Quantum which-way information and fringe visibility when the detector is entangled with an ancilla

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Quantum-mechanical wave-particle duality is quantified in terms of a trade-off relation between the fringe visibility and the which-way distinguishability in an interference experiment. This relation was recently generalized by Banaszek *et al.* [Nat. Commun. **4**, 2594 (2013)] when the particle is equipped with an internal degree of freedom such as spin. Here, we extend the visibility-distinguishability trade-off relation to quantum interference of a particle possessing an internal degree of freedom, when the *which-way* detector state is entangled with an ancillary system. We introduce an *extended which-way distinguishability* $\mathcal{D}_E$ and the associated *extended fringe visibility* $\mathcal{V}_E$, satisfying the inequality $\mathcal{D}_E^2 + \mathcal{V}_E^2 \leqslant 1$ in this scenario. We illustrate, with the help of three specific examples, that while the which-way information inferred solely from the detector state (without ancilla) vanishes, the *extended distinguishability* retrievable via measurements on the detector-ancilla entangled state is nonzero. Furthermore, in all the three examples, the *extended visibility* and the *generalized visibility* (which was introduced by Banaszek *et al.*) match identically with each other.

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I. INTRODUCTION

Visibility of fringes in a single-quantum particle interference experiment sets limits on the which-way information [1–4], thus demonstrating wave-particle duality. Very recently, Banaszek *et al.* [5] analyzed the trade-off between interference visibility and which-path distinguishability for a quantum particle possessing an internal structure (such as spin or polarization). In this setting, an interaction of the internal spin state with the detector system is shown to offer nontrivial identifications. The internal structure could play a manipulative role in controlling the information about which path in the interferometer arms is taken by the particle. The trade-off between the amount of which-way information encoded in the detector system and the fringe visibility is captured in terms of a generalized complementarity relation [5], by extending the notion of fringe visibility in terms of the internal spin states as well as their interaction with the detector.

To place things in order, we outline the basic scenario [4], where a single-quantum particle (quanton) Q travels through a two-path interferometer (double-slit or Mach-Zehnder interferometer), with the paths being equiprobable. Let us denote the initial state of the quanton and the detector system by $\rho_{QD}^{(\rm in)} = \rho_Q^{(\rm in)} \otimes \rho_D^{(\rm in)}$. When the quanton takes either path 0 or 1 of the interferometer arms, the detector state correspondingly gets transformed into

$$\rho_D^{(i)} = U_D^{(i)} \rho_D^{(\rm in)} U_D^{(i)\dagger}, \quad i = 0,1, \tag{1}$$

where $U_D^{(i)}$ denote unitary transformations on the detector states corresponding to the paths of the quanton. [The interaction is constrained such that the quanton paths cannot get transferred into one another due to interaction. The final detector state after the interaction is then given by $\rho_D^{(\rm fin)} = \frac{1}{2}\,\rho_D^{(0)} + \frac{1}{2}\,\rho_D^{(1)}$.]

Which-way information is quantified in terms of *distinguishability* $0 \leqslant \mathcal{D} \leqslant 1$, which is the trace distance between the detector states $\rho_D^{(0)}$ and $\rho_D^{(1)}$:

$$\mathcal{D} = \tfrac{1}{2}\left\|\rho_D^{(0)} - \rho_D^{(1)}\right\|. \tag{2}$$

(Here, $||A|| = \mathrm{Tr}[\sqrt{A^\dagger A}]$ denotes the trace norm of A).

It may be noted that the distinguishability is the maximum of the difference of probabilities of the correct and incorrect decisions about the paths [6]. The paths of the quanton cannot be distinguished when $\mathcal{D} = 0$ [i.e., when $\rho_D^{(0)} \equiv \rho_D^{(1)}$] whereas they are perfectly distinguishable when $\mathcal{D} = 1$ [i.e., when $\rho_D^{(0)}$ and $\rho_D^{(1)}$ are orthogonal]. Consequently, the fringe visibility $\mathcal{V}$ is given by [4]

$$\mathcal{V} = \left|\mathrm{Tr}\left[U_D^{(0)} \rho_D^{(\rm in)} U_D^{(1)\dagger}\right]\right|. \tag{3}$$

