Decohering *d*-dimensional quantum resistance

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(Received 16 September 2007; revised manuscript received 19 November 2007; published 7 February 2008)

The Landauer scattering approach to four-probe resistance is revisited for the case of a *d*-dimensional disordered resistor *in the presence of decoherence*. Our treatment is based on an invariant-embedding equation for the evolution of the coherent reflection amplitude coefficient in the length of a one-dimensional disordered conductor, where decoherence is introduced at par with the disorder through an outcoupling, or stochastic absorption, of the wave amplitude into side (transverse) channels, and its subsequent incoherent reinjection into the conductor. This is essentially in the spirit of Büttiker's reservoir-induced decoherence. The resulting evolution equation for the probability density of the four-probe resistance in the presence of decoherence is then generalized from the one-dimensional to the *d*-dimensional case following an anisotropic Migdal-Kadanoff-type procedure and analyzed. The anisotropy, namely, that the disorder evolves in one arbitrarily chosen direction only, is the main approximation here that makes the analytical treatment possible. A qualitative result is that arbitrarily small decoherence reduces the localization-delocalization transition to a crossover making resistance moments of all orders finite.

DOI: 10.1103/PhysRevB.77.064201 PACS number(s): 72.10.−d, 05.60.Gg, 05.10.Gg, 03.65.Yz

I. INTRODUCTION

Electron localization,[1,2] strong as well as weak, and the associated metal-insulator transition and conductance fluctuations[3] are due essentially to the time-persistent interference of the complex wave amplitudes that result from multiple elastic scattering on randomly distributed defects in the conductor with quenched potential disorder. Similar interference effects also manifest in mesoscopic systems as various phase-sensitive phenomena, e.g., the well-known persistent ring currents and the Aharonov-Bohm oscillations. Clearly, these one-electron phase-sensitive phenomena can get suppressed by decoherence. Microscopically,[4] decoherence can arise from incoherent processes involving, e.g., the inelastic electron-phonon or the electron-electron scattering, as also from an entanglement with the environmental degrees of freedom, which remain undetected or unmeasured. (While coherent inelastic scattering is, in principle, possible, as indeed in the case of neutron scattering, it is not relevant to the case of coherent multiple scattering of electrons in a disordered conductor). The question now is how to incorporate decoherence phenomenologically in an analytical treatment of the otherwise Hamiltonian system such as the system of noninteracting electrons moving in a lattice, or a continuum with random elastic scatterers, e.g., the Anderson model system for metal-insulator transition in random lattices.[1] Decoherence has often been included theoretically and probed experimentally through a phase breaking or dephasing cutoff length scale introduced on physical grounds.[2,5] It is clearly desirable, however, to have a phenomenology for introducing the degree of decoherence in the analytical treatment of elastic scattering in a disordered conductor. A highly successful and widely used approach to decoherence was pioneered by Büttiker and co-workers[6–9] through the idea of reservoir-induced decoherence. The latter could be introduced naturally in the scattering approach of Landauer[10] to quantum transport, e.g., the four-probe resistance. For the reservoir-induced decoherence, one inserts a scattering matrix with appropriately chosen side (transverse) channels and thereby outcouples a partial wave amplitude into an electron reservoir. The amplitude reemitted from the reservoir is then reinjected back into the conductor, adding necessarily incoherently to the transmitted amplitude along the conductor (the longitudinal channel) that carries the transport current. The chemical potential of the reservoir is, of course, tuned so as to make the net current in the side channel vanish on the average. (This is clearly analogous to the "potentiometric" probe of Engquist and Anderson.[11]) The net result is the introduction of decoherence, or partial coherence, which can be readily parametrized. It describes, in particular, the quantum-to-classical crossover of a series combination of conductors[7] with increasing strength of the coupling to the intervening reservoirs. While used extensively in the context of mesoscopic (zero-dimensional) systems,[12] the reservoir-induced decoherence has also been invoked by many workers for treating partial coherence in quantum transport on tight-binding lattices—without disorder,[13–15] and with weak disorder,[16–18] as also in a disordered continuum.[19] These studies are, however, confined to one-dimensional conductors.

