Nonlocal Pancharatnam phase in two-photon interferometry

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We propose a polarized intensity interferometry experiment, which measures the nonlocal Pancharatnam phase acquired by a pair of Hanbury-Brown–Twiss photons. The setup involves two polarized thermal sources illuminating two polarized detectors. Varying the relative polarization angle of the detectors introduces a two-photon geometric phase. Local measurements at either detector do not reveal the effects of the phase, which is an optical analog of the multiparticle Aharonov-Bohm effect. The geometric phase sheds light on the three-slit experiment and suggests ways of tuning entanglement.

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The familiar two-slit experiment in quantum mechanics describes the interference of a single particle with itself. However, there are also quantum processes that describe the interference of a pair of particles with itself. As shown by Hanbury-Brown and Twiss (HBT) [1] about 50 years ago, such interference is observed in the coincidence counts of photons. Their original motivation was to measure the diameters of stars, replacing Michelson interferometry by intensity interferometry. Their work was initially met with skepticism because the quantum mechanical interpretation of the proposed experiment was unclear at the time. The resulting controversy led to the birth of the new field of quantum optics. Intensity interferometry is now routinely used in a variety of fields, from nuclear physics [2] to condensed matter [3].

In the 1980s, Berry discovered [4] the geometric phase in quantum mechanics, which has now been applied and studied in various contexts [5]. It was soon realized that Berry's discovery had been anticipated by Pancharatnam's work [6] on the interference of polarized light, Pancharatnam's work is now widely recognized as an early precursor of the geometric phase [7], with a perspective that is far more general [8] than the context in which it was discovered by Berry.

Büttiker [9] noted in the context of electronic charge transport that two-particle correlations can be sensitive to a magnetic flux even if the single-particle observables are flux insensitive. The effect of the flux is visible only in current cross correlations and is a genuinely nonlocal and multiparticle Aharonov-Bohm effect [10]. This effect has been experimentally seen in intensity interferometry experiments carried out using edge currents in quantum Hall systems [3]; the theory was further developed in [11,12] and the possibility of controlled orbital entanglement and the connection to Bell inequalities mentioned.

In this paper, we propose an experiment with polarized light, which shows geometric phase effects *only in the intensity correlations* $\mathcal{G}^2$ and not in the lower-order correlations $\mathcal{G}^1$. The two-photon Pancharatnam phase effect is also nonlocal in the precise sense that it cannot be seen by local measurements at either detector. Coincidence detection of photons in two detectors yields counts which are modulated by a phase that has a geometric component as well as the expected dynamical (or propagation) phase. Unlike in earlier studies [13,14], the effects of the geometric phase are seen *only* in the *cross* correlation counts of two detectors. Neither the count rate nor self-correlation of the individual detectors shows any such geometric phase effects. The phase is given by half the solid angle enclosed on the Poincaré sphere by the total circuit of a *pair* of HBT photons and as expected, is *achromatic*.

The experimental setup is described below and then a theoretical analysis is given. Finally we conclude with a discussion and a comparison with previous work.

The experiment consists of having two thermal sources S_1 and S_2 illuminate two detectors D_3 and D_4 (Fig. 1). This setup is very similar to the HBT experiment [1]. The only difference is in the use of analyzers (shown in red online), which select a particular state of polarization. The source S_1 is covered by an analyzer P_R, which permits only right-hand circular light to pass through it, while the source S_2 is covered by an analyzer P_L, which permits only left-hand circular light to pass through. The light is incident on detectors D_3 and D_4 after passing through polaroids P_3 and P_4 , respectively, that permit only linearly polarized light to pass through (linear analyzers). The angle φ_{34} between the axes of P_3 and P_4 and the detector separation d_D can be continuously varied in the experiment. The measured quantity is the coincidence count $\mathcal{C}$ of photons received at detectors D_3 and D_4,

$$\mathcal{C} = \mathcal{G}^2 = \frac{\langle N_3 N_4\rangle}{\langle N_3\rangle\langle N_4\rangle}, \tag{1}$$

where N_3 and N_4 are the photon numbers detected at D_3 and D_4 per unit time (per unit bandwidth). As in the HBT interferometer, we would expect the coincidence counts to vary with the propagation phases and so the counts would depend on the detector separation d_D and the wavelength λ of the light. The additional effect that is present in the polarized version is that we expect the coincidence counts to also depend on φ_{34} and to be modulated by a geometric phase of half the solid angle on the Poincaré sphere shown in Fig. 2.