Visibility $0 \leqslant \mathcal{V} \leqslant 1$ characterizes the ability of the quanton, distributed between two paths, to interfere after getting combined at the exit of the interferometer. Wave-particle duality in the quantum interference experiment is expressed in terms of the trade-off relation [4] between visibility and distinguishability:

$$\mathcal{D}^2 + \mathcal{V}^2 \leqslant 1. \tag{4}$$

In a more general setup, explored recently [5], the quanton is equipped with an internal degree of freedom such as spin (characterized by a d_S level quantum system) and it is recognized that there is an intricate relation between the which-way information $\mathcal{D}$ on the initial preparation of the internal spin state, in addition to the specific details of its interaction with the detector. Banaszek *et al.* [5] demonstrated a stringent bound on distinguishability in terms of *generalized fringe visibility*, which depends on the initial preparation of

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the spin state as well as on the nature of its interaction with the detector system. The generalized trade-off inequality reads as [5]

$$\mathcal{D}^2 + \mathcal{V}_G^2 \leqslant 1, \tag{5}$$

where the distinguishability $\mathcal{D} = \frac{1}{2}||\rho_D^{(0)} - \rho_D^{(1)}||$ captures the leakout of which-way information to the detector [here, $\rho_D^{(i)} = 2\,{}_Q\langle i|\mathrm{Tr}_S[U_{QSD}\rho_{QS}^{(\mathrm{in})} \otimes \rho_D^{(\mathrm{in})} U_{QSD}^\dagger]|i\rangle_Q$, $i = 0,1$ are the detector states corresponding to the quanton paths]; the unitary interaction U_{QSD} is constrained to be of the form $U_{QSD} = \sum_{i=0,1} |i\rangle_Q\langle i| \otimes U_{SD}^{(i)}$ such that the which-way interaction does not shift the quanton between interferometer arms.

The generalized fringe visiblity $\mathcal{V}_G$ in (5) is given by [5]

$$\mathcal{V}_G = d_S \big\| (\mathbb{1} \otimes \Lambda_{01})\big[\big(I_S \otimes \sqrt{\rho_{S0}^{(\mathrm{in})}}\big) \times |\Phi_+\rangle\langle\Phi_+|\big(I_S \otimes \sqrt{\rho_{S1}^{(\mathrm{in})}}\big)\big]\big\|, \tag{6}$$

where the unitary interaction of the detector with the spin subsystem corresponds to the action of a quantum channel [7] Λ on the internal spin state as explained below: Let $\rho_{QSD}^{(\mathrm{in})} = \rho_{QS}^{(\mathrm{in})} \otimes \rho_D^{(\mathrm{in})}$ denote the initial quanton path spin (denoted by QS) and the detector (denoted by D) states. A unitary interaction between the detector and the internal spin results in the final state $\rho_{QSD}^{(\mathrm{fin})} = U_{QSD}\rho_{QSD}^{(\mathrm{in})}U_{QSD}^\dagger$. This unitary interaction on the initial state $\rho_{QSD}^{(\mathrm{in})}$ may be viewed as the action of a quantum superoperator Λ on the input state $\rho_{QS}^{(\mathrm{in})}$, i.e., $\Lambda(\rho_{QS}^{(\mathrm{in})}) = \mathrm{Tr}_D[\rho_{QSD}^{(\mathrm{fin})}]$. Further, as the quanton does not get switched between the interferometer arms 0 and 1 as a result of the interaction, the channel Λ must be of the form

$$\Lambda(|i\rangle_Q\langle j| \otimes \sigma_S) = |i\rangle_Q\langle j| \otimes \Lambda_{ij}(\sigma_S), \quad i,j = 0,1, \tag{7}$$

where σ_S corresponds to any operator in the spin space. Further, in (6), the states $\rho_{S0}^{(\mathrm{in})},\rho_{S1}^{(\mathrm{in})}$ are the initial spin states along the paths 0,1 and are given by $\rho_{S0}^{(\mathrm{in})} = 2\,{}_Q\langle 0|\rho_{QS}^{(\mathrm{in})}|0\rangle_Q$, $\rho_{S1}^{(\mathrm{in})} = 2\,{}_Q\langle 1|\rho_{QS}^{(\mathrm{in})}|1\rangle_Q$; I_S denotes the identity operator in the spin space and $\mathbb{1}$ denotes the identity channel; the state $|\Phi_+\rangle = \frac{1}{\sqrt{d_S}}\sum_{\alpha=0}^{d_S-1}|\alpha\rangle_S|\alpha\rangle_{S'}$ is a maximally entangled state of two replicas of the spin system.

