Dynamo Action in a Linear Shear Flow with Random Stirring: Numerical Studies

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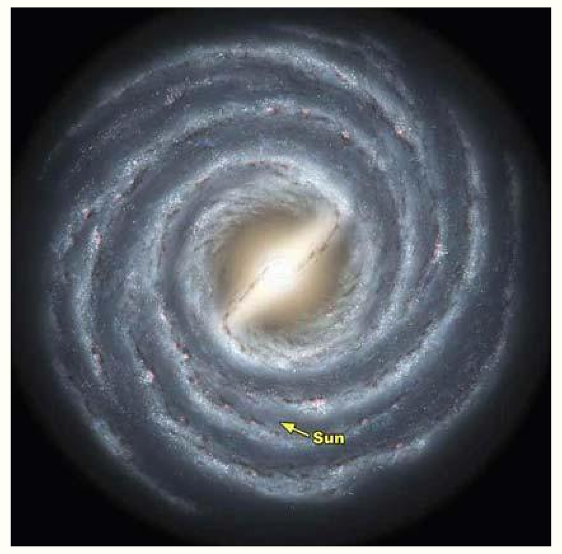
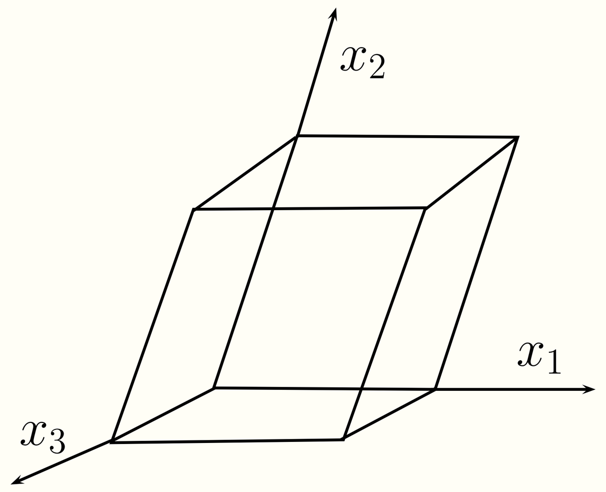
Motivation



It is well known fact that the large–scale magnetic fields in many astrophysical systems like sun, galaxy etc., are thought to originate from the dynamo action of the plasma in these objects. The standard explanation involves dynamo action of seed magnetic field due to turbulent flows which have helicity combined with shear. But recent work [1] shows the large–scale dynamo action for non-helical turbulence with shear, theory for which is yet to come. This motivated us to explore the dynamo action in different regimes of Re and Rm.

Model set up



It is a general practice in a simulation to simulate the local patch of the astrophysical systems like galaxy. So, we consider to do the simulation in a cubical box with shear in the x_2–direction. Using notation $\boldsymbol{x} = (x_1, x_2, x_3)$ for the position vector and t for time, the sheared–velocity field is written as $(Sx_1\boldsymbol{e}_2 + \boldsymbol{v})$, where S is the rate of shear parameter and $\boldsymbol{v}(\boldsymbol{x}, t)$ is the velocity deviation from the background shear flow. We have performed numerical simulations using the PENCIL CODE which is a weakly compressible MHD code.

Incompressible MHD equations

$$\left(\frac{\partial}{\partial t} + Sx_1\frac{\partial}{\partial x_2}\right)\boldsymbol{v} + Sv_1\boldsymbol{e}_2 + (\boldsymbol{v}\cdot\boldsymbol{\nabla})\boldsymbol{v} = -\frac{1}{\rho}\boldsymbol{\nabla}P + \frac{\boldsymbol{J}\times\boldsymbol{B}}{\rho} + \nu\nabla^2\boldsymbol{v} + \boldsymbol{f} \qquad (1)$$

$$\left(\frac{\partial}{\partial t} + Sx_1\frac{\partial}{\partial x_2}\right)\boldsymbol{B} - SB_1\boldsymbol{e}_2 = \boldsymbol{\nabla}\times(\boldsymbol{v}\times\boldsymbol{B}) + \eta\nabla^2\boldsymbol{B} \qquad (2)$$