The geometric phase is achromatic, unlike the propagation phases mentioned above. Note that the path traversed on the Poincaré sphere is not traced by a *single* photon, but by a *pair* of HBT photons. Thus the experiment explores an another

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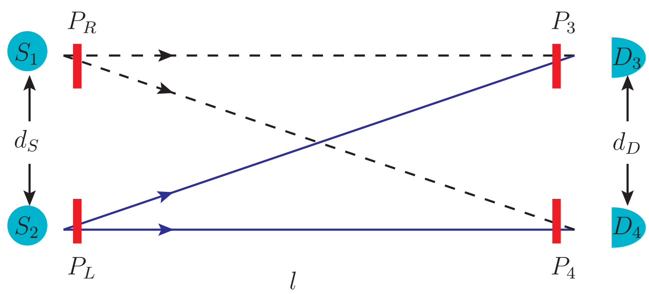

FIG. 1. (Color online) Schematic diagram of the proposed experiment: S_1 and S_2 are thermal sources, covered by circular analyzers which pass only right and left circular light, respectively. The two detectors D_3 and D_4 receive only linear polarizations. The angle φ_{34} between the axes of the linear polarizers can be continuously varied. The dashed and solid lines represent photons from the two sources S_1 and S_2, respectively. The separation between the detectors is d_D and that between the sources is d_S.

avatar of the geometric phase in the context of intensity interferometry.

We now calculate the expected coincidence counts for the detectors D_3 and D_4 and show that these counts depend on the geometric phase. For ease of calculation we suppose that we are dealing with a single-frequency, i.e., a quasimonochromatic beam. In fact the detectors will have a finite acceptance bandwidth, which has to be incorporated in a more realistic calculation. The principle of the effect comes across better in the present idealized situation.

We write a_1^α and a_2^α for the destruction operators of the photon modes at the sources S_1 and S_2 where α runs over the two polarization states. The modes just after the analyzers P_R and P_L are represented by projections $a_R^\alpha = P_R^{\alpha\beta}\, a_1^\beta$ and $a_L^\alpha = P_L^{\alpha\beta}\, a_2^\beta$ where a sum over repeated Greek indices is understood and the projection matrices P_R and P_L onto the right and left circular states represent the action of the analyzers. The modes at the detectors are characterized by the destruction operators a_3^α and a_4^α. We suppose that the separation l between the sources and the detectors is much larger than the separation d_S between the sources and the separation d_D between the detectors i.e., $l \gg d_S, d_D$. When light is emitted by a source and received by a detector, it suffers propagation phases and decrease of its amplitude inversely with distance. These effects are captured in the functions $u_{ij} = \frac{1}{l}\exp\{i[k(|\vec{r}_i - \vec{r}_j|) - \omega t]\}$, where ω is the frequency of the light, k is the wave vector, and $\vec{r}_i$ and $\vec{r}_j$ the locations of the detector and source. With this notation, we express a_b^α (where $b = 3,4$) as $a_b^\alpha = P_b^{\alpha\beta}[\, P_L^{\beta\gamma}\, a_2^\gamma\, u_{b2} + P_R^{\beta\gamma}\, a_1^\gamma\, u_{b1}\,]$ and its Hermitian conjugate $a_b^{\dagger\alpha}$ as $a_b^{\dagger\alpha} = [\, \bar{u}_{b2}\, a_2^{\dagger\gamma}\, P_L^{\gamma\beta} + \bar{u}_{b1}\, a_1^{\dagger\gamma}\, P_R^{\gamma\beta}\,]P_b^{\beta\alpha}$ where the overbar stands for complex conjugation and we use the fact that the 2×2 Hermitian projection matrices P satisfy $P^2 = P$ and $\bar{P}^{\alpha\beta} = P^{\beta\alpha}$.