Banaszek *et al.* [5] analyzed specific examples to demonstrate the intricate role played by the internal spin state preparation corresponding to specific interaction channels Λ_{01}. Specifically, when there is no interaction with the detector, the channel reduces to $\Lambda_{01} = \mathbb{1}$, and after simplification of (6) one obtains $\mathcal{V}_G = \sqrt{\mathrm{Tr}[\rho_{S0}^{(\mathrm{in})}]\mathrm{Tr}[\rho_{S1}^{(\mathrm{in})}]} = 1$, irrespective of the preparation of the initial spin state (i.e., when the detector gains no information about the path, visibility $\mathcal{V}_G$ is 1 and the distinguishability $\mathcal{D}$ is 0). When the interaction channel is given by $\Lambda_{01}(\sigma_S) = \mathrm{Tr}[\sigma_S]\Sigma_S$, with Σ_S being a fixed unit-trace Hermitian operator, the generalized visibility reduces to the fidelity [8,9] between the spin states $\rho_{S0}^{(\mathrm{in})},\rho_{S1}^{(\mathrm{in})}$, i.e., $\mathcal{V}_G = \mathrm{Tr}[\sqrt{\sqrt{\rho_{S0}^{(\mathrm{in})}}\rho_{S1}^{(\mathrm{in})}\sqrt{\rho_{S0}^{(\mathrm{in})}}}] = F(\rho_{S0}^{(\mathrm{in})},\rho_{S1}^{(\mathrm{in})})$. Thus, the which-way information can be blocked by preparing identical spin states for both the paths, i.e., $\rho_{S0}^{(\mathrm{in})} = \rho_{S1}^{(\mathrm{in})}$, so that the generalized visibility takes its maximum value 1. In yet another interesting example of the interaction channel, defined through $\Lambda_{01}(\sigma_S) = \sigma_S^T/d_S$ (where σ_S^T is the transpose of the spin operator σ_S), the generalized visibility (6) gets simplified to $\mathcal{V}_G = ||\sqrt{\rho_{S0}^{(\mathrm{in})}}||\,||\sqrt{\rho_{S1}^{(\mathrm{in})}}||/d_S$. The spin states in both the paths, prepared initially in a completely mixed state $\rho_{S0}^{(\mathrm{in})} = \rho_{S1}^{(\mathrm{in})} = I_S/d_S$, would lead to the generalized fringe visibility $\mathcal{V}_G = 1$ and hence the which-path information to the detector can be blocked.

In the present article, we explore the trade-off between the fringe visibility and the which-way information retrievable when the detector is entangled with an ancilla. Basically, the which-way information corresponds, in particular, to the discrimination of the detector states $\rho_D^{(0)}$ and $\rho_D^{(1)}$, when the quanton chooses path 0 or path 1. One may view the states $\rho_D^{(0)}$ and $\rho_D^{(1)}$ as the outputs of completely positive, trace-preserving quantum channels [10] Φ_0,Φ_1, acting on the input state $\rho_D^{(\mathrm{in})}$. The problem of gaining which-way information via distinguishing the two detector states $\rho_D^{(0)}$ and $\rho_D^{(1)}$ can then be linked with that of discriminating the two quantum channels Φ_0,Φ_1. A useful measure of distinguishability of two quantum channels is given by their trace distance,

$$\tfrac{1}{2}||\Phi_0 - \Phi_1|| = {}^{\max}_{\rho}\ \tfrac{1}{2}||\Phi_0(\rho) - \Phi_1(\rho)||, \tag{8}$$

where the maximum is taken over all pure input states ρ [11,12]. However, an optimal approach to maximize the observable difference between the two channels is to prepare the input state entangled with another auxiliary system, apply one of the channels (chosen randomly) to the input state (with the ancillary subsystem being an idler), and then measure the resulting output bipartite state to identify which of the channels was applied. It has been established that channel inputs, which are entangled with an ancilla, could offer remarkable improvements in distinguishing some pairs of channels [11–21]. In fact, there are examples of channels [11] that can be distinguished *perfectly* when they are applied to one part of a maximally entangled state, while they are *indistinguishable* if the auxiliary system is not employed.

