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Capturing Statistical Isotropy Violation with Generalized Isotropic Angular Correlation Functions of Cosmic Microwave Background Anisotropy

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Abstract

The exquisitely measured maps of fluctuations in the cosmic microwave background (CMB) present the possibility of systematically testing the principle of statistical isotropy of the Universe. A systematic approach based on strong mathematical formulation allows any nonstatistical isotropic (nSI) feature to be traced to the nature of physical effects or observational artifacts. Bipolar spherical harmonics (BipoSH) representation has emerged as an overarching general formalism for quantifying the departures from statistical isotropy for a field on a 2D sphere. We adopt a little-known reduction of the BipoSH functions, dubbed minimal harmonics in the original paper by Manakov et al. We demonstrate that this reduction technique of BipoSH leads to a new generalized set of isotropic angular correlation functions referred to here as minimal BipoSH functions that are observable quantifications of nSI features in a sky map. This paper presents a novel observable quantification of deviation from statistical isotropy in terms of generalized angular correlation functions that are compact and complementary to the BipoSH spectra that generalize the angular power spectrum of CMB fluctuations.

Unified Astronomy Thesaurus concepts: Two-point correlation function (1951); Cosmic microwave background radiation (322)

1. Introduction

The cosmic microwave background (CMB) anisotropy measurements by the WMAP (Hinshaw et al. 2009) and Planck (Aghanim et al. 2020) space missions have ushered in the precision era of cosmology, enabling cosmologists to pose queries beyond the statistically isotropic two-point correlation function predicated by the fundamental assumption of homogeneity and isotropy widely referred to as the cosmological principle. Current observations are in good agreement with CMB temperature anisotropies being Gaussian (Aghanim et al. 2020). In such a case, all the information encoded in the CMB temperature field can be specified by a two-point correlation function. However, the WMAP and Planck collaboration data release claimed intriguing hints of deviations from statistical isotropy in CMB maps beyond known effects. Bipolar spherical harmonics (BipoSH) provide an elegant and general formalism for the two-point correlation function between two directions $\hat{n}_1$ and $\hat{n}_2$ for a random field on a 2-sphere. In this formalism, the statistical isotropy component corresponds to the $L=0$ element and $L>0$ represents the nonstatistical isotropic (nSI) feature in the BipoSH basis (Hajian & Souradeep 2003). In this paper, we extend the BipoSH formalism to a new θ-dependent ($\cos\theta = \hat{n}_1.\hat{n}_2$) irreducible representation that is applicable in real (angular) space instead of a harmonic basis.

Departures from statistical isotropy can have their roots in known or yet-to-be-discovered physical effects, as well as observational artifacts. Some well-known effects include Doppler boost, weak lensing of CMB photons by large-scale structure, and systematics, such as noncircular beam, which have been extensively studied in the BipoSH representation (Mitra et al. 2004; Joshi et al. 2010; Mukherjee et al. 2014; Kumar et al. 2015). With upcoming missions that offer even greater precision, novel studies of violations of statistical isotropy using generalized mathematical constructs could have far-reaching implications in cosmology.

2. BipoSH Formalism

BipoSH representation provides a general formalism for quantifying the departure from the statistical isotropy of the CMB temperature field. BipoSH functions form a complete and orthonormal basis in $S^2 \times S^2$ and thus have bidirectional dependence. The most general two-point correlation function for a field defined on the sphere can be obtained in terms of a BipoSH basis as

$$C(\hat{n}_1, \hat{n}_2) = \sum_{L,M,l_1,l_2} A^{\mathrm{LM}}_{l_1,l_2} \{Y_{l_1}(\hat{n}_1) \otimes Y_{l_2}(\hat{n}_2)\}_{\mathrm{LM}}, \tag{1}$$

where $A^{\mathrm{LM}}_{l_1,l_2}$ are BipoSH coefficients and $\{Y_{l_1}(\hat{n}_1) \otimes Y_{l_2}(\hat{n}_2)\}_{\mathrm{LM}}$ are BipoSH basis functions. BipoSH functions are a tensor product of two spherical harmonics (SH) functions that can be expanded as