The quantities of interest[1] are $\langle N_3\rangle,\langle N_4\rangle$, being the photon counts per unit time (per unit bandwidth) at the two detectors (D_3 and D_4) and $\langle : N_3 N_4 : \rangle$, the coincidence counts, where the $::$ stands for normal ordering which has to be applied to the number operator product in the numerator of Eq. (1). N_b is given by $N_b = a_b^{\dagger\alpha} a_b^\alpha = [\bar{u}_{b2}\, a_2^{\dagger\alpha}\,(P_L P_b)^{\alpha\beta} + \bar{u}_{b1}\, a_1^{\dagger\alpha}\,(P_R P_b)^{\alpha\beta}](P_L^{\beta\gamma}\, a_2^\gamma u_{b2} + P_R^{\beta\gamma}\, a_1^\gamma u_{b1})$. We find

$$\langle N_b \rangle = \bar{u}_{b1}\, u_{b1}\, (P_R P_b P_R)^{\alpha\beta} \left\langle a_1^{\dagger\alpha} a_1^\beta \right\rangle + \bar{u}_{b2}\, u_{b2}\, (P_L P_b P_L)^{\alpha\beta} \left\langle a_2^{\dagger\alpha} a_2^\beta \right\rangle. \tag{2}$$

From the thermal nature of the sources, $\langle a_1^{\dagger\alpha} a_1^\beta \rangle = \langle a_2^{\dagger\alpha} a_2^\beta \rangle = \delta^{\alpha\beta}\, n_B$ where n_B is the Bose function $[\exp(\beta\hbar\omega) - 1]^{-1}$ and β the inverse temperature. And we arrive at $\langle N_3\rangle = \langle N_4\rangle = n_B/l^2$. The computation of $\langle : N_3 N_4 : \rangle$ is slightly more involved but straightforward. The product $N_3 N_4$ is a product of four brackets each of which has two terms. When the four brackets are expanded, there are sixteen terms, of which ten vanish. The six nonzero terms combine to give

$$\langle : N_3 N_4 : \rangle = n_B^2 \left(\frac{3}{2\, l^4} + \bar{u}_{32}\, u_{31}\, \bar{u}_{41}\, u_{42}\, \mathrm{Tr}(P_L P_3 P_R P_4 P_L) + \bar{u}_{31}\, u_{32}\, \bar{u}_{42}\, u_{41}\, \mathrm{Tr}\,(P_R P_3 P_L P_4 P_R) \right). \tag{3}$$

Only the second and third terms in Eq. (3) contain the propagation and geometric phases. The sequence of projections can be viewed as a series of closed-loop quantum collapses [7,8] given by $\langle R|3\rangle\langle 3|L\rangle\langle L|4\rangle\langle 4|R\rangle$,

$$\mathrm{Tr}\,(P_R P_3 P_L P_4 P_R) = \frac{1}{4}\exp\left(i\frac{\Omega}{2}\right), \tag{4}$$

where Ω is the solid angle subtended by the geodesic path $|R\rangle \to |3\rangle \to |L\rangle \to |4\rangle \to |R\rangle$ at the center of the Poincaré sphere. Apart from the phase, the projections also result in an amplitude factor of 1/4 [6] since projections are nonunitary operations leading to a loss in intensity. The final theoretical expression for $\mathcal{C}$ in the limit $l \gg d_S, d_D$ is

$$\mathcal{C} = \frac{3}{2} + \frac{1}{2}\cos\left(\vec{d}_D \cdot (\vec{k}_2 - \vec{k}_1) + \frac{\Omega}{2}\right), \tag{5}$$