II. WHICH-WAY INFORMATION USING ENTANGLED DETECTOR-ANCILLA ENTANGED STATE

Our purpose here is to investigate the enhancement of which-way information, given that the detector is entangled with an ancilla D' of the same dimension as that of the detector system [22]. We define *extended distinguishability* achievable with an entangled detector-ancilla initial state by

$$\mathcal{D}_E = \tfrac{1}{2}\big|\big|(\Phi_0 \otimes \mathbb{1})\big(\rho_{DD'}^{(\mathrm{in})}\big) - (\Phi_1 \otimes \mathbb{1})\big(\rho_{DD'}^{(\mathrm{in})}\big)\big|\big| = \frac{1}{2}\big|\big|\rho_{DD'}^{(0)} - \rho_{DD'}^{(1)}\big|\big|, \tag{9}$$

where

$$\rho_{DD'}^{(i)} = (\Phi_i \otimes \mathbb{1})\big(\rho_{DD'}^{(\mathrm{in})}\big) = 2\,{}_Q\langle i|\mathrm{Tr}_S[U_{QSD} \otimes I_{D'}\rho_{QS} \otimes \rho_{DD'}U_{QSD}^\dagger \otimes I_{D'}]|i\rangle_Q$$

denote the final detector-ancilla states corresponding to quanton paths $i = 0,1$.

Using the inequality [23] $D(\varrho,\tau) \leqslant \sqrt{1 - F^2(\varrho,\tau)}$, relating the trace distance $D(\varrho,\tau) = \frac{1}{2}||\varrho - \tau||$ and the fidelity

$F(\varrho,\tau) = \mathrm{Tr}[\sqrt{\sqrt{\varrho}\,\tau\sqrt{\varrho}}]$ between the two density operators ϱ and τ, we obtain the following relation for the extended distinguishability:

$$\mathcal{D}_E \leqslant \sqrt{1 - F^2\big(\rho^{(0)}_{DD'},\rho^{(1)}_{DD'}\big)}. \quad (10)$$

Expressing the extended distinguishability as $\mathcal{D}_E \leqslant \sqrt{1-\mathcal{V}_E^2}$, we define the corresponding *extended fringe visibility* by

$$\mathcal{V}_E = F\big(\rho^{(0)}_{DD'},\rho^{(1)}_{DD'}\big). \quad (11)$$

We now proceed to analyze three specific examples of interaction to illustrate that the extended which-way distinguishability $\mathcal{D}_E$ can assume nonzero values, even when the distinguishability $\mathcal{D}$ inferred by measuring only the detector states vanishes. In the meanwhile, we also find that the extended visibility and the generalized visibility [5] agree identically with each other in these examples.

III. EXAMPLES

Let us consider a quanton—with two-dimensional internal spin states $|0\rangle_S,|1\rangle_S$—traveling through a Mach-Zehnder interferometer. We consider a pure entangled detector-ancilla input state

$$|\Psi\rangle_{DD'} = \frac{1}{\sqrt{2}}\,(|0\rangle_D|0\rangle_{D'} + |1\rangle_D|1\rangle_{D'}). \quad (12)$$

Here, $\{|k\rangle_D, k=0,1\}$ and $\{|l\rangle_{D'}, l=0,1\}$ denote orthogonal states of the detector D and ancilla D'. Let $|0\rangle_Q,|1\rangle_Q$ denote the state of the quanton in path 0 and 1, respectively. Internal spin states in paths 0,1 are denoted by $|\psi_0\rangle_S = a_{00}|0\rangle_S + a_{01}|1\rangle_S$ and $|\psi_1\rangle_S = a_{10}|0\rangle_S + a_{11}|1\rangle_S$ (the coefficients $a_{\alpha\alpha'}$ obey the normalization condition $\sum_{\alpha'=0,1}|a_{\alpha\alpha'}|^2 = 1, \alpha = 0,1$).

We consider the unitary interaction on the quanton spin with the detector D along the paths 0,1 to be of the form

$$U^{(0)}_{SD} = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}, \quad U^{(1)}_{SD} = \begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}, \quad (13)$$

when expressed in the basis $\{|0\rangle_S|0\rangle_D, |0\rangle_S|1\rangle_D, |1\rangle_S|0\rangle_D, |1\rangle_S|1\rangle_D\}$. Under this unitary interaction with the detector the initial quanton path-spin state $|\zeta\rangle_{QS} = \frac{1}{\sqrt{2}}(|0\rangle_Q\,|\psi_0\rangle_S + |1\rangle_Q|\psi_1\rangle_S)$ and the detector-ancilla state $|\Psi\rangle_{DD'}$ [see (12)] get transformed to $|\zeta\rangle_{QS}|\Psi\rangle_{DD'} \to |\varphi\rangle_{QSDD'}$, which is given explicitly by

$$|\varphi\rangle_{QSDD'} = \frac{1}{\sqrt{2}}[(a_{00}|0\rangle_Q|0\rangle_S + a_{11}|1\rangle_Q|1\rangle_S)|\Psi\rangle_{DD'} + (a_{01}|0\rangle_Q|1\rangle_S + a_{10}|1\rangle_Q|0\rangle_S)|\Psi_\perp\rangle_{DD'}], \quad (14)$$

