Estimating the anti-symmetric and symmetric part of the circular polarization in pulsar profiles using simple modeling

A REPORT SUBMITTED FOR THE

Summer Research Fellowship Programme

TO THE

Indian Academy of Sciences

by

Prajwal V P

Application No. : PHYS1109-I

UNDER THE SUPERVISION OF

Prof. Avinash A. Deshpande

Raman Research Institute , Bangalore

May-July 2014

Acknowledgements

I would like to thank the Indian Academy of Sciences for providing this opportunity to work at Raman Research Institute through the Summer Research Fellowship Programme. I would like to express my sincerest gratitude to Dr. Avinash Deshpande for his motivation, patience and constant support in explaining key concepts throughout the programme. Special thanks to Ankit Singh for providing me with crucial insight into this project.I would also like to thank the supporting staff for providing resources throughout the tenure.

Abstract

This report provides an overview of the goals, background, and important concepts of the project. It describes the aim of the project and the concepts involved to achieve the necessary tasks.

This project deals with the analysis of the circular polarization in pulsar profiles. The aim of the project is to separate the symmetric and anti-symmetric components from the pulsar profile. This involves some simple modeling of symmetric and anti-symmetric profiles.

Initially the Stokes Parameters I and V are studied and analyzed with actual data collected from the European Pulsar Network (EPN) in order to observe the trend in percentage circular polarization in pulsars.

Some simple integral measures of Stokes V is exploited to find a means of representing the degree of symmetric and anti-symmetric nature in the pulsar profile. Having obtained the components, the anti-symmetric component is analyzed for dependence with the frequency of radio emission of the pulsar.

Based on several past and ongoing studies , scientists believe that the anti-symmetric component in a pulsar profile is an intrinsic property of the pulsar radiation mechanism itself and not influenced by propagation effects in the interstellar medium. This analysis indirectly helps in studying the anti-symmetric component with the height of radio emission from the surface of the pulsar . This is indirectly estimated by using different frequencies of radio emission.

MATLAB has been used to carry out the project which is a high-level technical computing language and is useful for data visualization and analysis. The data used for analysis has been obtained from a standard pulsar database known as the European Pulsar Network (EPN).

Contents

1 Introduction

1.1 A brief introduction to pulsars

Pulsars are celestial bodies described as highly magnetized fast rotating neutron stars. They emit light through two cones corresponding to the two poles of the magnetic dipole. If this beam of the pulsar sweeps over the Earth as the pulsar rotates, periodic pulses are obtained with a frequency corresponding to the frequency of rotation. The first pulsar was discovered in 1967 by Jocelyn Bell and Anthony Hewish at the Cambridge Radio Astronomy observatory. Since then, many pulsars have been discovered and their properties still studied. The precise periods of pulsars make them useful tools. Observations of a pulsar in a binary neutron star system were used to indirectly confirm the existence of gravitational radiation and certain types of pulsars rival even atomic clocks in their accuracy in keeping time.

1.2 Formation of pulsars

Cores of stars, towards the end of their lifetime become highly unstable resulting in a supernova explosion . The outcome of the explosion could either be a black hole(formed by stars with masses greater than 15-20 solar masses), white dwarf(formed by stars with masses less than 8 solar masses) or a neutron star. Neutron stars are created by the collapse of cores of stars more massive than 8 solar masses when they become supernovae. They have an estimated radius between 10 to 16 kilometres, a mass around 1.4 solar masses and a density in the order of 10^{14} times higher that the Suns.

Stars towards the end of their lifetime, collapse due to their own gravity resulting in a supernova explosion since the electron pressure cannot support the gravitational pressure of the collapsing star. At this point of extreme density, relativistic electrons combine with protons in and around the stellar core via a process called **neutronization** (also known as inverse beta decay) resulting in neutrons and electron neutrinos .

During neutronization many protons are converted to neutrons and vast amounts of neutrino energy is released. Neutron degeneracy pressure in a similar way as electron degeneracy pressure halts further core collapse. A neutron star is created at this point; also described as a supernova by-product or stellar corpse. The supernova is powered by neutrino energy released from inverse beta decay The newly created neutron star is extremely small, highly magnetised (magnetic field approximately 10^{12} Gauss) and fast spinning (spin period is generally between 0.25 and 2 seconds) compared to its pre-supernova stellar progenitor. This is a pulsar.