In this work, we have considered the case of a *d*-dimensional conductor for $d \geqslant 1$ in the presence of both the quenched disorder and decoherence. Our analytical treatment is based on the invariant-embedding approach developed earlier for a one-dimensional conductor with quenched disorder[20–22] and its subsequent generalization to higher dimensions for anisotropic disorder using the Migdal-Kadanoff technique.[23,24] Here, first, the elastic scatterers (resistances) are combined in series quantum mechanically along an arbitrarily chosen direction, and then classical Ohm's law is used to combine these resistances in parallel along the transverse directions. This is followed by a scaling transformation with an infinitesimal increase in scale at each step. The resulting "transverse" mixing up of disorder through the evolution equation is known to give a qualitatively correct description of the weak scattering regime in the absence of decoherence,[24] despite the assumption of anisotropic disor-

der, which is an approximation. In our approach, decoherence and elastic scattering (quenched disorder) are treated formally at par through a proper insertion of the scattering (S) matrices, i.e., transverse channels distributed over the conductor. Specifically, a side channel is to be viewed as causing a stochastic absorption—a coherent process by itself. The incoherent reinjection with zero net side current is, however, effectively realized through the use of the Landauer expression $|R(L)|^2/(1-|R(L)|^2)$ for the four-probe resistance, but with $|R(L)|^2$ now calculated as the coherent-only reflection coefficient. A physically robust argument is presented for the self-consistency of this procedure. The main results derived are (a) elimination of the metal-insulator transition (the unstable fixed point) for an arbitrarily small strength of decoherence [this is indeed expected on physical grounds, in as much as metal-insulator transition with increasing static disorder is essentially due to the coherent-backscattering[2] (where the backscattered amplitudes traversing the time-reversed paths add up in phase), while decoherence suppresses this phase coherent effect], (b) suppression of the four-probe resistance fluctuations with increasing decoherence strength making all the resistance moments finite, and (c) a correction to conductivity due to decoherence in the metallic limit that mimics the conventional phase cutoff length scale.

II. MODEL AND INVARIANT EMBEDDING: ONE-DIMENSIONAL CASE

Consider a model Hamiltonian H for the system of noninteracting electrons in a one-dimensional disordered conductor of length L,

$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x), \tag{2.1}$$

where $V(x), 0<x<L$, is a spatially random potential (quenched disorder) assumed to be delta-correlated Gaussian,

$$\langle V(x)V(x')\rangle = V_0^2\delta(x-x').$$

Let an electron wave of unit amplitude be incident at Fermi energy ($E_F=\hbar^2k_F^2/2m$) on the sample from right, and let $R(L)$ and $T(L)$, respectively, be the reflection and the transmission amplitude coefficients. Next, let the sample of length L be embedded invariantly in a supersample of length $L+\Delta L$ (Fig. 1). It is readily seen that the elastic scattering from the random potential in the interval ΔL with $k_F\Delta L \ll 1$ can be viewed as due to a delta-function potential of strength $V(L)\Delta L$, the corresponding scattering matrix being ΔS_E,

$$\Delta S_E = \begin{pmatrix} \dfrac{2mV\Delta L}{2i\hbar^2k_F} & 1+\dfrac{2mV\Delta L}{2i\hbar^2k_F} \\ 1+\dfrac{2mV\Delta L}{2i\hbar^2k_F} & \dfrac{2mV\Delta L}{2i\hbar^2k_F} \end{pmatrix}. \tag{2.2}$$

This gives an evolution equation for the S matrix in the sample length L. Specifically, we have for the amplitude reflection coefficient,[20–22]