where $|\Psi_\perp\rangle_{DD'} = \frac{1}{\sqrt{2}}(|0\rangle_D|1\rangle_{D'} + |1\rangle_D|0\rangle_{D'})$. The quanton path-spin final density operator $\rho^{(\mathrm{fin})}_{QS} = \mathrm{Tr}_{DD'}[|\varphi\rangle_{QSDD'}\langle\varphi|]$ is then found to be

$$\begin{aligned}\rho^{(\mathrm{fin})}_{QS} = \tfrac{1}{2}[&|0\rangle_Q\langle 0| \otimes (|a_{00}|^2|0\rangle_S\langle 0| + |a_{01}|^2|1\rangle_S\langle 1|)\\ &+ |1\rangle_Q\langle 1| \otimes (|a_{10}|^2\,|0\rangle_S\langle 0| + |a_{11}|^2|1\rangle_S\langle 1|)\\ &+ |0\rangle_Q\langle 1| \otimes (a_{00}a_{11}^*|0\rangle_S\langle 1| + a_{01}a_{10}^*|1\rangle_S\langle 0|)\\ &+ |1\rangle_Q\langle 0| \otimes (a_{00}^*a_{11}|1\rangle_S\langle 0| + a_{01}^*a_{10}|0\rangle_S\langle 1|)]. \quad (15)\end{aligned}$$

The states $\rho^{(i)}_{DD'}$, $i=0,1$ of the detector-ancilla system after the interaction are obtained by $\rho^{(i)}_{DD'} = {}_Q\langle i|\mathrm{Tr}_S[|\varphi\rangle_{QSDD'}\langle\varphi|]|i\rangle_Q$, and are explicitly given [in the basis $\{|0\rangle_D|0\rangle_{D'}, |0\rangle_D|1\rangle_{D'}, |1\rangle_D|0\rangle_{D'}, |1\rangle_D|1\rangle_{D'}\}$] by

$$\rho^{(0)}_{DD'} = \frac{1}{2}\begin{pmatrix}|a_{00}|^2&0&0&|a_{00}|^2\\0&|a_{01}|^2&|a_{01}|^2&0\\0&|a_{01}|^2&|a_{01}|^2&0\\|a_{00}|^2&0&0&|a_{00}|^2\end{pmatrix},$$

$$\rho^{(1)}_{DD'} = \frac{1}{2}\begin{pmatrix}|a_{11}|^2&0&0&|a_{11}|^2\\0&|a_{10}|^2&|a_{10}|^2&0\\0&|a_{10}|^2&|a_{10}|^2&0\\|a_{11}|^2&0&0&|a_{11}|^2\end{pmatrix}.$$

The which-way information retrievable from the *extended distinguishability* [see (9)] is given by

$$\mathcal{D}_E = \begin{cases}(|a_{00}|^2 - |a_{11}|^2) & \text{if } |a_{00}|^2 > |a_{11}|^2,\\ (|a_{11}|^2 - |a_{00}|^2) & \text{if } |a_{11}|^2 > |a_{00}|^2,\end{cases} \quad (16)$$

and $\mathcal{D}_E = 0$ when $|a_{00}|^2 = |a_{11}|^2$.

The extended visibility $\mathcal{V}_E$ [see (11)] gets simplified to the following:

$$\mathcal{V}_E = |a_{00}||a_{11}| + \sqrt{1-|a_{00}|^2}\sqrt{1-|a_{11}|^2}. \quad (17)$$

In Fig. 1 we have plotted the *extended which-way distinguishability* $\mathcal{D}_E$, the *extended fringe visibility* $\mathcal{V}_E$, and $\mathcal{D}_E^2 + \mathcal{V}_E^2$ as a function of the initial spin state parameters $|a_{00}|,|a_{11}|$. It is clearly seen that the extended distinguishability and visibility are complementary to each other and they obey the trade-off relation $\mathcal{D}_E^2 + \mathcal{V}_E^2 \leqslant 1$.

In the absence of the ancilla D', we find the detector states, $\rho^{(0)}_D = \mathrm{Tr}_{D'}[\rho^{(0)}_{DD'}] = \frac{1}{2}\left(\begin{smallmatrix}1&0\\0&1\end{smallmatrix}\right)$, $\rho^{(1)}_D = \mathrm{Tr}_{D'}[\rho^{(0)}_{DD'}] = \frac{1}{2}\left(\begin{smallmatrix}1&0\\0&1\end{smallmatrix}\right)$, are perfectly indistinguishable, leading to which-way distinguishability $\mathcal{D} = 0$ when the ancilla is not taken into consideration.