1.3 Coherent emission

High-energy photons produced by curvature radiation interact with the magnetic field and lower-energy photons to produce electron-positron pairs that radiate more high-energy photons. The final results of this cascade process are bunches of charged particles that emit at radio wavelengths.

The extremely high brightness temperatures are explained by coherent radiation. If the emission were to be incoherent , the equivalent blackbody temperature of the pulsar would have to be of the order 10^{23} K which is impossible given that celestial bodies are generally undergo self absorption and regulation beyond 10^{12}K.

The solution to this is by proposing a coherent emission model. The electrons do not radiate incoherently but instead as bunches of N electrons in volumes whose dimensions are less than a wavelength emit in phase as charges Ne. The radiation intensity can be N^2 times brighter than incoherent radiation from the same total number N of electrons. Hence the emission must be coherent.

1.4 Polarization in Pulsars

Polarization in general can be described as elliptical. This can be seen as a mixture of two types of polarization namely:

1. Linear
2. Circular

Unlike many celestial bodies, pulsars show a high degree of polarization in their radiation. This unique property of pulsars is a useful tool in putting forth plausible models for their mechanism of radiation.

From observations made over several years, It has been observed that the average percentage linear polarization is more than the average percentage circular polarization. The high degree of polarization in pulsars have led to one of the widely accepted models which attributes to curvature radiation from ultrarelativistic particles streaming along the field lines of a strong magnetic dipole present inside the pulsar. This rotating dipole is said to induce an electric field on the surface of the pulsar (order of 10^{16} V in MKS units) An electric field of this magnitude has the ability to accelerate charged particles(electrons and positrons) to ultrarelativistic speeds. As the charged particles accelerate away, the magnetic field also influences the particles leading to the charged particles to follow the magnetic field lines. The radiation that reaches us is produced by those charged particles which are accelerated by the electric field formed around the pulsar. There exists a quantity known as the radius of light cylinder i.e. the point beyond which the plasma cannot co-rotate with the pulsar.

Thus the electrons in the polar cap are accelerated to very high energies along the open but curved field lines, where the acceleration resulting from the curvature causes them to emit curvature radiation that is strongly polarized in the plane of curvature. As the radio beam of light sweeps across the line-of-sight, the plane of polarization is observed to rotate by up to 180 degrees, a purely geometrical effect.

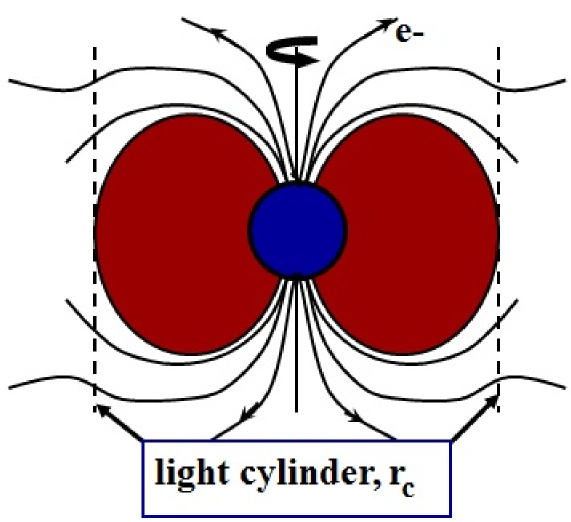


Figure 1: Blue body represents the rotating pulsar. The maroon region represents the co-rotating plasma

2 Stokes Parameters

2.1 Introduction

Polarization (linear or circular) of a beam can in general be visualized as an ellipse which has a mixture of linear and circular polarized light. Under certain unique cases , the beam of light which is analyzed can be totally linearly polarized or totally circularly polarized.

To describe the ellipse totally, one has to specify :

1. Semi major axis
2. Semi minor axis
3. Tilt of the ellipse itself
4. If the path along the ellipse is being traversed in a clockwise or anti-clockwise direction.

