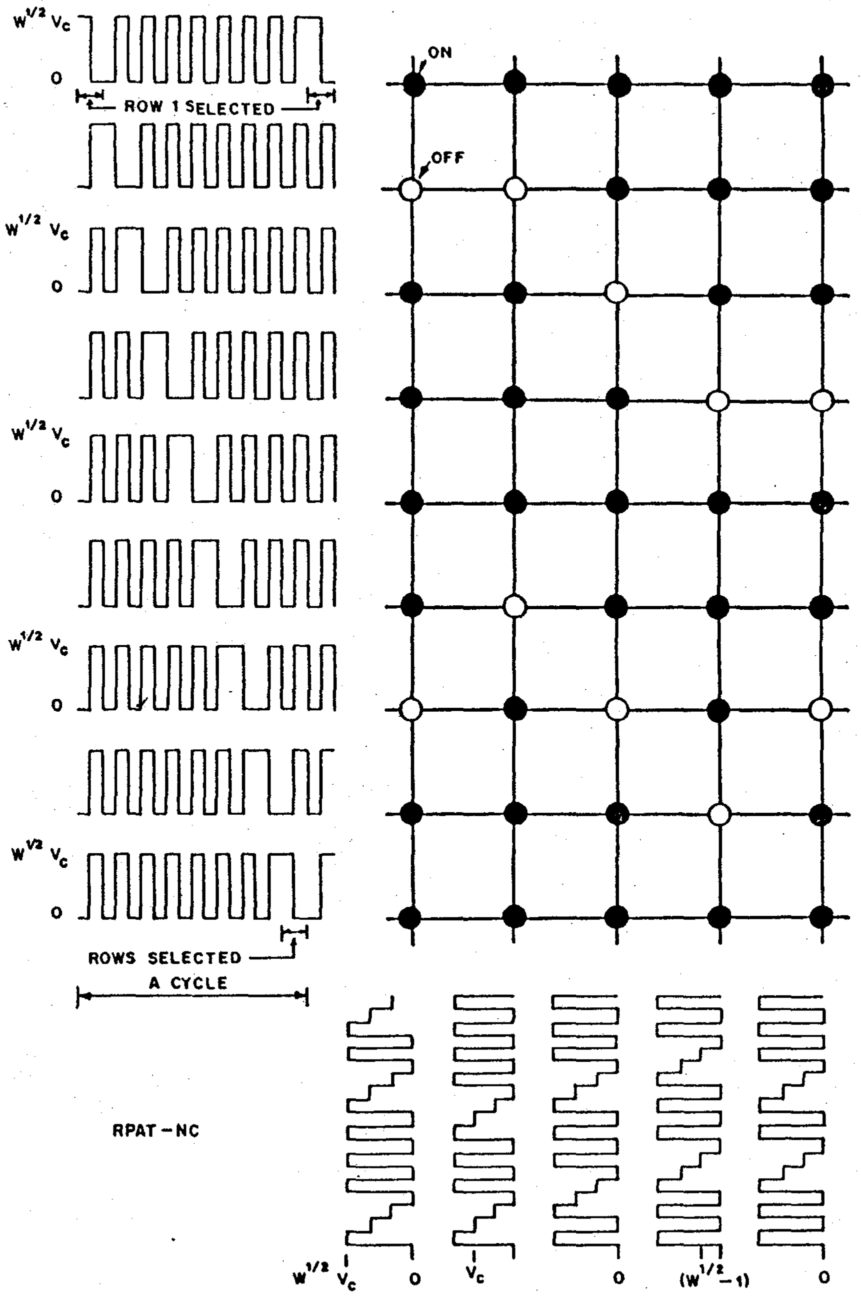

Fig. 3.10. Modified addressing waveforms of RPAT-NC with reduced supply voltage requirement.

