

Prospect of determining the Dirac/Majorana state of the neutrino by a Multi-OWL experiment

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Abstract

We consider the non-radiative two-body decay of a neutrino to a daughter neutrino with degraded energy and a very light particle (Majoron). Ultrahigh-energy neutrinos from an astrophysical source like a gamma ray burst undergoing this decay process are found to produce different number of events in the detector depending on whether they are Majorana or Dirac particles. The next generation large-scale experiments like Multi-OWL is expected to provide us an accurate determination of the flux of neutrinos from astrophysical sources and this may enable us to distinguish between the Dirac and Majorana nature of neutrino.

1. Introduction

The recent observations from Super-Kamiokande [1], KamLAND [2] and Sudbury Neutrino Observatory (SNO) [3] have established mixing between different neutrino flavours. There are, however, several questions which remain unanswered. The conventional model leads to three flavour states arising out of three mass eigenstates with $\Delta m_{12}^2 \approx 7 \times 10^{-5} \text{ eV}^2$ and $\Delta m_{23}^2 \approx 3 \times 10^{-3} \text{ eV}^2$. The Majorana/Dirac nature of the neutrino and the stability of all the states are still open questions. The Majorana/Dirac nature of neutrinos can be resolved unambiguously if neutrinoless double beta decay is observed. This difficult experiment is being carried out at various laboratories. Other alternative methods to establish the Dirac/Majorana nature of neutrinos would be desirable and this work is an attempt in that direction.

We have been motivated by the recent work of Beacom *et al* [4, 5], who have considered the possibility that some of the neutrinos could be unstable. Depending upon the decay pattern they have noted that high-energy neutrinos coming from astrophysical sources can have different flavour ratios. Such a decay of a 'heavy neutrino' to a 'light neutrino' could occur, say, by an emission of a spinless particle. For example, if the neutrino mass pattern is given by normal hierarchy and if only the lightest neutrino is stable then the flavour ratio of the astrophysical neutrinos reaching the Earth would be $\nu_e:\nu_\mu:\nu_\tau = 6:1:1$. An experiment like

IceCube [6], which could distinguish different neutrino flavours, may provide observational evidence for the different decay scenarios.

One has to consider detecting very high-energy ($>10^{15}$ eV) neutrinos to ensure the background from atmospheric neutrinos are negligible. Astrophysical sources which could emit high-energy neutrinos are active galactic nuclei (AGN) and gamma ray bursts (GRBs). Neutrino fluxes from such sources have been studied considering different theoretical models, and are expected to produce large number of events in experiments being planned, e.g., the Antarctic Impulsive Transient Antenna (ANITA) [7] and the Orbiting Wide-Angle Light-Collector (OWL) [8]. In this work, we explore the possibility of distinguishing the Dirac/Majorana nature of the neutrinos from these observations. The basic idea is this: if a ‘heavy neutrino’ decays to a ‘light neutrino’ with a spin flip then the light neutrino will be sterile if the neutrino is a Dirac particle. However, if the daughter neutrino is a Majorana particle, this would still be an active neutrino and would produce a signal in the detector. We consider the neutrinos from a single gamma ray burst. The nature of gamma ray bursts, the emitted photon spectrum in the keV–MeV energy range and the subsequent emission of afterglow have been subjects of intense study in the last few years. Based on a ‘fireball’ model of GRB, the spectrum of high-energy neutrinos from all GRBs has been obtained in [9]. The expected neutrino spectrum from a single GRB in the same model has been calculated in [10, 11].

We show that the number of active neutrino events would be different in the detector depending on whether the neutrinos are Dirac or Majorana particles. In particular, we have looked at upward-going tau neutrinos having energies above 3×10^{15} eV taking account of their regeneration as they traverse the Earth and look for signals of Dirac versus Majorana nature. The OWL [8] experiment will use the Earth’s atmosphere as a detector. Multi-OWL [12] would cover the Earth’s horizon with a field of view of 120° as it would view from an orbit of about 1000 km. The area on the Earth’s surface, which would be within the field of view of Multi-OWL, would be used as the effective target area for the upward-going neutrinos. Further, this experiment is suited for detecting tau neutrinos through observation of ultraviolet (UV) fluorescent light emitted by secondary charged particles. For energies above 10^{15} eV the main contribution would be from ν_τ for the upward vertical showers as ν_e and ν_μ would be absorbed in matter. The prospects of neutrino detection above energy 10^{15} eV by the OWL experiment have been discussed in [12].

In order to obtain results which do not depend sensitively on the model-dependent parameters of the GRB, we have attempted to consider ratios of events belonging to different energy ranges. We discuss the experimental accuracy needed to understand whether a neutrino is a Dirac or Majorana particle based on the event rates calculated for the Multi-OWL experiment.