An alternate way of representing these properties are what are known as the **Stokes Parameters**. Standard notation of the four Stokes Parameters are given by **I,Q,U,V**(also denoted by I,S_1, S_2, S_3) where: **I** is the intensity of the radiation,**Q** and **U** are measures of the orientation of the ellipse relative to the x axis, and **V** is the measure of the circularity of the polarization (i.e., right or left circularly polarized).

2.2 Poincare Sphere

A useful way of representing the Stokes parameters is through the **Poincare Sphere**

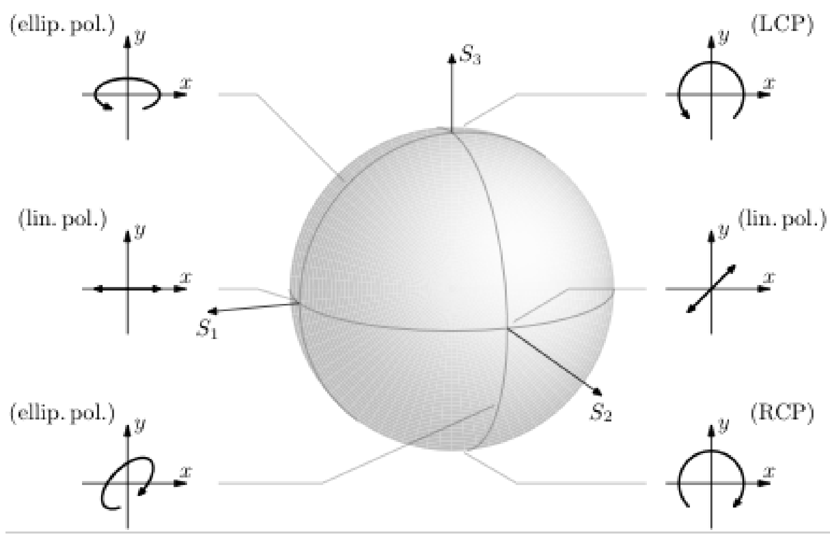


Figure 2: Poincare Sphere with various subcases.

The Poincare sphere (Figure 2) is a construction used to visualize the relationship between the Stokes parameters by using Q, U, and V as orthogonal vectors(S_1, S_2, S_3 in the figure). For example, completely left or right circularly polarized light is described by V of +1 or -1 (depending on your convention), and Q, U of 0. By taking a random point on the sphere which doesn't lie on any particular axis, the polarization is described to be elliptical.

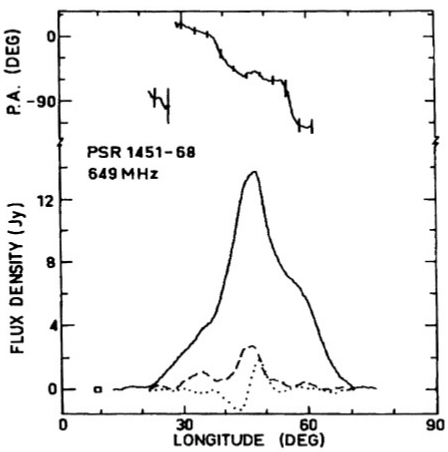
3 Stating the problem

Pulsars are known to show some degree of linear as well as circular polarization. This project deals with the circular polarization seen in pulsars. It has been observed that pulsars show two types of circular polarization itself.

a) A type where there is a change in sense of circular polarization i.e. RCP to LCP (or vice versa) within the pulsar profile.

b) A type where the sense remains same throughout the pulse.

The first type is known as the **anti-symmetric** component and the latter is known as the **symmetric** component.



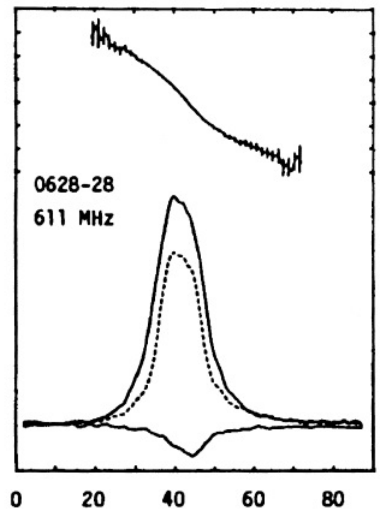


Figure 3(a) : Antisymmetric

Figure 3(b) : Symmetric

Types of circular polarization

A general pulsar profile could be a mixture of both the above mentioned components and the profile for V looks much more complicated.