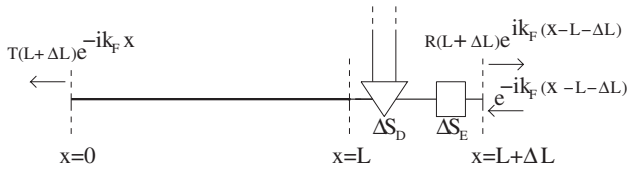

FIG. 1. Disordered sample of length L embedded invariantly in a supersample of length $L+\Delta L$. Also shown are the elementary matrices for the elastic (ΔS_E) and the decohering (ΔS_D) scatterings in ΔL, with the incident, the transmitted, and the reflected waves at Fermi wave vector k_F.

$$\frac{dR}{dL} = i\frac{k_F}{2}\xi(L)[1+R(L)]^2 + 2ik_FR(L), \tag{2.3}$$

with $\xi(L)=-[2mV(L)]/(\hbar^2k_F^2)$ and $\langle\xi(L)\xi(L')\rangle=\Lambda\delta(L-L')$. We are now in a position to introduce decoherence at par with the random elastic scattering within this approach. We recall the 4×4 S matrix with the side channels as introduced by Büttiker,[7]

$$S = \begin{pmatrix} 0 & \sqrt{1-\epsilon} & \sqrt{\epsilon} & 0 \\ \sqrt{1-\epsilon} & 0 & 0 & \sqrt{\epsilon} \\ \sqrt{\epsilon} & 0 & 0 & -\sqrt{1-\epsilon} \\ 0 & \sqrt{\epsilon} & -\sqrt{1-\epsilon} & 0 \end{pmatrix}. \tag{2.4}$$

Here, the outcoupling through the side channels is parametrized by ϵ, which must be of order ΔL in the present case. Accordingly, we use the 2×2 submatrix,

$$\Delta S_D = \begin{pmatrix} 0 & \sqrt{1-\epsilon} \\ \sqrt{1-\epsilon} & 0 \end{pmatrix}, \tag{2.5}$$

for insertion into the interval ΔL. It describes the outcoupling into the side channels, i.e., the stochastic absorption, as also the coherent transmission directly through the interval ΔL. (Its connection with the reservoir-induced decoherence will be clarified below later.) Figure 1 is a schematic depicting the insertion of the elementary ΔS_E and ΔS_D in the interval ΔL. Clearly, for $k_F\Delta L \ll 1$, the exact spatial order and the locations of the two insertions within the interval ΔL are not relevant. Combining these two elementary S matrices (ΔS_E and ΔS_D) for ΔL with the S matrix $[S(L)]$ for the sample of length L in series, we can read off the emergent quantities $R(L)$ and $T(L)$,

$$R(L+\Delta L) = \Delta R + \frac{\Delta T^2 e^{2ik_F\Delta L}R(L)}{1-(\Delta R)R(L)e^{2ik_F\Delta L}}, \tag{2.6}$$

with $\Delta R=(2mV\Delta L)/(2i\hbar^2k_F)$ and $\Delta T^2=1-\epsilon+(2mV\Delta L)/(i\hbar^2k_F)$. In the limit $\Delta L\to0$, we obtain the evolution equations for the amplitude reflection and/or transmission coefficients $R(L)$ and $T(L)$,

$$\frac{dR_c}{dL} = i\frac{k_F}{2}\xi(L)[1+R_c(L)]^2 + 2ik_FR_c(L) - \eta R_c(L), \tag{2.7}$$

$$\frac{dT_c}{dL} = i\frac{k_F}{2}\xi(L)[1+R_c(L)]T_c(L) + ik_F T_c(L) - \frac{\eta}{2}T_c(L), \tag{2.8}$$

where $\eta=\epsilon/\Delta L$, $\Delta L\to 0$ parametrizes decoherence. Here, we have introduced the subscript c just to emphasize that the reflection and/or transmission amplitude coefficients in Eq. (2.7) are coherent.