We also evaluate the generalized fringe visibility introduced in Ref. [5] [see (6)] in this example. From the final-state density operator $\rho^{(\mathrm{fin})}_{QS}$ [see (15)] of the quanton path-spin system, we identify that the action of the quantum channel Λ_{01} on spin states is to kill the diagonal elements of the spin operator. More explicitly,

$$\Lambda_{01}(|\alpha\rangle_S\langle\alpha'|) = \begin{cases}0, & \text{if } \alpha = \alpha',\\ 1, & \text{if} \alpha \neq \alpha',\end{cases} \quad (18)$$

where $\alpha,\alpha' = 0,1$. We simplify (6) to obtain the generalized fringe visibility

$$\mathcal{V}_G = |a_{00}||a_{11}| + \sqrt{1-|a_{00}|^2}\sqrt{1-|a_{11}|^2}, \quad (19)$$

which matches exactly with the extended visibility $\mathcal{V}_E$ of (17).

It may be noted that even though the which-way distinguishability $\mathcal{D}$ (obtained when the ancilla degree of freedom is ignored) is zero, the generalized visibility $\mathcal{V}_G$ does not take its maximum value 1 in this example. Variation of $\mathcal{V}_G$ does indeed reveal a leakage of which-way information. However, the trade-off relation turns out to be that between

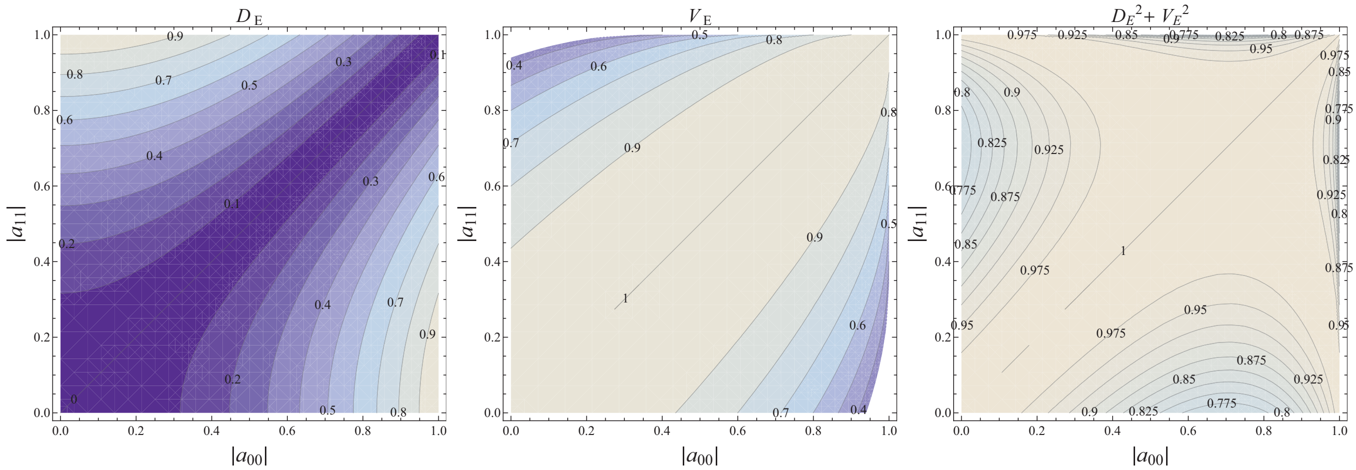

FIG. 1. (Color online) Contour plots of *extended distinguishability* $\mathcal{D}_E$, *extended visibility* $\mathcal{V}_E$ [see (16) and (17)], and $\mathcal{D}_E^2+\mathcal{V}_E^2$ as functions of $|a_{00}|,|a_{11}|$, the parameters of the initial spin preparation. It is clearly seen that the which-way distinguishability $\mathcal{D}_E$ and fringe visibility $\mathcal{V}_E$ obey the duality relation $\mathcal{D}_E^2+\mathcal{V}_E^2\leqslant 1$.

the visibility and the which-way information captured by the extended distinguishability $\mathcal{D}_E$—and not the one assimilated through $\mathcal{D}$.