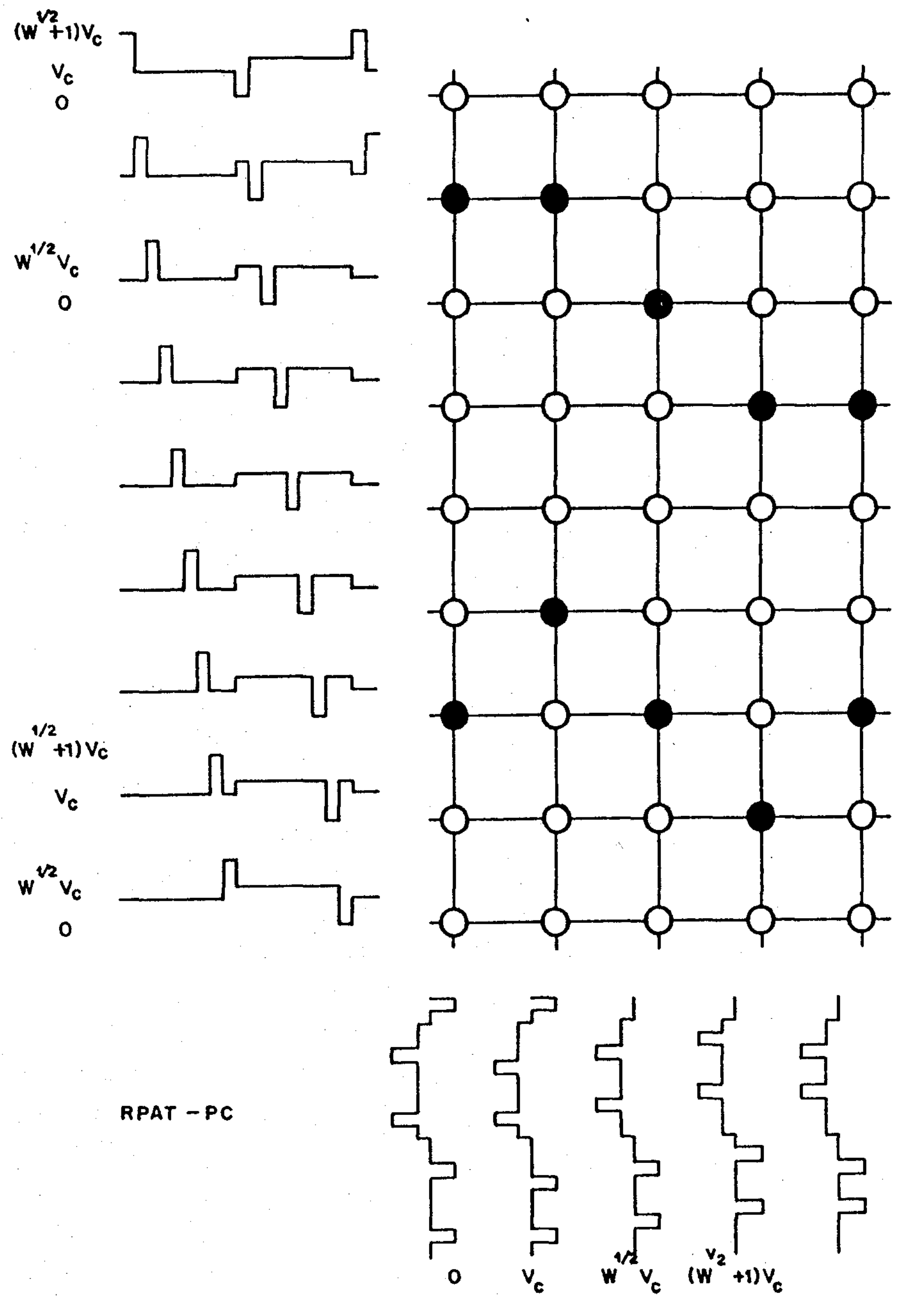

Fig. 3.11. Modified addressing waveforms of RPAT-PC with reduced supply voltage requirement.