It seems in order at this stage to clarify how decoherence is realized in relation to the sample resistance by the insertion of the side channel through ΔS_D. Clearly, the embedding Eq. (2.7) describes evolution of the coherent reflection amplitude $R_c(L)$. [Similarly, $T_c(L)$ is the coherent transmission amplitude, as depicted in Fig. 1. The embedding equation for $T_c(L)$, however, is not autonomous—it involves $R_c(L)$.] The outcoupling into the side channels corresponds to a stochastic absorption[25–27] in the interval ΔL. This, however, has to be reinjected now incoherently back into the conductor. In as much as this reinjected current necessarily flows down the chemical potential gradient, it contributes to the total transmitted current equal to (within constant of proportionality) $|T_c(L)|^2+|T_{in}(L)|^2\equiv|T_{tot}(L)|^2$, where the subscript "in" denotes incoherent. From the conservation of the total current flowing down the conductor, we must have $|T_c(L)|^2+|T_{in}(L)|^2=1-|R_c(L)|^2$. Now, recall that the Landauer resistance ($\rho^{(d,D)}$) formula, $\rho^{(d,D)}=(1-|T_{tot}|^2)/|T_{tot}|^2$, holds for arbitrary $|T_{tot}|^2$ (both coherent or incoherent). Here, the superscript (d,D) denotes the dimensionality d and the decoherence parameter D. Thus, we have $\rho^{(d,D)}=|R_c|^2/(1-|R_c|^2)$ given entirely in term of $R_c(L)$ which is calculable from Eq. (2.7). Thus, the four-probe resistance $|R_c(L)|^2/(1-|R_c(L)|^2)$ incorporates self-consistently the incoherent reinjection. Here, we must reemphasize that $|R_c(L)|^2$ is the coherent reflection coefficient given by and calculable from the embedding Eq. (2.7).

Our next step is to obtain the "Fokker-Planck" equation for the probability density of the reflection coefficient $r(L)=|R_c(L)|^2$ from the stochastic differential [Eq. (2.7)] which serves as the Langevin equation here. Following the now familiar procedure,[20–22,28] we obtain

$$\frac{\partial P^{(1)}(r,l)}{\partial l} = \frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}(1-r)^2P^{(1)}(r,l)\right] + D\frac{\partial}{\partial r}[rP^{(1)}(r,l)], \tag{2.9}$$

with $l=L/l_0$, $l_0=2/(\Lambda k_F^2)$, and $D=2\eta l_0$. This is clearly a two-parameter (l_0 and D) evolution equation. The two independent parameters l_0 and D are, of course, composed of the two basic independent parameters: Λk_F^2 (measure of disorder) and η (measure of decoherence). Thus, e.g., D may vary through η, while l_0 can remain constant.

Equation (2.9) in the limit of large length $L\gg l_0$ gives a steady-state distribution $P_\infty(r)$ for the reflection coefficient r,

$$P_\infty^{(1)}(r) = \frac{|D|\exp(|D|)\exp\left(-\dfrac{|D|}{1-r}\right)}{(1-r)^2}, \quad r\le 1. \tag{2.10}$$

Note that for $D=0$, the limiting distribution in Eq. (2.10) tends to the delta function, $\delta(1-r)$, and not to zero. (This can be readily seen by noting that the probability distribution is normalized to unity for all D.) This means that the reflection coefficient becomes unity with probability one, as it must for an infinitely long one-dimensional (1D) disordered wire without decoherence (all states being localized then, a well-known result from Anderson localization in one dimension). The corresponding resistance moments are all finite for $D\neq 0$. In particular, the limiting value of the average four-probe resistance in the presence of decoherence is

$$\rho_\infty^{(1,D)} = \frac{\pi\hbar}{e^2}\left\langle\frac{r}{1-r}\right\rangle = \frac{\pi\hbar}{e^2|D|}. \tag{2.11}$$

With this preparation [Eq. (2.9)] in hand, we now turn to the case of d dimensions.