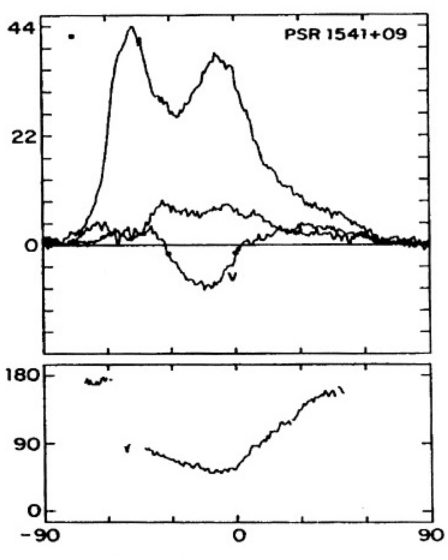


Figure 4: Pulsar profile showing a mixture of symmetric and anti-symmetric components

Pulsar profiles are in general represented by a plot of flux density against the longitude of the pulsar.

The observed diverse circular polarization properties may relate to the the pulsar emission mechanism or to propagation effects occurring in the pulsar magnetosphere (cf. Melrose 1995). In the widely adopted magnetic pole models , it has been a challenge to explain various circular polarization behaviours. Cheng and Ruderman (1979) have suggested that the expected asymmetry between the positively and negatively charged components of the magnetoactive plasma in the far magnetosphere of pulsars will convert linear polarization to circular polarization. On the other hand some have argued that the cyclotron instability, rather than the propagation effect, is responsible for the circular polarization of pulsars. Other authors have argued for its intrinsic origin . In summary, the observed circular polarization of pulsar radiation is not well understood.

This project deals with a perspective of intrinsic origin of the circular polarization. To understand the the properties of pulsars in a more detailed fashion, resolving the circular polarization of a pulsar into its symmetric and anti-symmetric components would be a useful tool since both can separately be studied and analyzed with different parameters associated with a pulsar.

Using some simple integral measures of the Stokes Parameter V, a series of operations has been made in order to get an estimate of the symmetric and anti-symmetric weightage in a pulsar profile for several profiles obtained at different frequencies . A special case has been studied where the frequency of radiation is checked for dependence with the anti-symmetric component.

4 Useful Parameters

Using the data of Stokes' Parameters for different pulsars at different frequencies from the EPN the following parameters have already been calculated previously. These parameters will play a crucial role in the aim to decompose the mixture anti-symmetric and symmetric components.

1. *s pulse* : Refers to the number of points in pulse region which is dependent on the bin size . Represented as N. (Not given in the tables)

2. *mean ipulse* : Refers to the sum of all I values calculated along the pulse profiles in certain intervals(bins) which are specified by rate of sampling the pulse during the observation. Represented as $\frac{1}{N}\sum_{i=1}^{N} I$

3. *mean v* : Refers to the sum of all V values calculated along the pulse profiles in certain intervals(bins) which are specified by rate of sampling the pulse during the observation. Represented as $\frac{1}{N}\sum_{i=1}^{N} V$

4. *mean modv* : Refers to the sum of all V values after taking absolute value of each V value calculated along the pulse profiles in certain intervals(bins) which are specified by rate of sampling the pulse during the observation.Represented as$\frac{1}{N}\sum_{i=1}^{N} |V|$

5. *sigma offpulse* : Refers to the standard deviation from the off pulse mean in the pulsar profile. Represented as σ. (Not given in the tables)

5 Histogram plots for percentage circular polarization

Data taken from the EPN database is for pulsars emitting in 5 different frequencies namely 400 MHz,600 MHz,1400 MHz,1600 MHz and 4000 MHz. The ratio of *sum v* and*sum ipulse* is taken. This yields a measure of percentage circular polarization in the pulsar. This ratio is calculated for all pulsars in the five different frequencies. A histogram is plotted of this measure. This just gives an idea of what amount of circular polarization is shown by pulsars in general.