Let the column voltage for a selected pixel be in-phase with the row-select voltage. Then the selected pixel gets a voltage $|V_r - V_c|$ when the corresponding row is selected. The voltage across this pixel is $|V_c|$ during $(W - 1)$ time intervals corresponding to the rest of the selected pixels in that column. The voltage across the pixel is zero during rest of the $(N - W)$ time intervals. Hence,

$$V_{selected}\,(rms) = \left[\frac{T\,.\,(V_r - V_c)^2 + (W-1)T\,.\,V_c^2}{NT}\right]^{1/2} \qquad (3.162)$$

The ratio $[V_{selected}(rms)/V_{background}(rms)]$ should be optimized in order to get a maximum discrimination between them. This ratio is optimum for the condition

$$\frac{V_r}{V_c} = \pm\, W^{1/2} \qquad (3.163)$$

Hence, the voltage across a background pixel under optimum condition is

$$V_{background}\,(rms) = \left[\frac{2W}{N}\right]^{1/2} .\,V_c \qquad (3.164)$$

The voltage across the selected pixels however depends on the relative phase between V_r and V_c. Hence, the rms voltage across a background pixel under the optimum condition is

$$V_{selected}\,(rms) = \left[\frac{2W - 2W^{1/2}}{N}\right]^{1/2} V_c \quad \text{for } V_r = +W^{1/2}V_c \qquad (3.165)$$

$$V_{selected}\,(rms) = \left[\frac{2W + 2W^{1/2}}{N}\right]^{1/2} V_c \quad \text{for } V_r = -W^{1/2}\,V_c \qquad (3.166)$$

Case 1: RPAT-NC

From eqns. (3.164) and (3.165), it is clear that the selected pixels get a lower voltage as compared to the background pixels when V_c is in-phase with V_r. Hence, the background pixels are ON and the selected pixels are OFF. Hence, the selection ratio can be arrived at from eqns. (3.164) and (3.165) as follows : -

$$R = \frac{V_{ON}\,(rms)}{V_{OFF}\,(rms)} = \left[\frac{W}{W - W^{1/2}}\right]^{1/2} = \left[\frac{W^{1/2}}{W^{1/2} - 1}\right]^{1/2} \qquad (3.167)$$

The N_{eq} here gives an idea of the selection ratio in terms of the standard IAPT. N_{eq} is obtained by equating this selection ratio to that of the standard form of IAPT as follows : -

$$R = \left[\frac{W^{1/2}}{W^{1/2} - 1}\right]^{1/2} = \left[\frac{N_{eq}^{1/2} + 1}{N_{eq}^{1/2} - 1}\right]^{1/2} \qquad (3.168)$$

or

$$N_{eq} = (2W^{1/2} - 1)^2 \qquad (3.169)$$

From eqns. (3.167) and (3.169), it is clear that the selection ratio is independent of the matrix size and depends just on the number of waveforms displayed. The selection ratio and the N_{eq} of the RPATs are compared with that of FMT (discussed in chapter 2) in Table 3.20. From this table it is evident that the selection ratio of RPAT-NC is higher than that of FMT for all values of W. Moreover, it is clear that if W waveforms are displayed using FMT then W^2 waveforms can be displayed using RPAT-NC, for the same selection ratio (Appendix 6.c).

Table 3.20. RPATs vs. FMT - A Comparison

Number of waveforms displayed (W)	Restricted Pattern Addressing Techniques				Multi-trace using FMT	
	RPAT - NC		RPAT - PC			
	Selection ratio $R = \left[\frac{W^{1/2}}{W^{1/2}-1}\right]^{1/2}$	$N_{eq} = (2W^{1/2}-1)^2$	Selection ratio $R = \left[\frac{W^{1/2}+1}{W^{1/2}}\right]^{1/2}$	$N_{eq} = (2W^{1/2}+1)^2$	Selection ratio $R = \left[\frac{W}{W-1}\right]^{1/2}$	$N_{eq} = (2W-1)^2$
1	Infinite	1.00	1.414	9.00	Infinite	1.00
2	1.848	3.34	1.307	14.66	1.414	9.00
3	1.538	6.07	1.256	19.93	1.225	25.00
4	1.414	9.00	1.225	25.00	1.155	49.00
5	1.345	12.06	1.203	29.94	1.118	81.00
6	1.300	15.20	1.187	34.80	1.095	121.00
7	1.268	18.42	1.174	39.58	1.080	169.00
8	1.244	21.69	1.163	44.31	1.069	225.00
12	1.186	35.14	1.135	62.86	1.044	529.00
16	1.155	49.00	1.118	81.00	1.033	961.00
25	1.118	81.00	1.095	121.00	1.021	2401.00