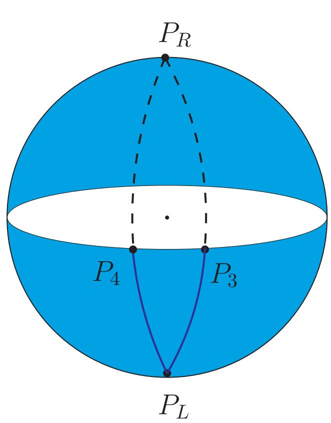

FIG. 2. (Color online) The path on the Poincaré sphere that determines the geometric phase. The angle φ_{34} between the linear polaroids determines the width of the lune on the Poincaré sphere and the geometric phase.

[1] For any general operator $\hat{O}$, $\langle\hat{O}\rangle = \mathrm{Tr}(\varrho\,\hat{O})$ where ϱ is the normalized thermal density matrix $\varrho = \exp(-\beta H)/Z$ with $Z = \mathrm{Tr}[\exp(-\beta H)]$ and $H = (a^\dagger a + 1/2)\omega$.

where $\vec{k}_i = k\hat{r}_i$ is the wave vector of light seen in the ith detector. [The propagation phases in Eq. (5) can also be written in an equivalent form with the sources and detectors exchanged.] It is also easily seen that the self-correlation $\langle : N_3 N_3 : \rangle$ ($\langle : N_4 N_4 : \rangle$) can be obtained by replacing 4 by 3 (3 by 4) in Eq. (3) above. In this case, the sequence of projections $\mathrm{Tr}\,(P_R P_3 P_L P_3 P_R)$ [$\mathrm{Tr}\,(P_R P_4 P_L P_4 P_R)$] subtends a zero solid angle and the geometric contribution to the phase vanishes. Thus neither the photon counts $\langle N_3 \rangle$,$\langle N_4 \rangle$ in individual detectors nor the self-correlations $\langle : N_3 N_3 : \rangle$, $\langle : N_4 N_4 : \rangle$ reveal the geometric phase. This supports our claim that the effect described here is present only in the cross correlations and not in the self-correlations.

$\mathcal{C}$ depends on the experimentally tunable parameters d_D and φ_{34}. The geometric part is achromatic and depends only on φ_{34}. The propagation part in the phase carries the dependence on d_D as well as on the wavelength. By changing the angle φ_{34} between the axes of the two polaroids, we can conveniently modulate the geometric component Ω. If the propagation and geometric phases are set to zero, we find that the correlation $\mathcal{C}$ takes the value 2, just as in the original HBT interferometry.

We have proposed a simple generalization of the HBT experiment which uses the vector nature of light to produce a geometric phase. The only difference between the proposed experiment and the HBT experiment is the presence of polarizers at the sources and detectors. These polarizers cause a geometric phase to appear in the coincidence counts of two detectors which receive linearly polarized light. Neither the count rates nor the self-correlations of individual detectors show any geometric phase effects. These appear solely in the *cross* correlations in the count rates of the detectors. The appearance of the geometric phase cannot be attributed or localized to any single segment joining a source (S_1, S_2) to a detector (D_3, D_4). It appears only when one considers the *two-photon path* (Fig. 2) on the Poincaré sphere in its entirety. Our experiment brings out a result of a conceptual nature, which may not have been guessed without our present understanding of the Pancharatnam phase. The experiment proposed here would be a good demonstration of a *purely multiparticle and nonlocal* geometric phase in optics. We hope to interest experimentalists in this endeavor. Apart from verifying the theoretical expectation, our proposed experiment suggests further lines of thought concerning multiparticle and nonlocal effects which may be stimulating to research in this area. We mention two of these, the first an application of our ideas to generating controlled entanglement and the second of a more conceptual nature regarding the role of probabilities in quantum mechanics.