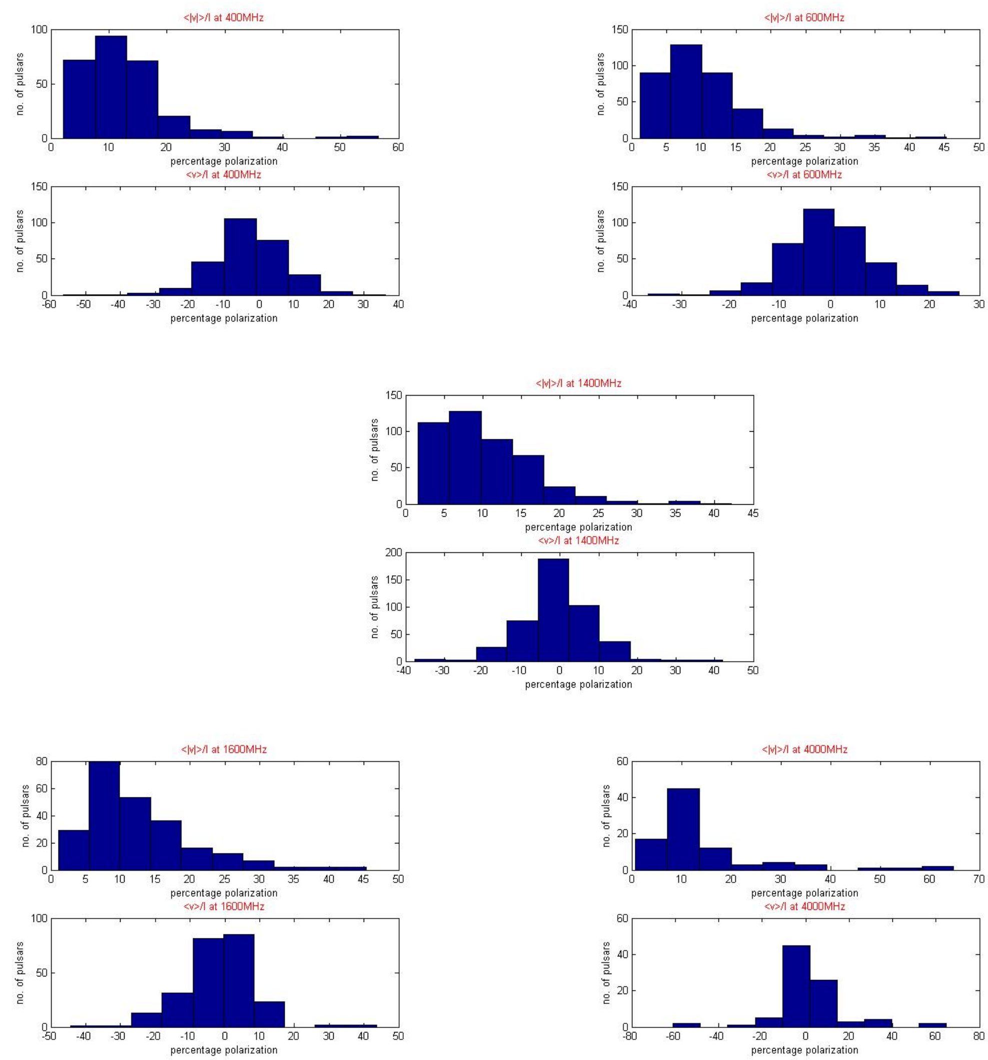


Figure 5: Histogram plots for 5 different frequencies.

It is easy to see that independent of the frequency of emission , most of the pulsars show about 10-20 % circular polarization.

6 A simple model to decompose the components

6.1 Model

The symmetric component which doesn't change sense can in general be represented in the form of a Gaussian curve whereas the anti-symmetric component can be represented a simple sinusoidal curve .

Another interesting way of representing the anti-symmetric component would be to modulate a Gaussian over the sinusoid. It would still yield a shape that changes has some degree of anti-symmetric properties.