$$Y^{l_1,l_2}_{\mathrm{LM}}(\hat{n}_1, \hat{n}_2) = \{Y_{l_1}(\hat{n}_1) \otimes Y_{l_2}(\hat{n}_2)\}_{\mathrm{LM}} = \sum_{m_1 m_2} C^{\mathrm{LM}}_{l_1,m_1,l_2,-m_2} Y_{l_1,m_1}(\hat{n}_1) Y_{l_2,m_2}(\hat{n}_2), \tag{2}$$

where $C^{\mathrm{LM}}_{l_1 m_1 l_2 - m_2}$ are Clebsch–Gordon coefficients (Varshalovich et al. 1988). The indices of Clebsch–Gordon coefficients satisfy the triangularity conditions as $|l_1 - l_2| \leqslant L \leqslant l_1 + l_2$ and $m_1 + m_2 = M$.

BipoSH coefficients are the natural generalization of the CMB angular power spectrum. The $L=0$ BipoSH coefficient gives the isotropic part, and the coefficients at higher L values represent the strength of the nSI effect in the CMB sky at the corresponding bipolar multipole. BipoSH coefficients carry crucial signatures of statistical isotropy violation, whose description necessarily invokes direction-dependent statistics of the CMB sky. Since the two-point correlation function is a real measurable, BipoSH is widely used to characterize different known sources of nSI effects and systematically probe for nSI from the CMB maps.

3. Reduction Technique for Bipolar Harmonics

In this section, we present a mathematical construct for the reduction of the BipoSH basis along the lines studied by Manakov et al. (1996). In the BipoSH basis, the rank L has values from 0, 1, 2, 3 ..., and the internal ranks l, l' run over all values from 0 to infinity for a given rank L, constrained by the triangularity relations obeyed by the Clebsch–Gordon coefficients. In other words, the information at a given bipolar multipole L could be spread over the entire or a large angular spectral range l. We show that the reduction to minimal bipolar spherical harmonics (mBipoSH) limits the spectral spread to L isotropic angular correlation functions with θ-dependence ($\cos\theta \equiv \hat{n}_1 \cdot \hat{n}_2$).

This above reduction follows from the detailed proof in Manakov et al. (1996), that any irreducible tensor of rank L can be constructed using L vectors of its arguments. Any BipoSH with any possible internal rank $l + l'$ can be constructed using a combination of L mBipoSH basis functions (originally referred to as minimal harmonics in Manakov et al. 1996) as

$$\mathcal{Y}^{k}_{\mathrm{LM}}(\hat{n}_1, \hat{n}_2) = Y^{L-k,k}_{\mathrm{LM}}(\hat{n}_1, \hat{n}_2), \text{ where } k = 0, 1 \,....., L. \quad (3)$$

The above relation reduces our analysis to only a few internal ranks up to L. The tensor with rank $l + l' \geqslant L$ can be written from L mBipoSH basis functions and its coefficients depending upon l, l' and $\theta = \cos^{-1}(\hat{n}_1.\hat{n}_2)$. The mathematical representation of the mBipoSH basis functions can be written as

$$Y^{l_1 l_2}_{\mathrm{LM}}(\hat{n}_1, \hat{n}_2) = \sum_{\lambda=\lambda_p}^{L} a_\lambda(l_1, l_2, L, \cos\theta)\, Y^{\lambda, L+\lambda_p-\lambda}_{\mathrm{LM}}(\hat{n}_1, \hat{n}_2), \quad (4)$$

where

$$\lambda_p = \begin{cases} 0 & \text{for even } (l_1 + l_2 - L) \\ 1 & \text{for odd } (l_1 + l_2 - L). \end{cases} \quad (5)$$