2. The flux of neutrinos from a single gamma ray burst

We consider a typical gamma ray burst (GRB), whose parameters are inferred from observations, as our source of ultrahigh-energy (UHE) neutrinos and calculate the number of tau neutrino events from it in Multi-OWL. There are discussions on the parameters of a GRB and the mechanism of neutrino production in GRBs in the fireball model in appendix A. We have used the neutrino spectrum from a single GRB in the fireball model [11, 13]. The ultrahigh-energy neutrino spectrum in the source (GRB) rest frame is given as

$$\frac{dN_\nu(E_\nu)}{dE_\nu} \propto \frac{1}{E_\nu^2} \begin{cases} (E_\nu/E_\nu^b)^{\alpha_1} & E_\nu < E_\nu^b \\ (E_\nu/E_\nu^b)^{\alpha_2} & E_\nu^b < E_\nu < E_\nu^s \\ (E_\nu/E_\nu^b)^{\alpha_2} (E_\nu/E_\nu^s)^{-2} & E_\nu > E_\nu^s \end{cases} \quad (1)$$

where E_ν^b is the neutrino spectrum break energy, $\alpha 1$ and $\alpha 2$ are the neutrino spectral indices below and above the neutrino spectrum break energy (E_ν^b), respectively. E_ν^b is due to the break energy in the low-energy photon spectrum of a GRB (see appendix A for discussion). E_ν^s is the break energy in the neutrino spectrum due to the synchrotron energy loss by high-energy pions.

The final neutrino spectrum on Earth can be derived in the following way. Assuming that the GRB is occurring at a redshift z , the energy per unit time and unit area to be measured by a detector on Earth is [14]

$$F_{\nu \text{ obs}} = \frac{L_\nu}{4\pi d_L^2} \quad (2)$$

where d_L is the luminosity distance of the source. The luminosity in the source rest frame L_ν can be expressed in terms of the number of neutrinos per unit energy and unit time in that frame.

$$L_\nu = \int_{E_{\nu \text{ min}}}^{E_{\nu \text{ max}}} E_\nu \frac{dN'_\nu(E_\nu, t_\nu)}{dE_\nu dt_\nu} dE_\nu \quad (3)$$

where $\frac{dN_\nu(E_\nu)}{dE_\nu} = \int_0^{t_d} \frac{dN'_\nu(E_\nu, t_\nu)}{dE_\nu dt_\nu} dt_\nu$ and t_d is the duration of the burst in the source rest frame. We denote the observed neutrino spectrum on Earth per unit area, unit energy and unit time by $\frac{dM'_\nu(E_{\nu \text{ obs}}, t_{\nu \text{ obs}})}{dE_{\nu \text{ obs}} dt_{\nu \text{ obs}}}$. From equations (2) and (3) we can write

$$\int_{E_{\nu \text{ min, obs}}}^{E_{\nu \text{ max, obs}}} E_{\nu \text{ obs}} \frac{dM'_{\nu \text{ obs}}(E_{\nu \text{ obs}}, t_{\nu \text{ obs}})}{dE_{\nu \text{ obs}} dt_{\nu \text{ obs}}} dE_{\nu \text{ obs}} = \frac{1}{4\pi d_L^2} \int_{E_{\nu \text{ min}}}^{E_{\nu \text{ max}}} E_\nu \frac{dN'_\nu(E_\nu)}{dE_\nu dt_\nu} dE_\nu. \quad (4)$$

Hence, using $E_{\nu \text{ obs}} = E_\nu/(1+z)$ and $d_L = (1+z)d_z$,

$$\frac{dM'_{\nu \text{ obs}}(E_{\nu \text{ obs}}, t_{\nu \text{ obs}})}{dE_{\nu \text{ obs}} dt_{\nu \text{ obs}}} = \frac{1}{4\pi d_z^2} \frac{dN'_\nu(E_\nu, t_\nu)}{dE_\nu dt_\nu} \quad (5)$$

where d_z is the comoving distance. The number of neutrinos received on Earth per unit area and unit energy is

$$\frac{dM_{\nu \text{ obs}}(E_{\nu \text{ obs}})}{dE_{\nu \text{ obs}}} = \frac{1}{4\pi d_z^2} (1+z) \frac{dN_\nu(E_\nu)}{dE_\nu} \quad (6)$$

where $dt_{\nu \text{ obs}} = (1+z) dt_\nu$, $\frac{dM_{\nu \text{ obs}}}{dE_{\nu \text{ obs}}} = \int_0^{t_{d \text{ obs}}} \frac{dM'_{\nu \text{ obs}}(E_{\nu \text{ obs}}, t_{\nu \text{ obs}})}{dE_{\nu \text{ obs}} dt_{\nu \text{ obs}}} dt_{\nu \text{ obs}}$ and $t_{d \text{ obs}} = t_d(1+z)$. In a spatially flat universe we calculate the comoving distance of a source using $\Omega_\Lambda = 0.73$, $\Omega_m = 0.27$ and $H_o = 71 \text{ km s}^{-1} \text{ Mpc}^{-1}$ from [15]. The source spectrum contains ν_e and ν_μ in the ratio 1:2. As the electron and muon neutrinos propagate from the source to the observer, the electron neutrinos oscillate to muon neutrinos which in turn can oscillate to tau neutrinos. If we assume $U_{e3} = 0$, the solar neutrino mixing angle θ_\odot to be 30° and the atmospheric neutrino mixing angle θ_{atm} to be 45° , then due to the effect of neutrino oscillations alone, we expect an equal number of ν_e , ν_μ and ν_τ on Earth. Inclusion of the neutrino decay scenario causes the flavour ratio of the three active neutrino species to deviate from 1:1:1 [4]. We assume the decay scenario of neutrinos where one neutrino decays to a daughter neutrino and a very light or massless particle. The daughter neutrinos do not disappear and the neutrino mass spectrum is hierarchical. The energy-dependent neutrino decay term is proportional to $\exp(-dm_\nu/E_\nu t)$, where d is the distance of the source from the detector, m_ν is mass of neutrino and t is its rest-frame lifetime. We assume the neutrino decays to be complete when