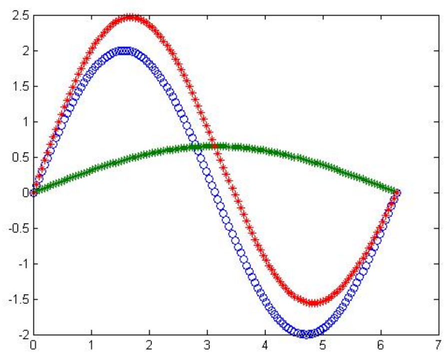


Figure 6: The curve in red represents the modulation of a Gaussian(green) with a sinusoid(blue)

The curves representing the components have a Gaussian embedded in them . On taking a ratio of the symmetric and anti-symmetric components, the Gaussian component would cancel out leading to a model where a sinusoid is superposed over a shift from the X axis. A measure $\mathbf{V_c}$ has been used to represent the shift above the X axis. Similarly $\mathbf{V_a}$ would represent the amplitude of the sinusoidal counterpart representing the anti-symmetric component.

From the parameters already given at hand , the aim is to be able to estimate the values of $\mathbf{V_c}$ and $\mathbf{V_a}$.

For this purpose , a ratio of *mean v* and *mean modv* is taken.

To get an idea of how the ratio would behave with varying $\mathbf{V_c}$ and $\mathbf{V_a}$ a curve has been simulated where a set of points has been taken from a curve formed by adding $\mathbf{V_c}$ (dc shift) and $\mathbf{V_a}$. This would give a sinusoid which is shifted either up or down.

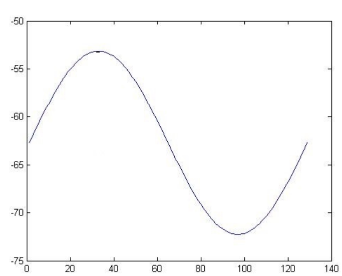


Figure 7: A sinusoid which is shifted down

Using the values of these set of points , 2 quantities are calculated namely :

- *mean v* : the values are added up
- *mean modv* : the modulus of each is taken before the values are added up.

On taking the modulus of the ratio of *mean v* and *mean modv* and plotting it against the ratio of $\mathbf{V_c}$ and $\mathbf{V_a}$ the following plot is obtained. The quantity $\mathbf{V_a}$ has been kept fixed and $\mathbf{V_c}$ is varied to obtain the following plot :

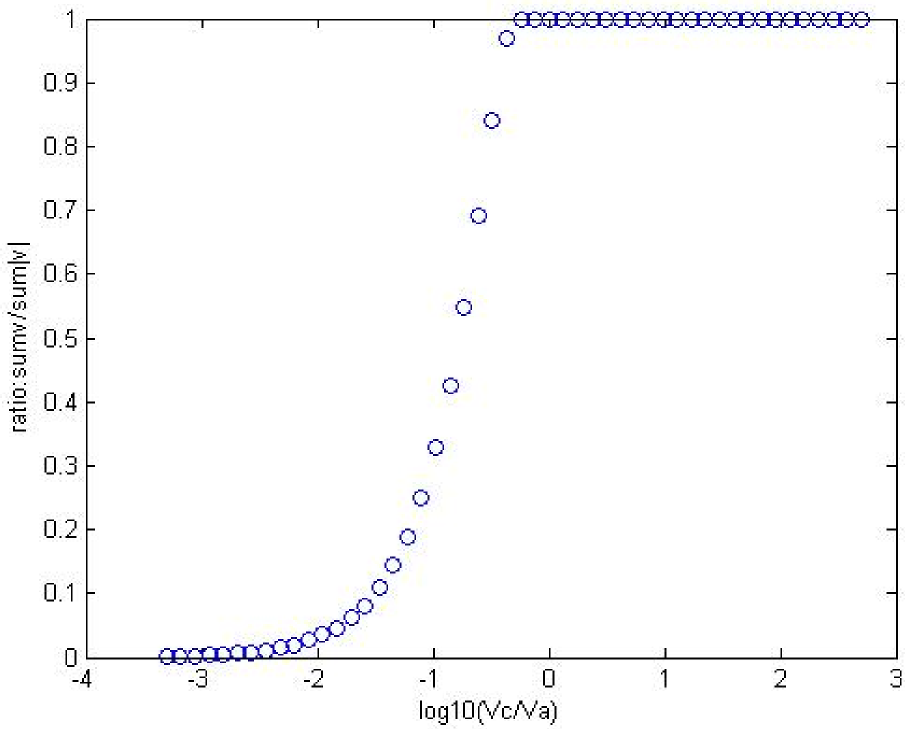


Figure 8: Plot of ratio of *sumv* and *sum modv* against ratio of $\mathbf{V_c}$ and $\mathbf{V_a}$. The log10 represents log_{10} of the ratio which helps giving a broader range of values.