The parameter λ_p describes the space inversion property that carries information about the parity of BipoSH functions, as discussed in Book et al. (2012). It specifies that the BipoSH function $Y^{l_1 l_2}_{\mathrm{LM}}$ is a tensor for even parity and a pseudo tensor for odd parity. Further, the coefficients a_λ follow the symmetry relation given as

$$a_\lambda(l_1, l_2, L, \cos\theta) = a_{L-\lambda_p-\lambda}(l_2, l_1, L, \cos\theta). \quad (6)$$

The set of mBipoSH basis functions, after the above transformation, also forms a complete basis for BipoSH. The coefficients a_λ are functions of the angle θ between two directions, and the BipoSH functions with dependence on higher L values can be constructed using these coefficients with a finite set of mBipoSH basis functions. With the completeness property of BipoSH functions, the final expression for a_λ can be written as

$$a^{\lambda_p}_{\lambda}(l_1, l_2, L, \cos\theta) = i^{\lambda_p}(-1)^{l_2}\left[\frac{2^L(2l_1+1)(2l_2+1)(2L+1)!(l_1+l_2-L)!\lambda!(L-\lambda-\lambda_p)!}{q!(L-l_1+l_2)!(L-l_2+l_1)!(2\lambda+1)!!(2L-2\lambda-2\lambda_p+1)!!}\right]$$
$$* \sum_{t=0}^{t_{max}}(-1)^{\lambda+t}\begin{pmatrix}\frac{(j-l_1)}{2}\\ t\end{pmatrix}\begin{pmatrix}L-\lambda_p-t\\ \lambda\end{pmatrix}\frac{(q-\lambda_p)!!}{(q-\lambda_p-2t)!!}P^{(L-t)}_{j-t-\lambda}(\cos\theta), \quad (7)$$

where $q = l_1 + l_2 + L + 1$, $j = L + l_2 - \lambda_p$, $t_{\max} = \min\left[L - \lambda_p - \lambda, \frac{j-l_1}{2}\right]$, $\begin{pmatrix}m\\ n\end{pmatrix}$ denotes the binomial coefficient, and $P^{(m)}_n(x)$ is the mth derivative of the Legendre polynomial $P_n(x)$. As a_λ coefficients follow the symmetry relation in Equation (6), Equation (7) is only referred to for coefficients having $\lambda \geqslant \frac{L}{2}$ to construct the mBipoSH basis functions, and the remaining coefficients can be computed using the symmetry relation. This provides us with a complete set of a_λ coefficients for the reduction of bipolar harmonics with any rank L. Further analysis of the aforementioned set of mBipoSH basis functions reveals that the tensor $Y^{l_1 l_2}_{\mathrm{LM}}$ has $L + 1 - \lambda_p$ different basis components. Using this reduction mechanism for bipolar harmonics, we can construct equivalent compact and complementary θ-dependent mBipoSH basis functions for any given value of multipole L.

4. Application to CMB Sky Maps

By employing the mathematical reduction of the bipolar harmonics basis described in the previous section, we construct a representation of the nSI CMB temperature anisotropy sky maps, $\Delta T(\hat{n})$, in terms of a set of angular correlation functions[5]. The most general two-point correlation function $C(\hat{n}_1, \hat{n}_2) \equiv \langle \Delta T(\hat{n}_1)\Delta T(\hat{n}_2)\rangle$ can be expanded using Equations (1) and (4) in the form of reduced minimal BipoSH basis functions as follows

$$C(\hat{n}_1, \hat{n}_2) = \sum_{L,M,l_1,l_2} A^{\mathrm{LM}}_{l_1,l_2}\sum_{\lambda=\lambda_p}^{L} a_\lambda(l_1, l_2, L, \cos\theta)\, Y^{\lambda, L+\lambda_p-\lambda}_{\mathrm{LM}}(\hat{n}_1, \hat{n}_2)$$
$$= \sum_{L,M}\sum_{\lambda=\lambda_p}^{L}\left[\sum_{l_1,l_2} A^{\mathrm{LM}}_{l_1,l_2} a_\lambda(l_1, l_2, L, \cos\theta)\right] Y^{\lambda, L+\lambda_p-\lambda}_{\mathrm{LM}}(\hat{n}_1, \hat{n}_2). \quad (8)$$