The maximum voltage swing in the addressing waveforms can be reduced to V_r from $2V_r$ for the following reasons:-

- The voltage across any pixel in the display does not exceed $|V_r|$ at any given instant, and
- The polarity of the row-select voltage is either positive or negative at a given instant.

The modifications in the addressing waveforms of Fig.3.8 to reduce the supply voltage requirement are given below: -

- The row and column voltages are shifted by $+V_r$, when the polarity of the row-select voltage is negative ;
- The row and column voltages are the same (unaltered) when the polarity of the row-select voltage is positive. This transformation does not alter the selection ratio, since the resultant waveforms across the pixels are the same as before.

The OFF pixels are usually biased near V_{th} in order to achieve a good contrast in the display. The column voltage V_c can be obtained in terms of V_{th} as follows:-

$$V_{OFF} = V_{th} = \left[\frac{2(\omega - \omega^{1/2})}{N}\right]^{1/2} V_c \qquad (3.170)$$

or

$$V_c = \left[\frac{N}{2(\omega - \omega^{1/2})}\right]^{1/2} V_{th} \qquad (3.171)$$

Hence, the supply voltage for this technique is

$$V_{supply}\,(RPAT\text{-}NC) = V_r = W^{1/2}\,V_c = \left[\frac{N\,W}{2(W-W^{1/2})}\right]^{1/2} V_{th}$$

.... (3.172)

Since most of the pixels in the display are ON in this case, these pixels can as well be biased near V_{SAT} instead of biasing the OFF pixels to V_{th}. The column voltage in terms of V_{SAT} is obtained as follows:-

$$V_{ON} = V_{SAT} = \left[\frac{2W}{N}\right]^{1/2} V_c \qquad (3.173)$$

or

$$V_c = \left[\frac{N}{2W}\right]^{1/2} V_{SAT} \qquad (3.174)$$

The supply voltage requirement here is

$$V_{supply}\,(RPAT\text{-}NC) = V_r = W^{1/2}.V_c = \left[\frac{N}{2}\right]^{1/2} V_{SAT} \qquad (3.175)$$

It is clear from the above equation that the supply voltage is independent of the number of waveforms (W) and depends only on the number of address lines (N) in the matrix. However the ratio (V_r/V_c) depends on W as in eqn. (3.163).

<u>Case 2:</u> <u>RPAT-PC</u>

It is evident from eqns. (3.164) and (3.166) that the selected pixels get a higher rms voltage as compared to the background pixels, when V_c is out-of-phase with V_r. Hence, the selection ratio in this case is

$$R = \frac{V_{ON}\,(rms)}{V_{OFF}\,(rms)} = \left[\frac{W + W^{1/2}}{W}\right]^{1/2} = \left[\frac{W^{1/2} + 1}{W^{1/2}}\right]^{1/2} \qquad (3.176)$$

Here again, the N_{eq} gives an idea of the selection ratio obtained in this technique, in terms of the standard IAPT. Hence,

$$R = \left[\frac{W^{1/2}+1}{W^{1/2}}\right]^{1/2} = \left[\frac{N_{eq}^{1/2}+1}{N_{eq}^{1/2}-1}\right]^{1/2} \quad (3.177)$$

or

$$N_{eq} = (2W^{1/2} + 1)^2 \quad (3.178)$$

It is clear from eqns. (3.176) and (3.178) that the selection ratio here too is independent of the matrix size and depends just on W, the number of waveforms displayed. The selection ratio of RPAT-PC is lower than that of RPAT-NC as shown in the Table 3.20. However, the RPAT-PC has a higher selection ratio as compared to FMT, when the number of waveforms is greater than 2. But, the FMT (discussed in chapter 2) leads to a negative contrast mode while RPAT-PC operates in the positive contrast.