we receive them on Earth. This assumption is a reasonable one because there is a lower limit on the value of d/E_ν set by the shortest distances of the sources (typically hundreds of Mpc) and the maximum energy that will be visible in OWL. The daughter neutrinos would be slightly deflected from the direction of propagation of the parent neutrinos. Since, we assume that the source is emitting isotropically, the flux received on Earth would not change due to the angular deflection of the decay products. If neutrinos are Dirac particles then daughters, which flip helicity, are sterile and undetectable. In the case of Majorana neutrinos, neutrinos and antineutrinos of both the helicity states are active and there is no distinction between a neutrino and an antineutrino. The expression for the neutrino spectrum after the appearance of daughter neutrinos with degraded energies is given in equation (7) of [4]. The energy dependence of the decay rate of neutrinos has been explicitly given in appendix B. $W(E', E)$ is the energy distribution function of the daughter neutrinos where E' and E are the energies of the parent and daughter neutrinos, respectively. This distribution function is different for Dirac and Majorana nature of neutrinos because in the Dirac case the neutrinos with flipped helicities are sterile in contrast to the Majorana neutrinos. We have also verified that in the case of radiative decay of neutrinos the energy dependence of the decay rate remains identical to the scalar decay scenario of neutrinos if one considers a coupling of the type $\sigma_{\mu\nu} F^{\mu\nu}$.

3. Propagation of UHE neutrinos through the Earth

The propagation of ν_τ through the Earth's matter at very high energy does not diminish their number [16]. The vertically upward-going ν_e s and ν_μ s are absorbed in Earth's matter above 10^{16} eV and cannot produce signals in the detector. This is because the lifetime of μ lepton is large (10^{-6} s) and it loses all its energy before decaying. This is in contrast to a τ lepton whose lifetime is 10^{-13} s, hence it decays much before energy loss occurs due to electromagnetic interaction. The ν_τ s produce τ leptons in charged current interactions with matter nuclei. A τ lepton can decay in many ways but in all of these decay processes another ν_τ is produced in the final state. The ν_τ regenerated in this way carries one-third of the energy of the primary ν_τ [17]. Also there are neutral current interactions and the ν_τ generated in these interactions carries about half of the primary ν_τ energy [16]. High energy ν_τ s initiate a series of interactions in the Earth and ultimately there is generation of ν_τ s of lower energies. Once the mean free path of interaction of ν_τ with nucleon of Earth's matter becomes comparable to the radius of the Earth, it directly propagates to the detector. There are discussions on propagation of tau neutrinos through the Earth and their event rates in detectors in a series of recent papers [18–23]. The procedure we have followed to calculate the probability of τ lepton emission from the surface of the Earth is discussed in appendix C. The area on the surface of the Earth, which would be covered by the field of view of Multi-OWL, would act as the effective target area for the upward-going tau neutrinos. One may consult [12] for a detailed discussion on how the larger field of view of Multi-OWL compared to the OWL experiment would increase the effective target area on the Earth's surface for the upward-going neutrinos. The τ leptons emitted from the surface of the Earth would produce UV fluorescent light in air while passing through the atmosphere. If they decay before reaching the atmosphere, there will be charged particles in their decay products which can produce UV fluorescent light in the atmosphere. UV fluorescent light will be emitted isotropically, hence OWL would be able to detect the charged particles from any direction [24]. The GRB ν_τ spectrum on Earth folded with the probability of τ emission from the surface of the Earth, calculated in appendix C, and the target area of the upward-going neutrinos on the Earth's surface gives the number of τ signals in the detector.

Table 1. Ratio of the numbers of vertically upward-going ν_τ events in Multi-OWL above energies 3×10^{15} eV and 2.2×10^{16} eV in the detector with mass ratio of daughter to parent neutrino 0.001 and $\alpha_2 = -0.3$.

Unstable	Branchings	Ratio		
		$R_{\nu_\tau}^D$	$R_{\nu_\tau}^M$	$(R_{\nu_\tau}^M - R_{\nu_\tau}^D)/R_{\nu_\tau}^D$
ν_2, ν_3	Irrelevant	27.53	29.99	0.09
ν_3	$B_{3 \rightarrow 1} = 1$	24.92	25.38	0.02
ν_3	$B_{3 \rightarrow 2} = 1$	25.84	27.04	0.05
ν_3	$B_{3 \rightarrow 1} = 0.5$	25.4	26.26	0.03
	$B_{3 \rightarrow 2} = 0.5$			