For the nSI effect at a specific multipole $L > 0$, the above Equation (8) reduces the representation of correlation function

[5] This can be readily recast for CMB polarization, weak lensing, and other cosmological random sky maps.

to BipoSH basis functions up to L. (Typically for most nSI effects, the bipolar spectral range in L is more compact than that in the angular spectral range of l). We refer to the expression inside the square bracket as mBipoSH angular correlations functions:

$$\alpha_{\lambda}^{L,M}(\cos\theta) = \sum_{l_1 l_2} A_{l_1,l_2}^{\mathrm{LM}} a_{\lambda}(l_1, l_2, L, \cos\theta). \tag{9}$$

For $L = 0$, the mBipoSH is the isotropic two-point correlation function $C(\hat{n}_1.\hat{n}_2) \equiv C(\theta)$ that has been extensively studied and measured in cosmology literature within the context of the homogeneous and isotropic cosmological model (Copi et al. 2010). It is noted that the nSI effects at multipole L with projection M are characterized by $L+1$ different angular correlation functions in the case of even parity $\lambda_p = 0$ and L correlation functions in the case of odd parity $\lambda_p = 1$.

Further, BipoSH coefficients can be expressed in terms of the harmonic space covariance matrix of CMB maps (Hajian & Souradeep 2003), leading to the following expression for the mBipoSH angular correlations functions,

$$\alpha_{\lambda}^{L,M}(\cos\theta) = \sum_{l_1 l_2}\sum_{m_1 m_2} \langle a_{l_1 m_1} a_{l_2 m_2}^{*}\rangle (-1)^{m_2} C_{l_1,m_1,l_2,-m_2}^{\mathrm{LM}} a_{\lambda}(l_1, l_2, L, \cos\theta). \tag{10}$$

This equation is expressed in terms of harmonic coefficients a_{lm} that are directly measurable from a CMB temperature fluctuation map. The above relations emphasize that mBipoSH angular correlations functions are a set of different θ-dependent correlation functions in the real space defined for a specific nSI feature in a CMB map.

Further, it is interesting to note this mathematical structure opens a new avenue toward a visual representation of statistical isotropic violations in a set of sky maps. The mBipoSH angular correlation functions can be summed over to construct the θ-dependent angular correlation function at each point on the 2D sphere.

$$\zeta_{\lambda}(\hat{n}_1, \cos\theta) = \sum_{\mathrm{LM}} \alpha_{\lambda}^{\mathrm{LM}}(\cos\theta) Y_{\mathrm{LM}}(\hat{n}_1). \tag{11}$$

These defined functions form a basis in $\mathbf{S}^2 \times \mathbf{S}^1$ and can minimally represent the underlying pattern in the nSI CMB map.

To summarize, in this paper, we present an extended set of θ-dependent mBipoSH correlation functions that can be employed to study nSI CMB maps and to completely capture deviations from statistical isotropy that have not been previously studied using correlation functions. This mBipoSH representation provides a complimentary compact avenue to study nSI CMB maps using higher bipolar multipole angular correlation functions.

5. Illustrative Example: Doppler Boost

Our motion today with respect to the cosmic rest frame causes a dipole anisotropy in the CMB temperature and polarization fields, with an inferred velocity ($\beta \equiv |\boldsymbol{v}|/c = 1.23 \times 10^{-3}$). The Doppler boost of the CMB sky in this moving observer frame leads to observable nSI features in the CMB due to well-known relativistic effects of modulation and aberration of the CMB temperature field. The Doppler boost nSI effect has been reliably measured by Planck collaboration (Planck Collaboration et al. 2014).

In the BipoSH representation, this effect induces a nonzero even parity dipolar ($L = 1$) BipoSH coefficient in the CMB map (Mukherjee et al. 2014). The Planck collaboration has employed the BipoSH formalism to measure this effect (Ade et al. 2016) using a quadratic estimator. More recently, a fully Bayesian approach utilizing publicly available Planck data has provided a 5σ confirmation of this nSI effect (Saha et al. 2021).