Random stirring

Stochastic velocity field is set up by a forcing function $\boldsymbol{f}$ in equation (1) which is taken to be *homogeneous*, *isotropic* and *delta–correlated–in–time*. Forcing is confined to a spherical shell of magnitude $|\boldsymbol{k}_f| = k_f$ where the wavevector $\boldsymbol{k}_f$ signifies the energy–injection scale ($l_f = 2\pi/k_f$). Simulations have been performed in a cubic box of size $L \times L \times L$ (i.e., $L_1 = L_2 = L_3 = L$) in which the forcing $\boldsymbol{f}$ at each time step is a single plane wave proportional $\boldsymbol{k}_f \times \boldsymbol{a}$ where the wavevector $\boldsymbol{k}_f$ is randomly chosen from a set of precalculated vectors and $\boldsymbol{a}$ is an arbitrary random unit vector not aligned with $\boldsymbol{k}_f$. The properties as described above that $\boldsymbol{f}$ should possess can be achieved if the size of the box is much larger as compared to the forcing–scale, i.e., $k_f/K \gg 1$ where $K = 2\pi/L$. The background stochastic field becomes almost statistically steady for acceptable values of k_f/K if the averaging is done over long times.

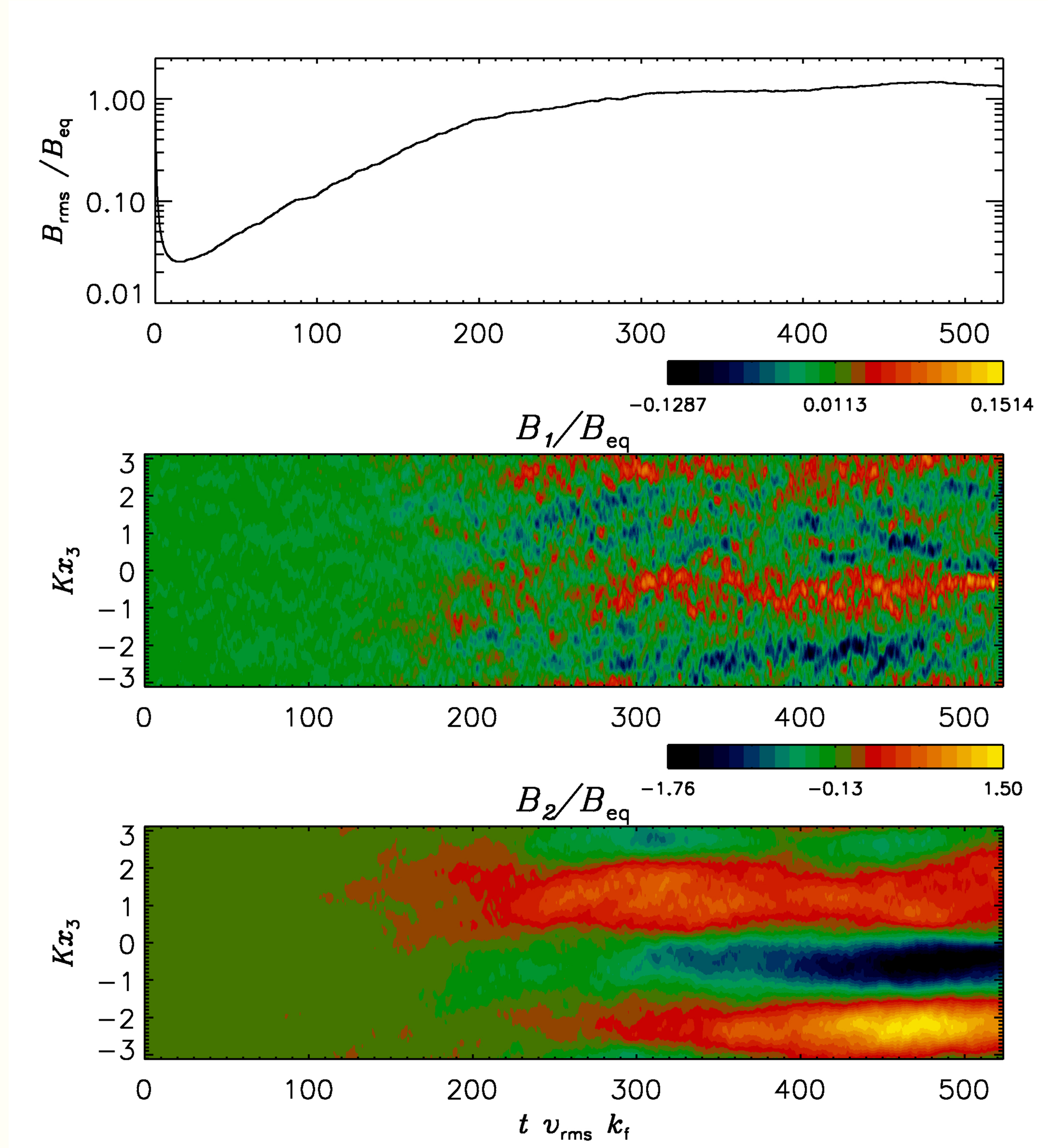
Boundary Conditions

We use "shear–periodic" boundary conditions to solve equations (1,2).
Sheared coordinates

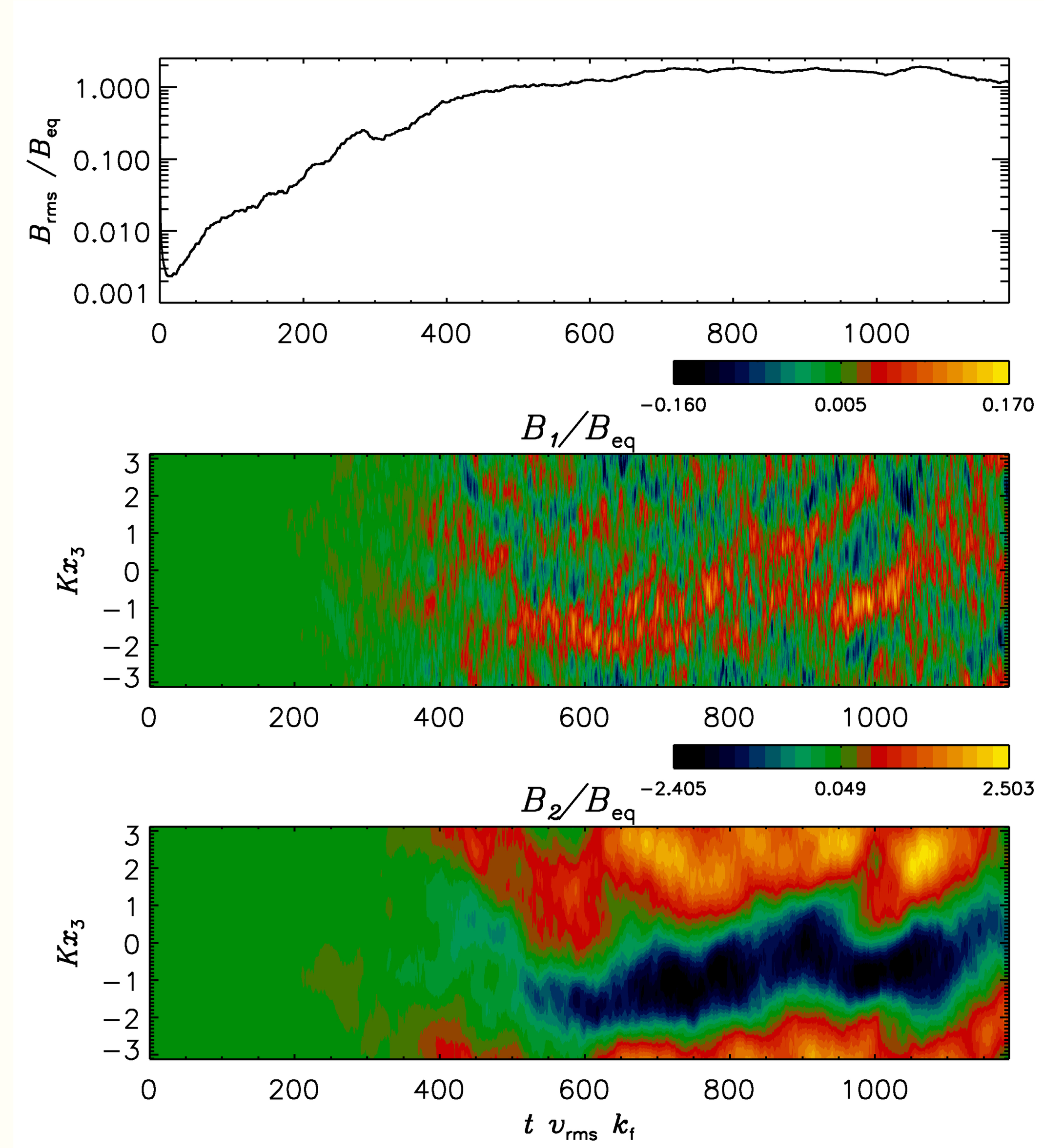
$$x_1^{\rm sh} = x_1\,, \qquad x_2^{\rm sh} = x_2 - Stx_1\,, \qquad x_3^{\rm sh} = x_3$$