4. Results

The GRB parameters that have been used in our calculations are the following: a Lorentz factor $\Gamma = 300$, total energy emitted in neutrinos 10^{51} erg, low-energy photon luminosity $L_\gamma = 10^{51}$ erg s $^{-1}$, neutrino spectral indices $\alpha_1 = 1.5$ and $\alpha_2 = -0.3, 0.05, 0.3$ in equation (1), variability time $t_v = 10^{-3}$ s, $\epsilon_e = 0.5$, $\epsilon_B = 0.05$. The values of these parameters have been inferred from low-energy γ -ray observations on GRBs. We have calculated neutrino events in Multi-OWL from a GRB at a redshift $z = 0.5$ and at a zenith angle $\theta = 180^\circ$. Since, the neutrino spectrum break energy E_ν^b is 2.26×10^{15} eV for the GRB parameters used in our calculation and we are considering the neutrino events in the detector above energy 3×10^{15} eV, the GRB neutrino spectrum above E_ν^b only contributes to the number of events in the detector. The numbers of neutrino events calculated for Multi-OWL above energy 3×10^{15} eV are of the order of hundred. If we take the ratio of the numbers of events in the detector above different energies our results become independent of the parameters of the GRB, e.g., Γ , z , $\epsilon_{\gamma, \text{MeV}}^b$ and depend only on α_2 .¹ We calculate the expected numbers of active neutrino events in the detector above energies 3×10^{15} and 2.2×10^{16} eV for the two cases: (i) the neutrino is a Dirac particle, (ii) the neutrino is a Majorana particle. The ratios of the numbers of events above different energies are useful quantities for our study. They can be compared in future with their measured values in experiments like Multi-OWL. All GRB parameters except α_2 have been fixed for our results displayed in tables 1–4.

In table 1, the unstable neutrino eigenstates/eigenstate and the decay modes considered have been mentioned in first and second columns, respectively. The third and fourth columns show the ratios of the numbers of neutrino events in the detector above energies 3×10^{15} and 2.2×10^{16} eV for Dirac and Majorana nature of neutrinos, respectively. The fifth column displays by what fraction the ratios differ in the two cases. The ratios of the numbers of active neutrino events in the detector for the Dirac and Majorana nature of neutrinos differ by 9% for the decay scenario when both ν_2 and ν_3 are unstable and by 2% for $B_{3 \rightarrow 1} = 1$. We have varied α_2 to study its effect on the ratios of the numbers of events above different energies and our results have been presented in tables 2 and 3.

Table 2 shows the ratios of the numbers of events in the detector above energies 3×10^{15} and 2.2×10^{16} eV for $\alpha_2 = 0.05$.

If we compare tables 1–3, we find that the percentages by which the ratios of the numbers of events in the Dirac and Majorana case differ for different decay modes are independent of the value of α_2 in the range $-0.3 \leq \alpha_2 \leq 0.3$. Also, the ratios of the numbers of neutrino events do not vary significantly as we vary α_2 . We can possibly observe neutrino events

¹ We thank P L Biermann for this suggestion.

Table 2. Ratio of the numbers of vertically upward-going ν_τ events in Multi-OWL above energies 3×10^{15} eV and 2.2×10^{16} eV in the detector with mass ratio of daughter to parent neutrino 0.001 and $\alpha_2 = 0.05$.

Unstable	Branchings	Ratio		$(R_{\nu_\tau}^M - R_{\nu_\tau}^D)/R_{\nu_\tau}^D$
		$R_{\nu_\tau}^D$	$R_{\nu_\tau}^M$	
ν_2, ν_3	Irrelevant	27.41	29.85	0.09
ν_3	$B_{3 \rightarrow 1} = 1$	24.83	25.28	0.02
ν_3	$B_{3 \rightarrow 2} = 1$	25.73	26.93	0.05
ν_3	$B_{3 \rightarrow 1} = 0.5$	25.3	26.15	0.03
	$B_{3 \rightarrow 2} = 0.5$			

Table 3. Ratio of the numbers of vertically upward-going ν_τ events in Multi-OWL above energies 3×10^{15} eV and 2.2×10^{16} eV in the detector with mass ratio of daughter to parent neutrino 0.001 and $\alpha_2 = 0.3$.

Unstable	Branchings	Ratio		$(R_{\nu_\tau}^M - R_{\nu_\tau}^D)/R_{\nu_\tau}^D$
		$R_{\nu_\tau}^D$	$R_{\nu_\tau}^M$	
ν_2, ν_3	Irrelevant	27.31	29.71	0.09
ν_3	$B_{3 \rightarrow 1} = 1$	24.74	25.19	0.02
ν_3	$B_{3 \rightarrow 2} = 1$	25.63	26.82	0.05
ν_3	$B_{3 \rightarrow 1} = 0.5$	25.2	26.05	0.03
	$B_{3 \rightarrow 2} = 0.5$			

Table 4. Number of vertically upward-going ν_τ events in Multi-OWL from neutrinos having energies above 3×10^{15} eV, ν_3 unstable, $B_{3 \rightarrow 1} = 0.5$, $B_{3 \rightarrow 2} = 0.5$ and $\alpha_2 = -0.3, 0.3$.

m_l/m_h	Dirac		Majorana	
	$\alpha_2 = -0.3$	$\alpha_2 = 0.3$	$\alpha_2 = -0.3$	$\alpha_2 = 0.3$
0.01	698	735	891	938
0.1	699	736	892	940
0.25	702	744	896	949
0.5	711	773	908	987
0.75	731	829	933	1058

from a large number of GRBs whose values of spectral index α_2 are in a range such that $(R_{\nu_\tau}^M - R_{\nu_\tau}^D)/R_{\nu_\tau}^D$ are equal for all of them. In principle, if $(R_{\nu_\tau}^M - R_{\nu_\tau}^D)$ is greater than the sum of the theoretical and experimental errors in the ratios ($R_{\nu_\tau}^D$ or $R_{\nu_\tau}^M$) and we find that for most of the GRBs the experimentally measured ratios of the numbers of neutrino events above energies 3×10^{15} and 2.2×10^{16} eV are closer to $R_{\nu_\tau}^D$ or $R_{\nu_\tau}^M$, then we would be able to say whether a neutrino is a Dirac or a Majorana particle. We have discussed about the percentages of error in our theoretical calculation and neutrino experiments in section 5.