The experimental setup described above can be used to make a source of photon pairs with a controlled degree of entanglement. Like many other elementary particles, the photon has spin (polarization) as well as orbital (spacetime) degrees of freedom. Our idea is to use the polarization degree of freedom to control the orbital entanglement of photons. Let us replace the two thermal sources of Fig. 1 by a single two-photon source producing a pair of oppositely circularly polarized photons. Each photon is then passed through an interferometric delay line which consists of a short and a long arm with time delays t_S and t_L. The relative amplitudes and phases of the two paths can be chosen to generate any state in the two-dimensional Hilbert space spanned by $|S\rangle$ and $|L\rangle$. By such means we can arrange for the incident state at P_R to be in a spin state of right circular polarization and in an orbital state $|\phi\rangle_1 = \alpha|S\rangle_1 + \beta|L\rangle_1$ and, similarly, the incident state at P_L to be in a spin state of left circular polarization and in an orbital state $|\psi\rangle_2 = \alpha'|S\rangle_2 + \beta'|L\rangle_2$, where α, β, etc., are complex numbers. The input state is therefore a direct product of states at P_R and P_L: $|\phi\rangle_1 \otimes |\psi\rangle_2$. By combining the amplitudes for the two photons to arrive at the detectors via the paths 1–3,2–4 and 1–4,2–3 (direct and exchange) we find that the state at the output is of the form $|\phi\rangle_3 \otimes |\psi\rangle_4 + \exp(i\Omega/2)|\psi\rangle_3 \otimes |\phi\rangle_4$, where the geometric phase factor $\exp(i\Omega/2)$ is the relative phase between the direct and exchange processes. This final two-photon state is entangled as it cannot in general be written as a direct product $|\Psi\rangle_3 \otimes |\Phi\rangle_4$ of photon states at 3 and 4. The entanglement is generated by particle exchange effects rather than interactions. The degree of entanglement can be tuned using the polaroid setting φ_{34}. The degree of entanglement can be quantified either using Bell's inequality or by the von Neumann entropy of the reduced density matrix after tracing over one of the subsystems (3 or 4). A straightforward calculation of the von Neumann entropy shows that it does depend on the geometric phase. Since the geometric phase is achromatic, we can apply the same phase over all the frequencies in the band of interest by tuning φ_{34} and generate entangled photon pairs with a degree of precision and control. This setup can be used as a source of entangled photon pairs for other experiments probing quantum entanglement.

Quantum mechanics is often introduced by a discussion of the two-slit experiment in which an electron is incident on an opaque barrier with two slits and then detected when it falls on a screen. The surprise of the quantum theory is that the outcome of the two-slit experiment is not determined by the outcome of one-slit experiments in which one or the other of the slits is blocked. This is in sharp contrast to classical random processes like Brownian motion. If one considers the passage of a Brownian particle through slits A and B, one finds that[2]

$$\mathcal{P}_{AB} = \mathcal{P}_A + \mathcal{P}_B,$$

where $\mathcal{P}_{AB}$ is the probability of detecting the particle when both slits are open and $\mathcal{P}_A$ and $\mathcal{P}_B$ are the corresponding detection probabilities in one-slit experiments. Thus classical probabilities are one-slit separable, but quantum probabilities are not: the equality above is not satisfied in the two-slit quantum experiment.

However, if one considers three slits A,B,C, one finds that, in quantum mechanics, the outcome of the three-slit experiment *is* determined by the outcomes of the one- and two-slit experiments. Mathematically,

$$\mathcal{P}_{ABC} = \mathcal{P}_{AB} + \mathcal{P}_{BC} + \mathcal{P}_{CA} - \mathcal{P}_A - \mathcal{P}_B - \mathcal{P}_C,$$

which follows easily from writing $\mathcal{P}_{ABC} = |\psi_A + \psi_B + \psi_C|^2$ where ψ_A, ψ_B, ψ_C are the amplitudes for passage through the slits. Thus quantum mechanics is two-slit separable [15]. This

[2]Throughout this discussion we neglect single-particle trajectories that recross the barrier and wind around multiple slits.

is why we do not find a discussion of the three-slit experiment in elementary quantum mechanics books: it brings in nothing new.