III. HIGHER-DIMENSIONAL CASE

Changing over to the four-probe resistance $\rho=r/(1-r)$ (measured in the unit of $\pi\hbar/e^2$) as the new variable with the associated probability density $P^{(1)}(\rho,l)$, Eq. (2.9) reduces to

$$\frac{\partial P^{(1)}}{\partial l} = \rho(\rho+1)\frac{\partial^2 P^{(1)}}{\partial\rho^2} + \{(2\rho+1)+D\rho(\rho+1)\}\frac{\partial P^{(1)}}{\partial\rho} + D(2\rho+1)P^{(1)}. \tag{3.1}$$

The corresponding nth resistance moment in one dimension is

$$\rho_n^{(1,D)} = \int_0^\infty P^{(1)}(\rho,l)\rho^n d\rho. \tag{3.2}$$

Multiplying both sides of Eq. (3.1) by ρ^n and integrating by parts on the right-hand side, we get the evolution equation for the one-dimensional moment,

$$\frac{\partial\rho_n^{(1,D)}}{\partial l} = n(n+1)\rho_n^{(1,D)} + n^2\rho_{n-1}^{(1,D)} - Dn\rho_n^{(1,D)} - Dn\rho_{n+1}^{(1,D)}, \tag{3.3}$$

which is hierarchical in nature (i.e., the equation for $\rho_n^{(1,D)}$ involves $\rho_{n-1}^{(1,D)}$ and $\rho_{n+1}^{(1,D)}$). For $D=0$, however, the equation for $\rho_n^{(1)}$ involves the lower-order moments only leading to a closure of the hierarchy. Thus, the presence of decoherence ($D\neq 0$) brings about a qualitative change in the structure of the coupled equations for the moments of different orders. For $D=0$, the solutions of Eq. (3.3) for the first and the second moments are readily obtained as

$$\rho_1^{(1,0)} = \frac{1}{2}(e^{2l}-1),$$

$$\rho_2^{(1,0)} = \frac{2}{3}(2\rho_1^{(1,0)^3} + 3\rho_1^{(1,0)^2}). \tag{3.4}$$

In writing the last equation above, we have eliminated the length l in favor of an implicit relation between $\rho_2^{(1,0)}$ and $\rho_1^{(1,0)}$. We have verified by iteration of Eq. (3.3) that this relation remains valid for $\rho_2^{(1,D)}$ and $\rho_1^{(1,D)}$ in a good approxi-

mation for $D \neq 0$ and will be used as such. Substituting for $\rho_2^{(1,D)}$ in terms of $\rho_1^{(1,D)}$ in Eq. (3.3) for $n=1$, and integrating, we obtain a relation between l and $\rho_1^{(1,D)}$,

$$l = \int_0^{\rho_1^{(1,D)}} \frac{d\tilde{\rho}_1^{(1,D)}}{-\frac{4}{3}D\tilde{\rho}_1^{(1,D)^3} - 2D\tilde{\rho}_1^{(1,D)^2} + (2-D)\tilde{\rho}_1^{(1,D)} + 1}. \tag{3.5}$$

(From now, dummy integration variable will be distinguished by a tilde.) Hereinafter, the superscript D in $\rho_1^{(1,D)}$ will be dropped except when required for the sake of clarity. Defining the associated moment generating function $\chi^{(1)}(x,l)$ and the cumulant generating function $K^{(1)}(x,l)$ of $P^{(1)}(\rho,l)$ as

$$\chi^{(1)}(x,l) \equiv \int_0^\infty e^{-x\rho} P^{(1)}(\rho,l) d\rho,$$

$$K^{(1)}(x,l) \equiv \ln \chi^{(1)}(x,l),$$

we derive from Eq. (3.3) their evolution equations,

$$\frac{\partial \chi^{(1)}}{\partial l} = (x^2 + Dx)\frac{\partial^2 \chi^{(1)}}{\partial x^2} + (2x - Dx - x^2)\frac{\partial \chi^{(1)}}{\partial x} - x\chi^{(1)}, \tag{3.6}$$

$$\frac{\partial K^{(1)}}{\partial l} = (x^2 + Dx)\frac{\partial^2 K^{(1)}}{\partial x^2} + (x^2 + Dx)\left(\frac{\partial K^{(1)}}{\partial x}\right)^2 + (2x - Dx - x^2)\frac{\partial K^{(1)}}{\partial x} - x. \tag{3.7}$$