On observing the above figure , it can be concluded that the ratio is a good measure to use since it is finite .But it saturates eventually on both sides due to excess of one parameter which is a drawback. For very low $\mathbf{V_c}$, the ratio saturates to zero and for a huge value of $\mathbf{V_c}$ compared to $\mathbf{V_a}$, the graph saturates to 1.

The region in between can be exploited. Here the ratio would lie between 0 and 1 . From the already existing data , ratio of *mean v* and *mean modv* for various pulsars can be calculated. And this can be used to read off the values of the corresponding $\mathbf{V_c}$ and $\mathbf{V_a}$ values. This idea can be explained using the following equation .

$$\frac{1}{N}\sum_{i=1}^{N} V \quad = \quad \mathbf{V_c} \tag{1}$$

where N is nothing but *spulse*. Data for the y axis values are already known. Using these values , the corresponding x-axis values can be calcuated from the graph shown in Figure 8.

Using equation (1), $\mathbf{V_a}$ itself can be calculated.

6.2 Advantage of this model

The symmetric component need not be calculated explicitly since *sum v* itself is a good measure of this component.

6.3 Disdvantage of this model

Given that the ratio saturates to 1, there could be many pulsars which could yield a ratio 1 since their respective *mean v* and *mean modv* values are the same. In this case , there is no unique x-axis value for the ratio being unity. Thus the anti-symmetric component cannot be uniquely determined.

6.4 Need for another parameter

From the disadvantage of this model as stated above, it is necessary to use an improvised model. From calculations , some pulsars have yielded a ratio of unity and thus the model cannot help in resolving the anti-symmetric component for these pulsars.

Clearly, another parameter needs to be used which could remove this problem. Hence a new parameter called *sum vsquared* has been calculated. This is represented as $\sum_{i=1}^{N} V^2$.

7 Error Analysis

An important part of this modelling is also specifying the uncertainty in measuring the components. All the quantities used up in finding the components, will have some level of uncertainty. This uncertainty gets carried on as the components are calculated.

It is important to calculate the uncertainty since it gives a window or a range of values within which the actual value should lie.

7.1 Estimation of standard deviation associated with uncertainty

A given pulsar profile is never totally smooth. There is always some amount of noise that creeps into it. This could be due to various reasons like dispersion from the interstellar medium , faulty equipment etc. In order to account for this noise , standard deviation gives a good measure of the amount of fluctuations shown by the profile from the mean pulse.

In order to estimate the error in calculating a quantity like Stokes' Parameter V an assumption needs to be made. The off pulse shift of V is much greater than the maximum height attained by V within the pulse which means that the system noise is much greater than the noise in the pulse itself. Thus *s*igma off pulse itself is a good measure to calculate the error.

Given that the error needs to be calculated for N points. The error propagates as

$$\sigma_{sum} = \sigma_{off}\sqrt{N} \tag{2}$$

This quantity can be used to calculate error in *mean v* and *mean modv* But what is needed is the uncertainty in calculating the ratio of the above 2 quantities. This can be solved for mathematically.

Let the ratio between *mean v* and *mean modv* be denoted as R. Of the N points there could be some points which have a positive value and the rest have negative values. Let these be denoted as N_+ and N_- respectively.

For an equation of the form

$$z = x/y$$

The error in z denoted as Δz can be given solved with the following equation

$$\Delta z/z = \Delta x/x + \Delta y/y$$

A general way of representing the square of the error would be

$$< |dz| >^2 = < (\tfrac{\partial z}{\partial x})^2 dx^2 > + < (\tfrac{\partial z}{\partial y})^2 dy^2 > + \tfrac{\partial z}{\partial x}\tfrac{\partial z}{\partial y} dxdy$$

The cross term in the above equation would cancel out if x and y are mutually exclusive.

In the given scenario , the points which have positive values are totally correlated while calculating *mean v* and *mean modv*. The points which are negative are totally anti-correlated . Hence the mathematics behind calculating error for the ratio can be represented int the following way .

