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Large Scale Magnetic Fields: Density Power Spectrum in Redshift Space

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Abstract. We compute the density redshift-space power spectrum in the presence of tangled magnetic fields and compare it with existing observations. Our analysis shows that if these magnetic fields originated in the early universe then it is possible to construct models for which the shape of the power spectrum agrees with the large scale slope of the observed power spectrum. However requiring compatibility with observed CMBR anisotropies, the normalization of the power spectrum is too low for magnetic fields to have significant impact on the large scale structure at present. Magnetic fields of a more recent origin generically give density power spectrum $\propto k^4$ which doesn't agree with the shape of the observed power spectrum at any scale. Magnetic fields generate curl modes of the velocity field which increase both the quadrupole and hexadecapole of the redshift space power spectrum. For curl modes, the hexadecapole dominates over quadrupole. So the presence of curl modes could be indicated by an anomalously large hexadecapole, which has not yet been computed from observation.

It appears difficult to construct models in which tangled magnetic fields could have played a major role in shaping the large scale structure in the present epoch. However if they did, one of the best ways to infer their presence would be from the redshift space effects in the density power spectrum.

Key words. Cosmology: theory—large-scale structure of the universe — magnetic fields—MHD.

1. Introduction

Magnetic fields play an important dynamical role in shaping most structures in the universe (see e.g., Parker 1979). The largest scale spatially coherent fields are seen in galaxies and galaxy clusters with coherence lengths $\simeq$ 10–100 kpc (for a recent review see Widrow 2002). Though there is also some evidence of coherent magnetic fields on super-cluster scales (Kim *et al.* 1989), the existence of magnetic fields at larger scales ($\gtrsim$ 1 Mpc) cannot generally be inferred from direct observations (for a summary of results see Kronberg 1994; Widrow 2002). The most direct method to infer the presence of intergalactic magnetic fields for $z \lesssim 3$ is to study the Faraday rotation of polarized emission from extra-galactic sources (Rees & Reinhardt 1972;

Kronberg & Simard-Normandin 1976; Vallée 1990; Blasi, Burles & Olinto 1999). The existence of these fields can also be constrained at the last scattering surface from CMBR anisotropy measurements and upper limits on the CMBR spectral distortion (Barrow, Ferreira & Silk 1997; Subramanian & Barrow 1998; Jedamzik, Katalinić & Olinto 2000). Large scale magnetic fields also cause Faraday rotation of the polarized component of the CMBR anisotropies (Kosowsky & Loeb 1996). If magnetic fields existed at even higher redshifts, they can also affect the primordial nucleosynthesis (see e.g., Widrow 2002 for detailed discussion).

The origin of these large scale fields is not clear. They could arise from dynamo amplification of small seed fields (see e.g., Parker 1979; Zeldovich, Ruzmaikin & Sokolov 1983; Ruzmaikin, Sokolov & Shukurov 1988) or these fields have their origin in very early universe and their flux-frozen evolution result in presently observed fields (see e.g., Turner & Widrow 1988; Ratra 1992).

Wasserman (1978) considered the effect of large scale magnetic fields on the formation of structures in the universe. This study showed that nano-gauss fields could provide initial conditions for density and velocity perturbations which could gravitationally collapse to form galaxies at the present epoch. Kim, Olinto & Rosner (1996) calculated the density power spectrum in the presence of magnetic fields. Sethi (2003) studied the effect of magnetic fields on the two-point correlation function of galaxies.

Two-point functions in real and Fourier space remain the most important tools to understand the formation of structures in the universe (see e.g., Peebles 1980). Recently large galaxy survey 2dF (Colless *et al.* 2001) has computed these functions with unprecedented precision. In particular one of the most important results from the 2dF survey is the unambiguous detection of anisotropy in the two-point functions, which is the best statistical evidence of the large scale velocity field (Peacock *et al.* 2001; Hawkins *et al.* 2002). The on-going survey Sloan digital sky survey (SDSS) is likely to improve upon this result owing to its larger size (York *et al.* 2000). The results of 2dF survey show good agreement with the theoretical predictions of variants of CDM models (see e.g., Lahav *et al.* 2002), in which initial density perturbations are produced at the time of inflation in the very early universe. Larger surveys like the on-going SDSS have the potential to uncover the small discrepancy between theory and observations.

In this paper we study the possibility that initial density and velocity perturbations were caused by tangled magnetic fields. In particular we estimate the density power spectrum in redshift space from these perturbations for two classes of models and compare with present observations. In one class of models we assume the magnetic fields to have originated in very early universe; we also consider simple models in which the magnetic fields could be of more recent origin and could have originated by astrophysical processes at $z \lesssim 10$. This study could be considered a continuation of the early studies of Kim *et al.* (1996) who calculated density power spectrum in real space and Sethi (2003) who computed the density two-point correlation function in redshift space.