We illustrate our new representation of this well-understood and quantified case of statistical isotropy violation in the CMB map. We explicitly derive the expressions to compute the two nonzero mBipoSH angular correlations functions expected in a Doppler-boosted CMB map for this even parity effect at $L = 1$. We also outline the derivation of an appropriate estimator for these mBipoSH angular correlation functions from a CMB map.

The nSI signature of the Doppler boost is captured by the BipoSH coefficients

$$\tilde{A}_{l,l+1|TT}^{1M} = \beta^{1M} D_l^{TT} \frac{\Pi_{l,l+1}}{\Pi_1} C_{l,0,l+1,0}^{10}, \tag{12}$$

where BipoSH spectra are defined as

$$D_l^{TT} = \frac{1}{\sqrt{4\pi}}[(l + b_\nu)C_l^{TT} - (l + 2 - b_\nu)C_{l+1}^{TT}], \tag{13}$$

where $\boldsymbol{\beta}$ refers to the boost velocity vector and b_ν captures the frequency dependence of the Doppler boost given by

$$b_\nu = \frac{\nu}{\nu_0}\coth\left(\frac{\nu}{2\nu_0}\right) - 1, \tag{14}$$

and the local velocity β_{1M} defined in the harmonic basis as

$$\beta_{1M} = \int \boldsymbol{\beta}.\hat{n} Y_{1M}^{*}(\hat{n}) d\hat{n} \tag{15}$$

with the notation $\Pi_{l_1 l_2 \ldots l_n} = \sqrt{(2l_1 + 1)(2l_2 + 1)\ \ldots\ (2l_n + 1)}$. The mBipoSH angular correlation functions for Doppler boost can be constructed using the method used in the previous section as

$$\alpha_{\lambda}^{1M}(\cos\theta) = \sum_{l} A_{l,l+1}^{1M} a_{\lambda}(l, l+1, 1, \cos\theta). \tag{16}$$

The above reduction process for the Doppler boost ($L = 1$) can be simplified in terms of the correlation function as

$$C(\hat{n}_1, \hat{n}_2) = C_{\mathrm{SI}}(\cos\theta) + C_{\mathrm{nSI}}(\hat{n}_1, \hat{n}_2), \tag{17}$$

where the nSI correlation function corresponding to Doppler boost ($L = 1$) along the $\boldsymbol{\beta}$ direction in terms of angular correlation functions as

$$C_{\mathrm{nSI}}(\hat{n}_1, \hat{n}_2) = \sum_{\lambda=0}^{1} C_{\lambda}(\cos\theta) f_{\lambda}(\hat{n}_1, \hat{n}_2). \tag{18}$$

The above Equation (18) provides an explicit expression to compute the angular correlation function $C_{\mathrm{nSI}}(\hat{n}_1, \hat{n}_2)$ for the Doppler-boosted CMB temperature map. The function $C_{\lambda}(\cos\theta)$ represents two distinct angular correlation functions, while $f_{\lambda}(\hat{n}_1, \hat{n}_2)$ are distinct functions of directions for $\lambda = 0, 1$. For the $L = 1$ case, the functions $f_{\lambda}(\hat{n}_1, \hat{n}_2)$ take the form of $f_0(\hat{n}_1, \hat{n}_2) \equiv [\hat{n}_1]_{10} = \hat{n}_1.\hat{d}$ and $f_1(\hat{n}_1, \hat{n}_2) \equiv [\hat{n}_2]_{10} = \hat{n}_2.\hat{d}$ (where the indices represent the zeroth component of the rank-1 tensor). These indices specify the projection of the respective vector along the boost direction ($\hat{d}$). We obtain the