We also study the effect of the mass ratio of daughter to parent neutrino on the number of events in the detector. We denote the masses of the daughter and parent neutrinos by m_l and m_h , respectively. The area covered by Multi-OWL on the Earth's surface would be 3.3×10^{13} m² [12]. Our results are displayed in table 4.

The number of events increases with increasing neutrino mass ratios and this increase is more pronounced in case of Majorana neutrinos.

5. Error analysis

The values of the parameters of a GRB are determined from γ -ray and afterglow observations on that GRB. These values inferred from observations certainly carry some errors due to the errors in the measured data by the experiments. They would be used in the theoretical prediction of the number of neutrino events from that GRB and the errors in the measurement of the values of the GRB parameters would propagate to the theoretical calculation of expected number of neutrino events. We have calculated the ratios of the numbers of neutrino events above different energies in the detector, which depend only on the value of the GRB parameter α_2 . If it is possible to determine α_2 accurately from the low-energy γ -ray spectrum emitted by the GRB then the error in our calculation would become negligible. For example, if the error in the measurement of α_2 , $\Delta\alpha_2$, is 0.005 or 0.001 then the theoretical values of the ratios of the numbers of neutrino events ($R_{\nu_\tau}^D$ or $R_{\nu_\tau}^M$) would contain 1% or 0.2% error, respectively, as $\frac{\Delta R_{\nu_\tau}^D}{R_{\nu_\tau}^D}$ or $\frac{\Delta R_{\nu_\tau}^M}{R_{\nu_\tau}^M} = \left(\frac{3 \times 10^{15}}{2.2 \times 10^{16}}\right)^{\Delta\alpha_2} - 1$. Experimentally obtained ratios of the numbers of the neutrino events above different energies from a large number of GRBs can be compared with their theoretically possible values $R_{\nu_\tau}^D$ or $R_{\nu_\tau}^M$. Detection of neutrinos from a large number of GRBs would reduce the statistical error in the measurement of the ratios of the numbers of neutrino events from them. If the systematic error of the detector in the measurement of number of neutrino events is 4% then the Dirac/Majorana nature of a neutrino would be distinguishable in that detector for the decay scenario with both ν_2 and ν_3 unstable using GRBs with $\Delta\alpha_2 \approx 0.001$ as sources of ultrahigh neutrinos.

6. Conclusion

We have calculated the ratios of the expected numbers of vertically upward-going ν_τ events from a single GRB in Multi-OWL experiment above energies 3×10^{15} and 2.2×10^{16} eV, including the oscillation and non-radiative two-body decay of neutrinos. The ratios of the numbers of active neutrino events in the detector would be different depending on whether the neutrino is a Dirac or a Majorana particle. These theoretically calculated ratios carry errors only from the error in the determination of α_2 , which is obtained from the low-energy photon data of the GRB. Measurement of the ratios of the numbers of neutrino events from a large number of GRBs would reduce the experimental statistical error. If the systematic error in detecting neutrino events is 4%, and the error in determination of α_2 is of the order of 0.001, then we expect the Dirac/Majorana nature of neutrinos to be revealed in Multi-OWL for the scenario of neutrino decay when both ν_2 and ν_3 are unstable.

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Appendix A. The parameters of a gamma ray burst

GRBs are short lived bursts of γ -rays of durations milliseconds to several minutes. After a burst emission of x-rays, optical and radio waves continue for several hours to months. This is called the afterglow phase of a GRB. The observed radiation is produced both during the burst phase and the afterglow phase. In the fireball model, GRBs are produced by the dissipation of

the kinetic energy of a relativistic expanding fireball. keV–MeV energy photons are expected to be produced by synchrotron shock accelerated electrons. A GRB occurs at a redshift z , it has a Lorentz factor Γ and it is expected to emit energy of the order of 10^{51} erg in keV to MeV energy γ -ray emission. The observed photon spectrum on the Earth from a GRB can be expressed as a broken power law. The data obtained by Burst and Transient Source Experiment (BATSE) can be fitted using the parametrization given by

$$\frac{dn_\gamma}{d\epsilon_{\gamma,\text{ob}}} \propto \begin{cases} \epsilon_{\gamma,\text{ob}}^{-\alpha_2-1} & \epsilon_{\gamma,\text{ob}} < \epsilon_{\gamma,\text{ob}}^b \\ \epsilon_{\gamma,\text{ob}}^{-\alpha_1-1} & \epsilon_{\gamma,\text{ob}} > \epsilon_{\gamma,\text{ob}}^b \end{cases} \quad (\text{A.1})$$