Next, we consider the initial quanton path spin to be prepared in the state $|\zeta\rangle_{QS}=\frac{1}{2}[|0\rangle_Q+|1\rangle_Q]\otimes|0\rangle_S$ and the detector-ancilla state is initially prepared in the maximally entangled state $|\Psi\rangle_{DD'}$ given by (12). The unitary interaction between the quanton spin and detector is chosen to be

$$U_{SD}^{(0)}=\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{pmatrix},\quad U_{SD}^{(1)}=\begin{pmatrix}0&0&0&1\\0&0&e^{-i\phi}&0\\0&e^{i\phi}&0&0\\1&0&0&0\end{pmatrix},\tag{20}$$

which are given in the spin-detector basis states $\{|0\rangle_S|0\rangle_D,|0\rangle_S|1\rangle_D,|1\rangle_S|0\rangle_D,|1\rangle_S|1\rangle_D\}$. After the interaction, the detector-ancilla states associated with the paths 0,1 of the quanton are given by

$$\rho_{DD'}^{(0)}=\frac{1}{2}\begin{pmatrix}0&0&0&0\\0&1&1&0\\0&1&1&0\\0&0&0&0\end{pmatrix},\quad \rho_{DD'}^{(1)}=\frac{1}{2}\begin{pmatrix}0&0&0&0\\0&1&e^{-i\phi}&0\\0&e^{i\phi}&1&0\\0&0&0&0\end{pmatrix}.\tag{21}$$

The which-way information extracted via the extended distinguishability is $\mathcal{D}_E=|\sin(\phi/2)|$, while the extended fringe visibility is identified to be exactly complementary [24], i.e., $\mathcal{V}_E=|\cos(\phi/2)|$. It is easy to see that the detector states $\rho_D^{(i)}=\mathrm{Tr}_{D'}[\rho_{DD'}^{(i)}]=I_S/2,\ i=0,1$ are indistinguishable.

In order to evaluate the generalized visibility $\mathcal{V}_G$, we first identify the action of the spin channel Λ_{01}:

$$\Lambda_{01}(|0\rangle_S\langle 0|)=\left(\frac{1+e^{i\phi}}{2}\right)|1\rangle_S\langle 1|,$$
$$\Lambda_{01}(|1\rangle_S\langle 1|)=\left(\frac{1+e^{-i\phi}}{2}\right)|0\rangle_S\langle 0|,$$
$$\Lambda_{01}(|0\rangle_S\langle 1|)=\left(\frac{1+e^{-i\phi}}{2}\right)|1\rangle_S\langle 0|,$$
$$\Lambda_{01}(|1\rangle_S\langle 0|)=\left(\frac{1+e^{i\phi}}{2}\right)|0\rangle_S\langle 1|.$$

We thus find that the generalized visibility (11) reduces to $\mathcal{V}_G=|\cos(\phi/2)|$, which is equal to the extended visibility $\mathcal{V}_E$ in this example, too.

Even here, the variation of $\mathcal{V}_G$ would indicate leakout of which-path information—but it is not retrievable from the detector alone—thus bringing out the significance of detection using the extended detector-ancilla system.

In the third example, we consider the unitary interaction between the quanton and the detector to be

$$U_{QSD}=\sum_{i=0,1}|i\rangle_Q\langle i|\otimes U_{SD}^{(i)},\quad U_{SD}^{(i)}=e^{-i\frac{\theta_i}{2}\sigma_z\otimes\sigma_x},\tag{22}$$

where $\sigma_z\otimes\sigma_x=(|0\rangle_S\langle 0|-|1\rangle_S\langle 1|)\otimes(|0\rangle_D\langle 1|+|1\rangle_D\langle 0|)$.

With an initial quanton path-spin state $|\zeta\rangle_{QS}=(\frac{|0\rangle_Q+|1\rangle_Q}{\sqrt{2}})(\frac{|0\rangle_S+|1\rangle_S}{\sqrt{2}})$ and the maximally entangled detector-ancilla state (12), we find that

$$\rho_{DD'}^{(i)}=\frac{1}{2}\begin{pmatrix}\cos^2\left(\frac{\theta_i}{2}\right)&0&0&\cos^2\left(\frac{\theta_i}{2}\right)\\0&\sin^2\left(\frac{\theta_i}{2}\right)&\sin^2\left(\frac{\theta_i}{2}\right)&0\\0&\sin^2\left(\frac{\theta_i}{2}\right)&\sin^2\left(\frac{\theta_i}{2}\right)&0\\\cos^2\left(\frac{\theta_i}{2}\right)&0&0&\cos^2\left(\frac{\theta_i}{2}\right)\end{pmatrix}\tag{23}$$