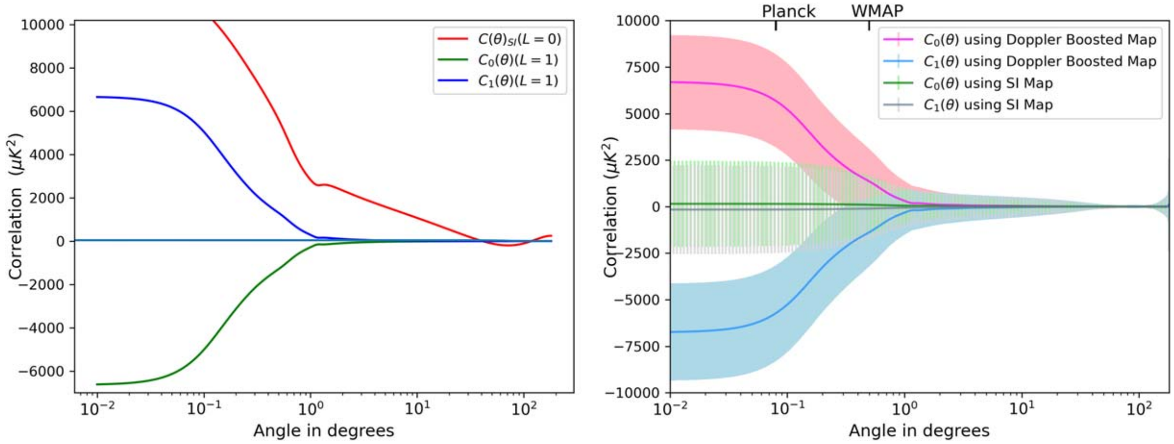

Figure 1. The left panel displays the mBipoSH correlation functions for the nSI Doppler boost effect, which are $C_0(\theta)$ and $C_1(\theta)$ with parameters $b_\nu = 3$ for $\nu = 217$ GHz with $\beta = 1.23 \times 10^{-3}$ along with the statistical isotropy correlation function. These plots are generated assuming angular power spectrum C_l^{TT} computed for the best-fit ΛCDM model parameters using CAMB (Lewis & Challinor 2011). The right panel presents the cosmic variance error bar for mBipoSH correlation functions corresponding to the Doppler boosted effect using 1000 simulated Doppler boost maps. We used the CoNIGS code to generate the simulated nSI maps for the Doppler boost (Mukherjee & Souradeep 2014).

complete expression for $C_{\rm nSI}(\hat{n}_1, \hat{n}_2)$ from Equation (18) as

$$C_{\rm nSI}(\hat{n}_1, \hat{n}_2) = C_0(\cos\theta)\hat{n_1}.\hat{d} + C_1(\cos\theta)\hat{n_2}.\hat{d}, \tag{19}$$

where

$$C_0(\theta) = \sum_l \beta^{10} D_l^{TT} \frac{\Pi_{l,l+1}}{\Pi_1} C_{l,0,l+1,0}^{10} \left[\frac{-\sqrt{3}}{4\pi\sqrt{l+1}}(-1)^l P_l^{(1)}(\cos\theta)\right] \tag{20}$$

$$C_1(\theta) = \sum_l \beta^{10} D_l^{TT} \frac{\Pi_{l,l+1}}{\Pi_1} C_{l,0,l+1,0}^{10} \left[\frac{-\sqrt{3}}{4\pi\sqrt{l+1}}(-1)^{l+1} P_{l+1}^{(1)}(\cos\theta)\right]. \tag{21}$$

It is interesting to note that both the angular correlation functions depend on the first derivative of Legendre polynomials $P_\ell^{(1)}(\cos\theta)$, indicating that the nSI signal is related to gradients induced in the temperature map. Furthermore, it should also be noted that the magnitudes of the two correlation functions are close but not identical, as evidenced by their explicit mathematical expressions.

Figure 1 shows the theoretical and simulated plots of the angular correlation functions for a CMB map with anisotropy corresponding to an $L = 1$ Doppler boost nSI effect. This representation makes it evident that the departures from statistical isotropy temperature fluctuations in a Doppler-boosted map are primarily correlated at small angular separations ($\theta \lesssim 1°$). Based on the cosmic variance error bars shown in the plot in the right panel of Figure 1, it can be more readily appreciated that the Planck mission (Planck Collaboration et al. 2014) could detect the Doppler boost velocity signal primarily due to its higher angular resolution compared to the previous full-sky measurements from WMAP (Hinshaw et al. 2013).