A function is said to be *shear–periodic* when it is a periodic function of $(x_1^{\rm sh}, x_2^{\rm sh}, x_3^{\rm sh})$ with periodicities (L_1, L_2, L_3), respectively.

Results: Re < 1 and Rm > 1



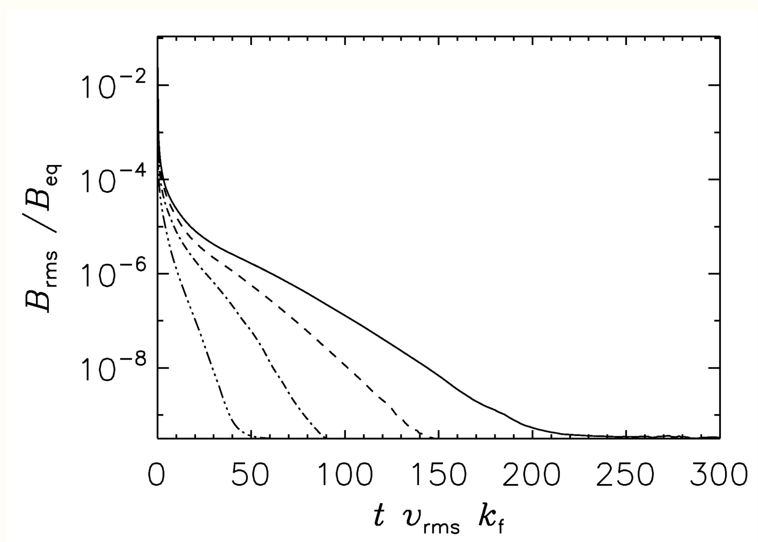
Re = 0.641, Rm = 32.039 (Pr = 50.0), $k_f/K = 5.09$ and $S_h = -0.60$



Re = 0.378, Rm = 15.135 (Pr = 40.0), $k_f/K = 3.13$ and $S_h = -1.01$

Figures (1–2) display the time dependence of root–mean–squared value of total magnetic field $\boldsymbol{B}$ and spacetime diagrams of $B_1(x_3, t)$ and $B_2(x_3, t)$ for two different combinations of Re and Rm obtained from a direct simulation. The top panel shows the initial exponential growth of mean magnetic field which saturates with time. The other two panels demonstrate the episodes of large scale feature in the x_3–direction, especially in the B_2 component.

Results: Re > 1 and Rm < 1



We explored this parameter regime in order to investigate the dynamo action when Re > 1 and Rm < 1. Figure displays the time dependence of root–mean–squared value of total magnetic field $\boldsymbol{B}$ for four different combinations of Re and Rm. This is done to check the results of the theory given in Reference [2] where kinematic theory of shear-dynamo was developed which is valid for low magnetic Reynolds number but places no restriction on the fluid Reynolds number.

Conclusions

In the present work we demonstrate that the dynamo action is possible in a background linear shear flow due to non–helical forcing when Re < 1 and Rm > 1.

There is no dynamo action for Re > 1 and Rm < 1.

References

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