$\epsilon_{\gamma,\text{ob}}$ is the photon energy and $\epsilon_{\gamma,\text{ob}}^b$ is the break energy of the photon spectrum as measured on Earth. The break energy is the characteristic observed energy of synchrotron photons produced by shock accelerated electrons. We denote the luminosity of a GRB in keV to MeV energy photon emission by L_γ . There is a detailed discussion on photon and neutrino production from GRBs in the fireball model in [25]. As the fireball expands relativistic shells come out from it with Lorentz factor $\Gamma \sim 100$ – 1000 . The variability time t_v of a GRB is the time interval between two successive emission of shells from it. When two shells collide with each other their kinetic energy is transformed into internal energy of the fireball. The fireball expands over a range of radii and reverse shocks are generated by the interaction of the expanding shells with the surrounding medium. Protons can be accelerated to very high energies by Fermi mechanism in internal and reverse shocks in the fireball. The low-energy (keV–MeV) photon spectral break energy in the source rest frame is $\epsilon_{\gamma,\text{MeV}}^b$, where $\epsilon_{\gamma,\text{MeV}}^b = \epsilon_{\gamma,\text{ob,MeV}}^b(1+z)$, and it depends on the equipartition parameters ϵ_e and ϵ_B , low-energy photon luminosity of the GRB L_γ , Lorentz factor Γ and variability time t_v . ϵ_e is the fraction of proton's shock thermal energy transformed to electrons and ϵ_B is the fraction of proton's shock thermal energy carried by the magnetic field in the fireball model. It is assumed that the wind luminosity carried by internal plasma energy L_{int} is related to the observed γ -ray luminosity through $L_{\text{int}} = L_\gamma/\epsilon_e$. This assumption is justified because the electron synchrotron cooling time is short compared to the wind expansion time, hence, electrons lose all their energy radiatively. The expression for photon spectral break energy can also be expressed as [25]

$$\epsilon_{\gamma,\text{MeV}}^b \approx \epsilon_B^{1/2} \epsilon_e^{3/2} \frac{L_{\gamma,52}^{1/2}}{\Gamma_{2.5}^2 t_{v,-2}} \quad (\text{A.2})$$

where $L_{\gamma,52} = L_\gamma/10^{52}$, $\Gamma_{2.5} = \Gamma/10^{2.5}$ and $t_{v,-2} = t_v/10^{-2}$. Inside the fireball protons interact with photons to produce pions which subsequently decay to produce neutrinos. There is a threshold energy for pion production in proton–photon interaction. For the energy of the proton–photon interaction to exceed the threshold energy for the Δ resonance, the proton energy in the source rest frame must satisfy the condition [11]

$$E_p^b \geq (1.4/\epsilon_{\gamma,\text{MeV}}^b) \Gamma_{2.5}^2 \times 10^7 \text{ GeV}. \quad (\text{A.3})$$

The reference frame which has been defined as the ‘observer’s rest frame’ in [13], is referred as the ‘source rest frame’ in this paper. From the expression of proton spectrum break energy one can obtain the neutrino spectrum break energy. We assume that on average 20% of the initial proton energy goes to the photo-pion. We also assume that the four final state leptons produced in the decay chain $\pi^+ \rightarrow \nu_\mu \mu^+ \rightarrow \nu_\mu e^+ \nu_e \bar{\nu}_\mu$ equally share the pion energy. Then the expression for the neutrino spectrum break energy in the source rest frame is as follows:

$$E_\nu^b = 7 \times 10^5 \frac{\Gamma_{2.5}^2}{\epsilon_{\gamma,\text{MeV}}^b} \text{ GeV}. \quad (\text{A.4})$$

The high-energy pions produced due to proton–photon interactions inside the fireball are expected to radiate energy due to their synchrotron emission. One can compare the synchrotron energy loss time of a high-energy pion with its lifetime for decay. Radiative energy loss of pions becomes important when their decay lifetime is larger than their synchrotron energy loss time. A break energy in the neutrino spectrum E_ν^s would appear due to the synchrotron radiation emission of the pions before they decay to produce neutrinos. Assuming that a neutrino or an antineutrino takes only one-fourth of the pion energy, one can write down the expression for E_ν^s from [11]:

$$E_\nu^s = 10^8 \epsilon_e^{1/2} \epsilon_B^{-1/2} L_{\gamma,52}^{-1/2} \Gamma_{2.5}^4 t_{v,-2} \text{ GeV}. \quad (\text{A.5})$$

The total energy emitted by the GRB in electron, muon neutrinos and antineutrinos is $E_{\nu,HE}$.

$$E_{\nu,HE} = \int_{E_{\nu,\min}}^{E_{\nu,\max}} E_\nu \frac{dN_\nu}{dE_\nu} dE_\nu \quad (\text{A.6})$$

where $E_{\nu,\min}$ and $E_{\nu,\max}$ are the minimum and maximum neutrino energy in the source rest frame. In the fireball model protons can be accelerated to a maximum energy of 10^{21} eV. Since, a neutrino on the average receives 5% of the proton energy, the maximum neutrino energy in the source rest frame $E_{\nu,\max}$ is approximately 10^{20} eV.