We now outline the derivation of an estimator of the real space mBipoSH angular correlation functions to capture nSI effects from a measured CMB map. The estimation of the mBipoSH angular correlation functions can be expressed as

$$\hat{\alpha}_\lambda^{\rm LM}(\cos\theta) = \alpha_\lambda^{\rm LM}(\cos\theta) + \Gamma_{\rm LM} G_\lambda^L(\cos\theta), \tag{22}$$

where $\hat{\alpha}_\lambda^{\rm LM}(\cos\theta)$ is the observed mBipoSH angular correlation function and $\alpha_\lambda^{\rm LM}(\cos\theta)$ is the mBipoSH angular correlation function for a single realization of the statistical isotropy map. The ensemble average of $\alpha_\lambda^{\rm LM}(\cos\theta)$ is zero for $L \neq 0$. $\Gamma_{\rm LM} G_\lambda^L$ represents a statistical isotropy violation, such as weak lensing, Doppler boost, etc. $G_\lambda^L(\cos\theta)$ is the shape factor related to the nSI effect and $\Gamma_{\rm LM}$ denotes the signal strength of the nSI effects. This estimator is based on the estimator defined by Hu & Okamoto (2002) and Hanson et al. (2009) for estimating the angular power spectrum.

Specifically, for the Doppler boost case, we define the estimator $\hat{\beta}_{\rm LM}$ for $L = 1$ as

$$\hat{\beta}_{1M} = \sum_\theta \frac{\alpha_\lambda^{1M}(\cos\theta)}{G_\lambda^1(\cos\theta)} + \beta_{1M}, \tag{23}$$

where the shape factor for $\lambda = 1$ is defined as

$$G_1^1(\cos\theta) = \sum_l \frac{\Pi_{l,l+1}}{\sqrt{12\pi}} [(l+b)C_l - (l+2-b)C_{l+1}] C_{l,0,l+1,0}^{10} \left[\frac{-\sqrt{3}}{4\pi\sqrt{l+1}}(-1)^l P_l^{(1)}(\cos\theta)\right]. \tag{24}$$

To arrive at the minimum variance estimator, we can write

$$\hat{\beta}_{1M} = \sum_\theta w_\lambda^1(\cos\theta) \frac{\alpha_\lambda^{1M}(\cos\theta)}{G_\lambda^1(\cos\theta)} + \beta_{1M}, \tag{25}$$

where $w_\lambda^1(\cos\theta)$ are the weights such that $\sum_\theta w_\lambda^L(\cos\theta) = 1$. The explicit expression for the weight factors that minimize the reconstruction noise can be readily derived.

6. Discussion

In this paper, we propose a natural generalization of the well-known isotropic angular correlation function that can also capture nSI features in random maps on a 2-sphere. We utilize a reduction technique for BipoSH that results in new basis functions called mBipoSH functions. In this new basis, new measures quantifying nSI features emerge as an additional set of the real space angular correlation functions that we refer to as mBipoSH angular correlation functions.

In the specific domain of our interest, which involves the detailed study of observed maps depicting the anisotropy in the CMB, these maps offer a new set of observables in real space that complement the harmonic space BipoSH representation used earlier in the literature. Introducing new approaches often helps shed new light on the nature of the phenomena underlying the observed nSI signals. As an illustrative example, we derive and plot the mBipoSH angular correlation functions for the well-known nSI effect induced in CMB maps due to the Doppler boost associated with the motion of the observer with respect to the cosmic rest frame of the CMB. We show that this representation readily reveals that the nSI effect on the two-point correlation function due to Doppler boost is strong only at small angular separations. We emphasize that the work presented in this paper can be effectively employed in the wider context to investigate random distributions on a 2D sphere, encompassing diverse applications ranging from celestial sky maps to geographical maps.

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