In the next section we discuss the magneto-hydrodynamics equations and the evolution of density and velocity fields in the presence of tangled magnetic fields. In section 3 we discuss the properties of spatial correlations of the density and velocity fields and their impact on redshift space power spectrum and give our main results. In section 4 we summarize our conclusions. Throughout this paper we use the currently-favoured background cosmological model: spatially flat with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$

(Perlmutter *et al.* 1999; Riess *et al.* 1998). For numerical work we use $\Omega_b h^2 = 0.02$ (Tytler *et al.* 2000) and $h = 0.7$ (Freedman *et al.* 2001).

2. Magneto-hydrodynamics equations

In co-moving coordinates, the equations of magneto-hydrodynamics in the linearized Newtonian theory are (Wasserman 1978):

$$\frac{d(a\mathbf{v}_b)}{dt} = -\nabla\phi + \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi\rho_b}, \tag{1}$$

$$\nabla \cdot \mathbf{v}_b = -a\dot{\delta}_b, \tag{2}$$

$$\nabla^2\phi = 4\pi G a^2(\rho_{\rm DM}\delta_{\rm DM} + \rho_b\delta_b), \tag{3}$$

$$\frac{\partial(a^2\mathbf{B})}{\partial t} = \frac{\nabla \times (\mathbf{v}_b \times a^2\mathbf{B})}{a}, \tag{4}$$

$$\nabla \cdot \mathbf{B} = 0. \tag{5}$$

In equation (1) the pressure gradient from matter is neglected as it is important at Jeans' length scales ($k \gg 1\,\mathrm{Mpc}^{-1}$ before re-ionization and $\simeq 1\,\mathrm{Mpc}^{-1}$ after re-ionization). Our interest here is to study scales at which the perturbations are linear at the present epoch, $\gtrsim 10\,\mathrm{h}^{-1}$ Mpc or $k \lesssim 0.2\,\mathrm{h}\,\mathrm{Mpc}^{-1}$. Equation (1) and equation (2) can be combined to give:

$$\frac{\partial^2\delta_b}{\partial t^2} + 2\frac{\dot{a}}{a}\frac{\partial\delta_b}{\partial t} - 4\pi G(\rho_{\rm DM}\delta_{\rm DM} + \rho_b\delta_b) = \frac{\nabla.\,[(\nabla \times \mathbf{B}) \times \mathbf{B}]}{4\pi a^2\rho_b}. \tag{6}$$

Here the subscript 'b' refers to the baryonic component and the subscript 'DM' refers to the dark matter component. Fluid equations for the evolution of dark matter perturbations can be obtained from the equations above by dropping the magnetic field terms (Peebles 1980). Wasserman (1978) showed that equation (6) admits a growing solution, i.e., tangled magnetic fields can provide initial conditions for the growth of density perturbations. These solutions are discussed in the next section. In equation (4) we have assumed the medium to have infinite conductivity. It can be simplified further by dropping the right hand side of the equation as it is of higher order, this gives:

$$\mathbf{B}(x, t)a^2 = \text{constant}. \tag{7}$$

We assume the tangled magnetic field to be a statistically homogeneous and isotropic vector random process. In this case the two-point correlation function of the field in Fourier space can be expressed as (Landau & Lifshitz 1987):

$$\langle B_i(\mathbf{q})B_j^*(\mathbf{k})\rangle = \delta_D^3(\mathbf{q} - \mathbf{k})\left(\delta_{ij} - q_i q_j/q^2\right) B^2(q). \tag{8}$$

In addition we assume the tangled magnetic fields to obey Gaussian statistics.

2.1 *Time evolution of density and velocity perturbations*

The space and time dependence in the solution of equation (6) can be separated. Equation (6) contains two source terms: dark matter perturbations and tangled magnetic fields. A similar equation for the dark matter perturbations contains baryonic perturbations as the source term.

$$\frac{\partial^2 \delta_b}{\partial t^2} = -2\frac{\dot{a}}{a}\frac{\partial \delta_b}{\partial t} + 4\pi G(\rho_{\rm DM}\delta_{\rm DM} + \rho_b\delta_b) + S(t, x),$$
$$\frac{\partial^2 \delta_{\rm DM}}{\partial t^2} = -2\frac{\dot{a}}{a}\frac{\partial \delta_{\rm DM}}{\partial t} + 4\pi G(\rho_{\rm DM}\delta_{\rm DM} + \rho_b\delta_b). \tag{9}$$