Here again the maximum swing in the addressing waveforms can be reduced to (V_r+V_c) instead of $2V_r$ as in Fig.3.9. This is possible for the following reasons:-

- The maximum voltage across a pixel in the display is $|V_r+V_c|$; and
- The polarity of the row-select voltage is either positive or negative at a given instant.

The modifications required to reduce the supply voltage requirement of RPAT-PC shown in Fig.3.9 are given below:-

- The row and column waveforms are shifted by $+V_c$, when the polarity of the row-select voltage is positive; and

- The row and column voltages are shifted by $+V_r$, when the polarity of the row-select voltage is negative. Here again the selection ratio is unaltered, since the resultant waveforms across the pixels are the same as before.

The OFF pixels are usually biased near V_{th} in order to get a good contrast in the display. The voltage V_c can be determined in terms of V_{th} as follows:-

$$V_{OFF}\ (rms) = V_{th} = \left[\frac{2W}{N}\right]^{1/2} V_c \qquad (3.179)$$

or

$$V_c = \left[\frac{N}{2W}\right]^{1/2} V_{th} \qquad (3.180)$$

The supply voltage required for this technique is determined as follows:-

$$V_{supply}\ (\text{RPAT-PC}) = (V_r + V_c) = (W^{1/2} + 1)\, V_c \qquad (3.181)$$

or

$$V_{supply}\ (\text{RPAT-PC}) = \left[\frac{N}{2W}\right]^{1/2} (W^{1/2} + 1)\ V_{th} \qquad (3.182)$$

From the above equation, it is evident that the supply voltage increases with N.

3.6.4. Discussion

The merits and demerits of RPATs are given in Table 3.21. The RPAT-NC is a generalized form of PCT (reviewed in Chapter 2). The addressing waveforms of RPAT-NC is the same as those of PCT, when $W = 1$. The selection ratios of these techniques are close to the theoretical limits given by eqn. (2.23), when the number of selected pixels in each column is small

Table 3.21. Merits and Demerits of RPATs

<u>Merits</u>

- Selection ratio higher than conventional techniques
- Selection ratio independent of matrix size.
- Simple addressing waveforms.

<u>Demerits</u>

- Supply voltage requirement proportional to matrix size.

as compared to the number of address lines (N) in a matrix display.

Legends are useful in Logic analyzer and Oscilloscopes. The alphanumeric information can be displayed as such at the top of the display with its inverse video at the bottom. This ensures the number of selected pixels in each column to be constant as required in RPATs. Two or more waveforms may intersect at a point when multiple waveforms are displayed. The number of selected pixels in the corresponding column will be less by (l-1) when l waveforms intersect at a point. This changes the rms voltage across the pixels in that column as compared to the pixels in the other columns. The non-uniformity arising due to the intersection of the waveforms can be compensated as follows :-

- By introducing (l-1) dummy rows to the matrix ; and
- By adding (l-1) selected pixels to the column with l waveforms intersecting at a point.

The RPATs can be extended to applications wherein the following conditions are satisfied:

- The number of selected pixels (s) is a variable with an upper bound, i.e., $0 \leq s \leq L$; and
- L is small as compared to N.

Since the selection ratio is independent of the matrix size L dummy rows can be added to the matrix and the number of pixels in each column can be made to be L by adding (L-s) selected pixels in the dummy rows in each column. The selection ratio depends on L and is obtained by substituting L in place of W in eqns. (3.166) and (3.175). The value of L should be taken into consideration in the calculation of the supply voltage by substituting (N + L) in place of N in equations (3.174) and (3.182).

Some new addressing techniques for displaying both general and resticted patterns have been proposed in this Chapter. These techniques have also been analysed and compared with the conventional techniques here. The practical implementation of some of these techniques are taken up in the next Chapter for the experimental verification of the analysis in this Chapter.