are the detector-ancilla final states, corresponding to the quanton paths $i=0,1$. Discrimination of these detector-ancilla states results in $\mathcal{D}_E = \frac{1}{2}||\rho^{(0)}_{DD'} - \rho^{(1)}_{DD'}|| = |\cos^2(\frac{\theta_0}{2}) - \cos^2(\frac{\theta_1}{2})|$. The extended fringe visibility is found to be $\mathcal{V}_E = |\cos(\frac{\theta_0}{2})\cos(\frac{\theta_1}{2})| + |\sin(\frac{\theta_0}{2})\sin(\frac{\theta_1}{2})|$. In this example, too, the detector states $\rho^{(i)}_D = \mathrm{Tr}_{D'}[\rho^{(i)}_{DD'}] = I_D/2$ after the interaction are indistinguishable and they are incapable of retrieving the which-way information.

We find that the generalized visibility reduces to the extended visibility in this example also. In order to see this, we first identify the operation of the spin interaction channel Λ_{01}:

$$\Lambda_{01}(|0\rangle_S\langle 0|) = \cos\left(\frac{\theta_0 - \theta_1}{2}\right)|0\rangle_S\langle 0|,$$

$$\Lambda_{01}(|0\rangle_S\langle 1|) = \cos\left(\frac{\theta_0 + \theta_1}{2}\right)|0\rangle_S\langle 1|,$$

$$\Lambda_{01}(|1\rangle_S\langle 0|) = \cos\left(\frac{\theta_0 + \theta_1}{2}\right)|1\rangle_S\langle 0|,$$

$$\Lambda_{01}(|1\rangle_S\langle 1|) = \cos\left(\frac{\theta_0 - \theta_1}{2}\right)|1\rangle_S\langle 1|.$$

After simplification of (6), we obtain the generalized visibility $\mathcal{V}_G = |\cos(\frac{\theta_0}{2})\cos(\frac{\theta_1}{2})| + |\sin(\frac{\theta_0}{2})\sin(\frac{\theta_1}{2})|$, which agrees perfectly with the extended visibility.

It is pertinent to point out here that in both the first and the third examples the trade-off between the which-way information and the visibilities turn out to be identical [which is evident by the parametrization $|a_{00}| = |\cos(\frac{\theta_0}{2})|$ and $|a_{11}| = |\cos(\frac{\theta_1}{2})|$]. While in the first example, the trade-off is realized for different initial spin preparations (by varying the initial spin parameters $|a_{00}|,|a_{11}|$), analogous trade-off is brought out by varying the parameters of the interaction channel.

IV. CONCLUSIONS

In a recent work on the interference of a quanton in a two-path interferometer, Banaszek *et al.* [5] showed that control over the leakage of which-way information can be achieved by appropriate initial preparation of the internal spin state of the quanton. In this article, we extended this analysis and showed that, with the help of an entangled detector-ancilla system, the amount of which-way information could get enhanced beyond that discerned solely from the detector. Our analysis generalizes the trade-off relation between the which-way distinguishability and the fringe visibility, when the detector is equipped with an ancillary degree of freedom. We considered three different examples of interaction between the quanton and the detector to demonstrate that the extended which-way distinguishability $\mathcal{D}_E$ can assume nonzero values even when the distinguishability $\mathcal{D}$ inferred only by the detector vanishes. In the meanwhile, we also find that the extended visibility $\mathcal{V}_E$ and the generalized visibility [5] $\mathcal{V}_G$ agree identically with each other in these examples. These illustrative examples analyzed here reveal that there are instances where the detector fails to gain any information on which-way distinguishability, but the corresponding generalized visibility does not attain its maximum value 1. In fact, variation of the generalized visibility $0 \leqslant \mathcal{V}_G \leqslant 1$—even when the which-way information transferred to the detector is completely erased—draws striking attention. These examples indeed bring forth the need for our extended analysis to explore how leakage of which-way information is captured by the entangled detector-ancilla system, but not by the detector state alone. However, the agreement of the extended visibility with generalized visibility in these specific examples appears to be coincidental. It would be of interest to investigate how both these fringe visibilities are related to each other in general. Furthermore, we leave open the question, "is it possible to represent the extended fringe visibility $\mathcal{V}_E$ in terms of quantities that could be controlled at the stage of preparation of quanton internal spin state, so that one can prevent the which-way information leakout to the combined detector-ancilla system?," for further exploration.

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