Here $S(t, x)$ is the source term from magnetic fields. The dark matter is not directly affected by the magnetic fields. To solve these equations, we define $\delta_m = (\rho_{\rm DM}\delta_{\rm DM} + \rho_b\delta_b)/\rho_m$ with $\rho_m = (\rho_{\rm DM} + \rho_b)$. This leads to:

$$\frac{\partial^2 \delta_b}{\partial t^2} = -2\frac{\dot{a}}{a}\frac{\partial \delta_b}{\partial t} + 4\pi G\rho_m\delta_m + S(t, x),$$
$$\frac{\partial^2 \delta_m}{\partial t^2} = -2\frac{\dot{a}}{a}\frac{\partial \delta_m}{\partial t} + 4\pi G\rho_m\delta_m + \frac{\rho_b}{\rho_m}S(t, x). \tag{10}$$

The second of these equations can be solved by the usual Green's function methods. Its solution is:

$$\delta_m(x, t) = A(x)D_1(t) + B(x)D_2(t) - D_1(t)\int_{t_i}^{t} dt' \frac{S(t', x)D_2(t')}{W(t')}$$
$$+ D_2(t)\int_{t_i}^{t} dt' \frac{S(t', x)D_1(t')}{W(t')}. \tag{11}$$

Here $W(t) = D_1(t)\dot{D}_2(t) - D_2(t)\dot{D}_1(t)$ is the Wronskian. $D_1(t)$ and $D_2(t)$ are the solutions of the homogeneous part of the δ_m evolution (Peebles 1980). These terms have the space dependence corresponding to initial, presumably originated during inflation, perturbations. There is no reason to expect that there will be any correlation between these perturbations and the tangled magnetic field-induced perturbations. And therefore in the two-point functions these two contributions will add in quadrature. We only consider magnetic field-induced perturbations for our analysis and drop the first two terms from equation (11). In equation (11), t_i corresponds to the epoch of recombination as compressional modes cannot grow before that epoch (see e.g., Subramanian & Barrow 1998). So our initial conditions are: $\delta(t_i) = \dot{\delta}(t_i)$, as is evident from equation (11). The solution to equation (11) can be readily calculated analytically for $\Omega_m = 1$ universe (Wasserman 1978). For the currently favoured cosmological model – spatially flat with non-zero cosmological constant – these solutions have to be found numerically. The evolution of δ_b can be solved from:

$$\frac{1}{a^2}\frac{\partial}{\partial t}\left(a^2\frac{\partial \delta_b}{\partial t}\right) = \frac{3}{2}H^2\delta_m + S(t, x). \tag{12}$$

Here we have used: $H^2 = (8\pi G/3)\rho_m$. At high redshifts, solutions to equation (12) can be found analytically and allow us some insight into the numerical solutions. For $z \gg 1$ the fastest growing solution of equation (12) is $\propto \Omega_b/\Omega_m^2 t^{2/3}$. It shows that in the presence of the dark matter, perturbations in baryonic matter are suppressed by a factor Ω_b/Ω_m^2.

Tangled magnetic fields give rise to both compressional and curl velocity fields. The time dependence of these two modes is different. The time dependence of the compressional velocity mode v_d can be found from the continuity equation (equation 2) and equation (11). For $\Omega_m = 1$ model, the compressional modes grow as $a^{1/2}$. In the presence of dark matter, their growth like the density mode is suppressed by a factor Ω_b/Ω_m^2.

The time evolution of the curl part of the velocity can be found by either taking the curl of equation (1) or in Fourier space project the transverse part of the velocity field (see below). The time dependence of the resulting equation is readily solved:

$$v_c(t) = a(t)^{-1} \int_{t_i}^{t} \frac{dt'}{a^{-3}(t')} S(t', x) \tag{13}$$

v_c doesn't have any growing mode. In the $\Omega_m = 1$ model $v_c \propto a^{-1/2}$. Unlike the density and compressional velocity modes it doesn't suffer any suppression in the presence of dark matter.

The time evolution of density and velocity fields is shown in Fig. 1. In case the tangled magnetic fields originated in the very early universe, the effect of the non-