Appendix B. Derivation of the factor $W(E', E)$

We assume the decay of a heavy neutrino ν_h of mass m_h to a light neutrino ν_l of mass m_l occurs through $\nu_h \rightarrow \nu_l + J$, where J is a massless scalar (Majoron). The decay is given by the interaction Lagrangian density

$$L_I = g_1 \bar{\nu}_l \nu_h J + g_2 \bar{\nu}_l \gamma_5 \nu_h J + \text{h.c.} \quad (\text{B.1})$$

The energy of the parent neutrino is E' and daughter neutrino is E . The total decay rate in the rest frame of the parent neutrino is

$$\Gamma_{\text{total}}^0 = \frac{m_h^2 - m_l^2}{16\pi m_h^3} [|\text{Re } g_1|^2 (m_l + m_h)^2 + |\text{Im } g_2|^2 (m_h - m_l)^2]. \quad (\text{B.2})$$

The above expression can be derived by standard methods [26]. In the laboratory frame the decay rate is $\Gamma_{\text{total}}(E') = \frac{m_h}{E'} \Gamma_{\text{total}}^0$. For Majorana neutrinos, we have contribution from both conserved helicity and flipped helicity amplitudes $\Gamma_{\text{total}}(E') = \Gamma_f(E') + \Gamma_{nf}(E')$. The differential decay rate is $\frac{d\Gamma_{\text{total}}(E', E)}{dE}$. The total decay rate can also be written as

$$\Gamma_{\text{total}}(E') = \int_{E_{\min}}^{E_{\max}} \frac{d\Gamma_{\text{total}}(E', E)}{dE} dE. \quad (\text{B.3})$$

Where $E_{\min} = \frac{m_l^2}{m_h^2} E'$ and $E_{\max} = E'$. We work in the laboratory frame where $E, E' \gg m_h, m_l$. We obtain the energy distribution function of daughter neutrinos in the case of Majorana neutrinos as follows:

$$W_{\text{Majorana}}(E', E) = \frac{1}{\Gamma_{\text{total}}(E')} \frac{d\Gamma_{\text{total}}(E', E)}{dE}. \quad (\text{B.4})$$

The above expression can be expressed as a function of neutrino masses:

$$W_{\text{Majorana}}(E', E) = \frac{m_h^2}{(m_h^2 - m_l^2) E'}, \quad \text{with} \quad \frac{m_l^2}{m_h^2} E' \leq E \leq E'. \quad (\text{B.5})$$

For Dirac neutrinos only the daughter neutrinos, which conserve helicity, are contributing to the active species. So we define

$$W_{\text{Dirac}}(E', E) = \frac{1}{\Gamma_{\text{total}}(E')} \frac{d\Gamma_{nf}(E', E)}{dE}, \quad (\text{B.6})$$

and we obtain the energy distribution function given as

$$W_{\text{Dirac}}(E', E) = m_h^2 \frac{m_h^2 E - m_l^2 E'}{(m_h^2 - m_l^2)^2 E'^2}, \quad \text{with} \quad \frac{m_l^2}{m_h^2} E' \leq E \leq E'. \quad (\text{B.7})$$

Radiative decay going through transition magnetic moment will also lead to a similar result².

Appendix C. Derivation of the probability $\text{Prob}(\nu_\tau \rightarrow \tau)$

The energy of the ν_τ is denoted by $E_{\nu_\tau}(0)$ before it enters the Earth. While ν_τ is propagating through the Earth it is losing energy by neutral current interactions. The expression for energy loss of ν_τ by neutral current interaction is

$$\begin{aligned} \frac{dE_{\nu_\tau}(x)}{dx} &= -\rho(x) N_A \int_0^{E_{\nu_\tau}(x)} (E_{\nu_\tau}(x) - E_f(x)) \frac{d\sigma_{\text{NC}}(E_{\nu_\tau}(x), E_f(x))}{dE_f(x)} dE_f(x) \\ &= -\rho(x) N_A \sigma_{\text{NC}}(E_{\nu_\tau}(x)) (\langle E_f(x) \rangle - E_{\nu_\tau}(x)) \\ &= -\frac{E_{\nu_\tau}(x)}{2\lambda_{\text{NC}}(E_{\nu_\tau}(x))}. \end{aligned} \quad (\text{C.1})$$

We have used the fact that in each neutral current interaction the ν_τ loses half of its energy [16] in writing the final expression in equation (19). $\sigma_{\text{NC}}(E_{\nu_\tau}(x))$ is the cross section and $\lambda_{\text{NC}}(E_{\nu_\tau}(x))$ is the mean free path of neutral current interaction. $E_f(x)$ is the energy of the neutrino after it loses energy by neutral current interaction. The expression for the probability of neutral current interaction of ν_τ is

$$\begin{aligned} P_{\nu_\tau \rightarrow \nu_\tau}^0(E_{\nu_\tau}(0), E_{\nu_\tau}(x), x) dE_{\nu_\tau}(x) \\ = \delta \left(E_{\nu_\tau}(x) - E_{\nu_\tau}(0) \exp \left(- \int_0^x \frac{dx'}{2\lambda_{\text{NC}}(E_{\nu_\tau}(x'))} \right) \right) dE_{\nu_\tau}(x). \end{aligned} \quad (\text{C.2})$$