It is already known that

$$\mathrm{R} = \frac{sumv}{summodv} \tag{3}$$

Also

$$sum\ v = \sum_{i=1}^{N_+} sumv_+ + \sum_{i=1}^{N_-} sumv_- \quad (4)$$

$$sum\ modv = \sum_{i=1}^{N_+} sumv_+ - \sum_{i=1}^{N_-} sumv_- \quad (5)$$

Thus error in R which is dR is given by

$$dR = \frac{\partial R}{\partial a} da + \frac{\partial R}{\partial b} db \quad (6)$$

where

$a = sumv_+$
$b = sumv_-$

On solving further and using the fact that

$$\sigma_R = \sqrt{< dR^2 >} \quad (7)$$

It can be shown that

$$\sigma_R = 2\frac{\sigma_{off}}{summodv^2}(a^2N_+ + b^2N_-) \quad (8)$$

8 Improved model

The only change from the previous model is that *mean modv* is replaced by square root of *sum vsquared* divided by N points. The disadvantage of the previous model is removed.

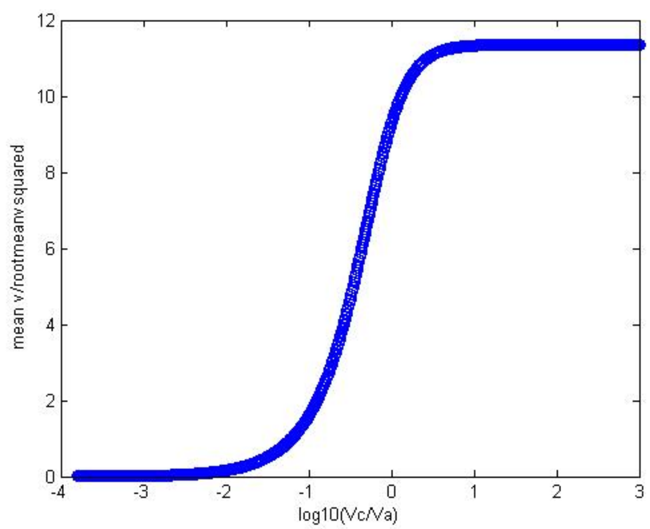


Figure 9 : Plot of ratio of *mean v* and square root of *mean vsquared* against ratio of $\mathbf{V_c}$ and $\mathbf{V_a}$.

In this case, the ratio shows an asymptotic behaviour where it tends to a certain value unlike the previous ratio which actually saturates to a particular value. Using the actual pulsar data this ratio has been calculated for the various pulsars and no value lies above the limiting value.

The procedure to find the anti-symmetric component remains the same as the previous model.

9 Conclusion and Future Prospects

- The anti-symmetric component has been uniquely analyzed for dependence on frequency. Of the 75 pulsars, which have been analyzed , 26 of them have shown a trend of the percentage of anti-symmetric circular polarization decreasing as frequency is increased.
- The anomalies are mainly seen between 1400MHz and 1600MHz. Since the frequencies are really close to each other , it isn't suitable to compare the anti-symmetric values at an absolute scale . The amount by which the component changes as a measure of factoring frequency is a useful measure.
- Based on the data , modelling of how exactly the anti-symmetric component varies with frequency can be done.
- Since the mixture has been decomposed into the necessary components, some parameters other than frequency like the angle between magnetic axis and the line of sight which is denoted as β.
- Based on the improved model , a much more accurate measure of the anti-symmetric component can be calculated. A corresponding error analysis would also follow.
- In the both the models, statistical bias needs to be removed from the *modv* and *vsquared* parameters, in order to neglect the shift of the entire profile due to system defects.

10 References

1. 'Toward an empirical theory of Pulsar emission V. On the Circular Polarization in Pulsar radiation' : Radhakrishnan and Rankin,1989

2. 'Circular polarization in pulsar integrated profiles' : J. L. Han, R. N. Manchester, R. X. Xu and G. J. Qiao 1998

3. 'Circular Polarization in Pulsars due to curvature radiation.' : R T Gangadhara

4. 'A theory of subpulse polarization patterns from radio pulsars ' : Cheng and Ruderman 1979

5. `www.astronomyonline.org`

11 Tables