Now, we proceed to generalize the above equations to the case $d>1$. For this, we closely follow the Migdal-Kadanoff procedure as in Ref. 23, assuming the quenched disorder to evolve along one chosen direction only. This anisotropic disorder is admittedly an approximation, but it is known to reproduce correctly the qualitative features of the Anderson transition in the absence of decoherence, as shown in the earlier works.[23,24] The probability density $P^{(d)}(\rho,l)$ of the resistance of a d-dimensional hypercubic sample is accordingly found to obey the integrodifferential evolution equations,

$$\frac{\partial \chi^{(d)}}{\partial \ln l} = -(d-1)x\frac{\partial \chi^{(d)}}{\partial x} + \left[(x^2 + Dx)\frac{\partial^2 \chi^{(d)}}{\partial x^2} + (2x - Dx - x^2)\frac{\partial \chi^{(d)}}{\partial x} - x\chi^{(d)}\right] l, \tag{3.8}$$

$$\frac{\partial K^{(d)}}{\partial \ln l} = -(d-1)x\frac{\partial K^{(d)}}{\partial x} + \left[(x^2 + Dx)\frac{\partial^2 K^{(d)}}{\partial x^2} + (x^2 + Dx)\left(\frac{\partial K^{(d)}}{\partial x}\right)^2 + (2x - Dx - x^2)\frac{\partial K^{(d)}}{\partial x} - x\right] l, \tag{3.9}$$

where l in the above equations is given by the integral in Eq. (3.5), but with $\rho_1^{(1)}$ in the integrand now reinterpreted as $\rho_1^{(d)}$. Clearly, in the limit $D=0$, the above equations for the generating functions reduce to the corresponding Eqs. (6) and (7) of Ref. 23. In particular, the fixed point probability distribution for $d=3$ obtained by setting $\partial \chi^{(d)}/\partial \ln l = 0$ and inverting the Laplace transform of the solution for $\chi^{(d)}$ is nothing but the known fixed point power-law distribution.[24]

In the presence of decoherence ($D \neq 0$), however, there is no fixed point even for arbitrarily small values of D for $d=3$. In order to see this, consider the evolution equation for the first cumulant $K_1^{(d)}(\equiv \rho_1^{(d)})$ obtained from the cumulant generating Eq. (3.9),

$$\frac{\partial K_1^{(d)}}{\partial \ln l} = -(d-1)K_1^{(d)} + [1 + 2K_1^{(d)} - DK_1^{(d)} - DK_1^{(d)^2} - DK_2^{(d)}] \times \int_0^{\rho_1^{(d)}} \frac{d\tilde{\rho}_1^{(d)}}{-\frac{4}{3}D\tilde{\rho}_1^{(d)^3} - 2D\tilde{\rho}_1^{(d)^2} + (2-D)\tilde{\rho}_1^{(d)} + 1}, \tag{3.10}$$

where we have replaced the length l in terms of $\rho_1^{(d)}$ as explained above. Carrying out the integration occurring in Eq. (3.10) numerically (using MATHEMATICA), we found no solution with $\partial K_1^{(d)}/\partial \ln l = 0$ for any nonzero value of D, however, small (down to $D \sim 10^{-6}$), confirming that there is no fixed point. This should, of course, be physically so in as much as the decoherence is expected to suppress quantum interference effects (and localization), in the limit of large sample size. For $D \neq 0$, however, we do expect the probability density to vary slowly in the vicinity of the $D=0$ fixed point, now become a crossover. Indeed, setting $\partial \chi^{(d)}/\partial \ln l \simeq 0$ for small nonzero D, we obtain for the quasi-fixed-point probability density of resistance,