The probability of a τ production with energy between $E_\tau(x)$ and $E_\tau(x) + dE_\tau(x)$ from a ν_τ of energy $E_{\nu_\tau}(x)$ at a distance between x and $x + dx$ is

$$\begin{aligned} P_{\nu_\tau \rightarrow \tau}^0(E_{\nu_\tau}(0), E_\tau(x), x) dE_\tau(x) dx = \rho(x) N_A dx dE_\tau(x) \int_{E_{\nu_\tau \text{ min}}}^{E_{\nu_\tau \text{ max}}} \frac{d\sigma_{\text{CC}}(E_{\nu_\tau}(x), E_\tau(x))}{dE_\tau(x)} \\ \times P_{\nu_\tau \rightarrow \nu_\tau}^0(E_{\nu_\tau}(0), E_{\nu_\tau}(x), x) dE_{\nu_\tau}(x), \end{aligned} \quad (\text{C.3})$$

where $\sigma_{\text{CC}}(E_{\nu_\tau}(x))$ is the cross section of charged current interaction for ν_τ with energy $E_{\nu_\tau}(x)$, $\rho(x)$ is the density of the medium at x and N_A is Avogadro's number. We assume that the τ lepton produced by charged current interaction carries 75% of the ν_τ energy [17]. The τ leptons are losing energy during their propagation. There is discussion on energy loss of τ leptons in [21]. The energy loss by τ leptons due to ionization is negligible compared to the other processes, e.g., bremsstrahlung, e^+e^- pair production and photonuclear processes for energies above 10^5 GeV. We have used the energy loss coefficient of the τ lepton

$$\beta(E_\tau(x)) = (1.6 + 6(E_\tau(x)/10^9 \text{ GeV})^{0.2}) \times 10^{-7} \text{ g}^{-1} \text{ cm}^2. \quad (\text{C.4})$$

² HSM would like to thank A Dutta for this observation.

The τ lepton may decay or undergo charged current interaction to produce a ν_τ . The ν_τ produced from τ decay carries 40% of the τ lepton energy. The expression for the probability of neutral current interaction including τ decay and τ charged current interaction is

$$\begin{aligned}
P_{\nu_\tau \rightarrow \nu_\tau}(E_{\nu_\tau}(0), E_{\nu_\tau}(x), x) dE_{\nu_\tau}(x) w &= P_{\nu_\tau \rightarrow \nu_\tau}^0(E_{\nu_\tau}(0), E_{\nu_\tau}(x), x) dE_{\nu_\tau}(x) \\
&+ dE_{\nu_\tau}(x) \int_0^x \int_0^{E_\tau(x_1)} P_{\nu_\tau \rightarrow \nu_\tau}^0(E_{\nu_\tau}(x_1), E_{\nu_\tau}(x), x - x_1) \\
&\times \frac{d\Gamma_\tau(E_{\nu_\tau}(x_1), E_\tau(x_1))}{dE_{\nu_\tau}(x_1)} \\
&+ \frac{d\sigma_{\text{CC}}^{\tau N \rightarrow \nu_\tau X}(E_{\nu_\tau}(x_1), E_\tau(x_1))}{dE_{\nu_\tau}(x_1)} \rho(x_1) N_A dE_{\nu_\tau}(x_1) dx_1 \\
&\times \int_0^{x_1} \int_0^{E_{\nu_\tau}(x_2)} P_{\nu_\tau \rightarrow \tau}(E_{\nu_\tau}(0), E_\tau(x_2), x_2) dE_\tau(x_2) dx_2. \tag{C.5}
\end{aligned}$$

The energy of the τ lepton at x_1 and x_2 are related by the energy loss equation of τ leptons.

$$\frac{dE_\tau(x)}{dx} = -\beta(E_\tau(x)) E_\tau(x). \tag{C.6}$$

The expression for the probability of charged current interaction is

$$\begin{aligned}
P_{\nu_\tau \rightarrow \tau}(E_{\nu_\tau}(0), E_\tau(x), x) dE_\tau(x) dx &= \rho(x) N_A dE_\tau(x) dx \\
&\times \int_{E_{\nu_\tau \text{min}}}^{E_{\nu_\tau \text{max}}} P_{\nu_\tau \rightarrow \nu_\tau}(E_{\nu_\tau}(0), E_{\nu_\tau}(x), x) \frac{d\sigma_{\text{CC}}(E_{\nu_\tau}(x), E_\tau(x))}{dE_\tau(x)} dE_{\nu_\tau}(x). \tag{C.7}
\end{aligned}$$

The τ leptons produced by charged current interaction lose energy before emerging from the surface of the Earth. The range of τ is approximately $R_\tau = 1/\beta(E_\tau(x)) \ln(E_\tau(x)/E_{\text{th}})$ from equation (24). E_{th} is the threshold energy of the detector. The range of tau leptons can be written as a function of $E_{\nu_\tau}(x)$ by replacing $E_\tau(x)$ with $0.75E_{\nu_\tau}(x)$. The final expression for the probability of emission of τ leptons from the surface of the Earth having energies above the threshold energy of the detector is obtained after integrating equation (25) over $E_\tau(x)$ and x .

$$\text{Prob}(\nu_\tau \rightarrow \tau) = \int_0^{R_\tau} P_{\nu_\tau \rightarrow \tau}(E_{\nu_\tau}(0), 0.75E_{\nu_\tau}(x), x) dx. \tag{C.8}$$

We have solved equations (23) and (25) iteratively to find the value of the probability on the left-hand side of equation (25). We use the charged current and neutral current interaction cross sections from [27] and the Earth's density profile from [28]. Our result converges after two iterations. Finally, we have used the probability calculated in equation (26) $\text{Prob}(\nu_\tau \rightarrow \tau)$ for our calculation of number of τ events in the detector.

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