The situation changes when one considers multiparticle and nonlocal processes of the kind exemplified by our experiment of Fig. 1. Consider a three-slit experiment in which three incoherent beams of light fall upon three slits A,B,C which are covered by analyzers P_A,P_B,P_C each of which allows a single state on the Poincaré sphere to pass. The light from the analyzers is then allowed to fall on three unpolarized detectors labeled 4,5,6. By considerations similar to our analysis of the experiment of Fig. 1, we find that the number correlations $\langle N_4 N_5 N_6 \rangle$ contain terms involving the geometric phase (half the solid angle subtended by the three polarization states A,B,C of the analyzers). Such an effect is not present in any of the two-slit or one-slit experiments, since two (or fewer) points on the Poincaré sphere do not enclose a solid angle. The effect is a genuinely three-slit effect, not decomposable in terms of two- and one-slit effects. Thus quantum theory contains effects which are not two-slit separable because of multiparticle entanglement. Our three-slit experiment involving the geometric phase brings out this point forcefully.

The question of whether a single particle crossing a barrier with slits obeys two-slit separability is ultimately an experimental one. The theoretical possibility of violations of two-slit separability in such experiments was noted by Sorkin [15], who proposed that there may be theories going beyond quantum mechanics which admit such effects. There have been attempts [16] to search for such effects in a three-slit experiment using photons. Since these experiments are null experiments, one has to be careful to rule out all possible three-slit effects that are present due to multiparticle entanglement. Geometric phase effects which involve three photons are an example of such three-slit effects. The experiment we propose here in Fig. 1 is just the simplest of a class of phenomena involving multiparticle entanglement, nonlocality, and the geometric phase. We hope to interest the quantum optics community in pursuing these ideas further.

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[1] R. Hanbury Brown and R. Q. Twiss, Nature (London) **177**, 27 (1956).
[2] G. Baym, Acta Phys. Pol. B **29**, 1839 (1998).
[3] I. Neder, N. Ofek, Y. Chung, M. Heiblum, D. Mahalu, and V. Umansky, Nature (London) **448**, 333 (2007).
[4] M. V. Berry, Proc. R. Soc. London, Ser. A **392**, 45 (1984).
[5] A. Shapere and F. Wilczek, *Geometric Phases in Physics* (World Scientific, Singapore, 1989).
[6] S. Pancharatnam, Proc. Ind. Acad. Sci. A **44**, 247 (1956).
[7] S. Ramaseshan and R. Nityananda, Curr. Sci. **55**, 1225 (1986).
[8] J. Samuel and R. Bhandari, Phys. Rev. Lett. **60**, 2339 (1988).
[9] M. Büttiker, Phys. Rev. Lett. **68**, 843 (1992).
[10] Y. Aharonov and D. Bohm, Phys. Rev. **115**, 485 (1959).
[11] P. Samuelsson, E. V. Sukhorukov, and M. Büttiker, Phys. Rev. Lett. **92**, 026805 (2004).
[12] J. Splettstoesser, M. Moskalets, and M. Büttiker, Phys. Rev. Lett. **103**, 076804 (2009).
[13] J. Brendel, W. Dultz, and W. Martienssen, Phys. Rev. A **52**, 2551 (1995).
[14] P. Hariharan, *Optical Interferometry* (Elsevier Science, New York, 2003).
[15] R. D. Sorkin, Mod. Phys. Lett. A **9**, 3119 (1994).
[16] U. Sinha, C. Couteau, T. Jennewein, R. Laflamme, and G. Weihs, Science **329**, 418 (2010).