$$P(\rho_1^{(d)}) = \frac{D^{1-\alpha} e^{-D(1+\rho_1^{(d)})}(1+\rho_1^{(d)})^{-\alpha}}{\Gamma(1-\alpha, D)}, \tag{3.11}$$

where $\Gamma(1-\alpha, D) \equiv \int_D^\infty e^{-u} u^{-\alpha} du$ and $\alpha = (d-1)/(l|_{\rho_1^{(d)}})$ Here, $\rho_1^{(d)}$ ($\simeq \rho_1^{(d)*} = 1.96$ for $d=3$) is the average resistance corresponding to the quasi-fixed-point probability density and $l|_{\rho_1^{(d)}}$ is the value of the integral [Eq. (3.5)] with the upper limit $\rho_1^{(d)}$. It is clear from Eq. (3.11) that a nonzero value of D (decoherence) makes all the resistance moments finite, that is, it cuts off the otherwise divergent resistance fluctuations. In the absence of decoherence ($D=0$), Eq. (3.11) gives a power-law probability distribution for resistance at the mobility edge as in Ref. 24. It is to be noted, however, that all numerical work on tight-binding Anderson model shows that the distribution of conductance in three dimensional at the mobility edge (the fixed point) is far from a power law.[29,30] We think that this could be for two reasons: first, the neglect of the transverse fluctuations in our anisotropic Migdal-Kadanoff procedure and second, as the numerical results are all for the ensemble averaged two-probe conductance ($\mathrm{Tr}\, tt^\dagger$) (where t is the transmission matrix), while we have calculated the ensemble averaged four-probe resistance. It is to be noted here that while the four-probe resistance is unbounded from above and can, therefore, have large fluctuations, the two-probe conductance is by definition bounded from above and can fluctuate relatively much less. Thus, e.g., in the 1D

case, Tr $tt^{\dagger} \leqslant 1$; but, of course, there is no fixed point in the 1D case. We would like to point out here that the invariant-embedding equation is, of course, known for d dimensional as also for the quasi-one-dimnsional case [see Ref. 22, Eq. (2.28)], but an analytical solution is lacking, making any comparison with the available results for the quasi-one-dimensional, $D=0$ conductance distribution[31,32] is impossible. Our immediate interest, however, lies in the fact that decoherence cuts off the resistance fluctuations exponentially.

Finally, we consider the asymptotic behavior of the resistance in three dimensions in the presence of decoherence in the metallic regime as the sample size tends to infinity. In three dimensions with $D \neq 0$, we expect the resistance to tend to a small value in the mean along with a narrow width (the variance) of the distribution. This motivates us to approximate the evolution [Eq. (3.10)] for the first moment as

$$\frac{\partial \rho_1^{(d)}}{\partial \ln l} = -(d-1)\rho_1^{(d)} + [1+(2-D)\rho_1^{(d)}] \int_0^{\rho_1^{(d)}} \frac{d\tilde{\rho}_1^{(d)}}{1+(2-D)\tilde{\rho}_1^{(d)}}. \tag{3.12}$$

Now, consider first the three-dimensional case ($d=3$) in the metallic regime starting with the resistance $\rho_0 = \rho_1^{(3)}(l_0)$ at a length scale l_0. Let this evolve through Eq. (3.12) to a length scale $l \gg l_0$ with $\rho_1^{(3)}(l) \equiv \rho \ll \rho_0$. Equation (3.12) then gives

$$\int_{\rho_0}^{\rho_1^{(3)}} \frac{d\tilde{\rho}_1^{(3)}}{-\tilde{\rho}_1^{(3)} + \frac{2-D}{2}\tilde{\rho}_1^{(3)2}} = \ln\left(\frac{l}{l_0}\right), \tag{3.13}$$

or in terms of the conductivity, $\sigma^{(3)}(l) \equiv g/l$, $g \equiv 1/\rho$, and $g_0 \equiv 1/\rho_0$, we have

$$\sigma^{(3)}(l) = \frac{g_0 - 1}{l_0} + \frac{1}{l} + \frac{D}{2}\left(\frac{1}{l_0} - \frac{1}{l}\right). \tag{3.14}$$

Equation (3.14) clearly shows that increasing decoherence (D) increases the metallic conductivity in three dimensions. Indeed, one can rewrite the correction $D/2l_0$ as $1/L_\phi$, with L_ϕ a phase-cut-off (dephasing) length scale as usual. Proceeding in similar way, we get for the two-dimensional case a logarithmic correction to the conductivity $\sigma^{(2)}(l)$ (noting that in two dimensions conductivity is the same as conductance),

$$\sigma^{(2)}(l) = \sigma_0 + \frac{D-2}{2}\ln\left(\frac{l}{l_0}\right), \tag{3.15}$$

where σ_0 is the conductivity (or the conductance) at the starting length scale l_0. Again, the conductivity $\sigma^{(2)}(l)$ is seen to increase with increasing decoherence D. This is qualitatively consistent with the negative temperature coefficient of resistance observed in disordered conductors at low temperatures in the weak localization regime, in particular, for two-dimensional systems.[2]

IV. DISCUSSION

We have extended the phenomenology of decoherence well known in the context of phase-sensitive systems, such as mesoscopic rings and one-dimensional quantum wires, to higher dimensions—specifically to a d-dimensional disordered conductor for $d=2$ and 3. Our treatment here follows the invariant-embedding approach developed earlier,[20–22] beginning with the 1D case. It treats decoherence and disorder formally at par in that the two are introduced through appropriately chosen and parametrized scattering matrices distributed over the conductor. The invariant-embedding approach is naturally suited to the problem on hand as it gives the evolution in length of the resultant emergent quantities such as the reflection coefficient related directly to the Landauer four-probe resistance of interest. Decoherence is realized specifically through stochastic absorption of the wave amplitude into distributed side (transverse) channels and the subsequent reinjection of the absorbed fraction back into the conductor so as to add incoherently to the (longitudinal) coherent transport. This is essentially in the spirit of Büttiker's reservoir-induced decoherence. A point to note here is that the current-conserving reinjection is realized here self-consistently through the use of the four-probe resistance which now needs to be calculated with the coherent-only reflection coefficient. Extension to higher dimensions has been carried out within the Migdal-Kadanoff procedure assuming the disorder to evolve only along an arbitrarily chosen direction for the current. This choice of anisotropic disorder is admittedly an approximation, but its innocuous nature is borne out *a posteriori* by the fact that this approximation had correctly given the unstable fixed point for the disorder induced Anderson (metal-insulator) transition for $d=3$ in the absence of decoherence. Its reasonableness may be attributed to the transverse mixing up of disorder by the evolution equation. Physically, moreover, the classicalization expected from decoherence should make the approximation even better. A nontrivial result of our work is the elimination of the unstable (Anderson) fixed point due to decoherence. Again, it is expected on physical grounds that the fixed point should get replaced by a crossover for $D \neq 0$, so is the finiteness of all moments, which is the suppression of resistance fluctuations due to decoherence, as is evident from our Eq. (3.11). A point to note is the decoherence correction to the quantum conductivity for $d=3$, where a cutoff length (dephasing length) appears naturally. Finally, we would like to point out here that the decoherence, through stochastic absorption into the transverse channels and the reinjection, does not cause scattering in the coherent longitudinal (transport) channel in the sense of momentum randomization that would have given additional resistance. Indeed, as is clear from our Eq. (2.7), in the absence of scattering by disorder, the reflection amplitude (R) remains identically zero for all lengths, independent of the value of η (that parametrizes decoherence). This is also obvious from Eq. (2.5). We would aptly like to call this a *pure* decoherence without any concomitant elastic scattering.

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