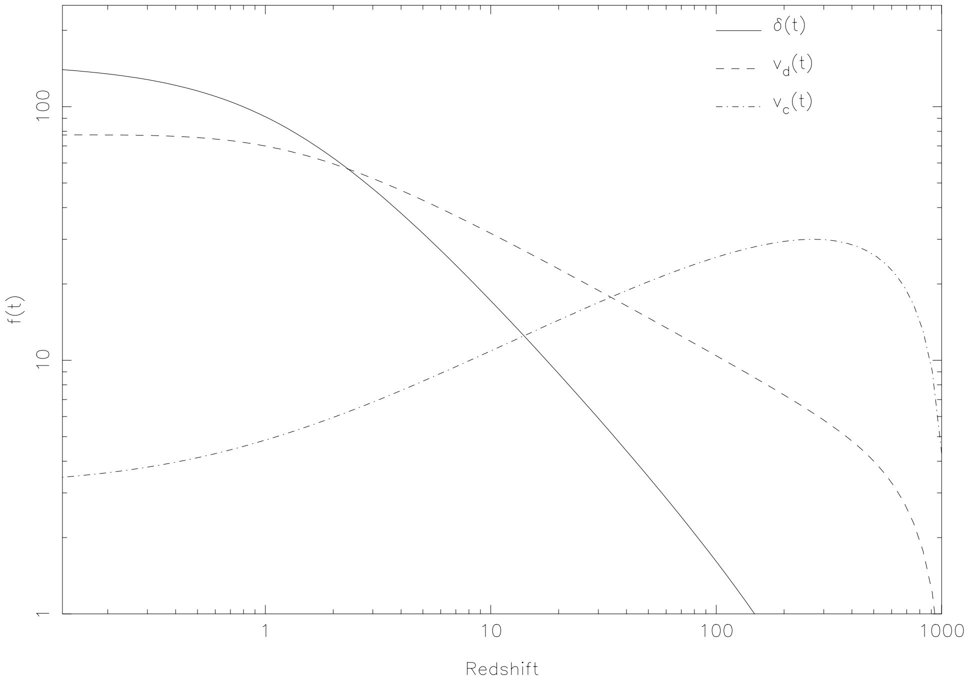

Figure 1. Evolution of density and velocity fields is shown if the tangled magnetic fields existed at the last scattering surface.

compressional modes generated by the magnetic fields would be negligible on the large scale structure at the present epoch, as these modes would have decayed by the present. Only if the magnetic fields are of more recent origin, these modes could have played an important part in the dynamics of large scale structure.

3. Density and velocity fields

The density and velocity fields are statistically homogeneous and isotropic random processes in real space. This allows one to define the power spectrum of the density field, $P(k)$, as (see e.g., Peebles 1980):

$$\langle \delta(\mathbf{k})\delta(\mathbf{k}')\rangle = (2\pi)^3 P(k)\delta_D^3(\mathbf{k}+\mathbf{k}'). \tag{14}$$

In redshift space both statistical homogeneity and isotropy of the density field break down (see e.g., Hamilton 1998). In the plane parallel approximation (Kaiser 1987), the density field is only statistically anisotropic. This is generally a good assumption in analysing large scale data (Hamilton 1998). We make this assumption in our analysis here. In linear theory and in the plane parallel approximation the observed density field, i.e., the redshift space density field, $\delta^s(\mathbf{r})$ can be written in terms of the real space density and velocity field as:

$$\delta^s(\mathbf{r},t) = \delta(\mathbf{r},t) - \hat{\mathbf{z}}.\nabla\hat{\mathbf{z}}.\mathbf{v}_b(\mathbf{r},t). \tag{15}$$

Here $\hat{\mathbf{z}}$ is taken to be the common line of sight to all the objects. In Fourier space equation (15) can be written as:

$$\delta^s(\mathbf{k},t) = \delta(\mathbf{k},t) + ik_z v_z(\mathbf{k},t). \tag{16}$$

Here $k_z = \hat{\mathbf{z}}.\mathbf{k}$ and $v_z = \hat{\mathbf{z}}.\mathbf{v_b}$. The velocity field in the case of tangled magnetic fields has both a divergence and a curl component.

$$\mathbf{v_b} = \mathbf{v_d} + \mathbf{v_c}. \tag{17}$$

Here $\mathbf{v_d}$ and $\mathbf{v_c}$ are the divergence and curl part of the velocity field. Their time evolution is already discussed in the last section. In Fourier space the divergence component points in the direction of the $\mathbf{k}$ vector, therefore it is convenient to decompose the velocity field parallel and perpendicular to the $\mathbf{k}$ vector, this gives the velocity field in the Fourier space as:

$$\mathbf{v_d}(\mathbf{k}) = \hat{\mathbf{k}}\hat{\mathbf{k}}.\mathbf{v}(\mathbf{k}), \tag{18}$$

$$\mathbf{v_c}(\mathbf{k}) = \mathbf{v}(\mathbf{k}) - \hat{\mathbf{k}}\hat{\mathbf{k}}.\mathbf{v}(\mathbf{k}), \tag{19}$$

$\mathbf{v_d}$ can readily be solved in terms of the density field using the continuity equation (equation 2):

$$\hat{\mathbf{z}}.\mathbf{v}_\mathrm{d}(\mathbf{k}) \equiv v_{dz} = -\frac{i\mu}{k}\delta(\mathbf{k})g_1(t). \tag{20}$$

Here $\mu = k_z/k$ is the angle between the Fourier mode and the line of sight and $g_1(t)$ is the time dependence of the divergence part of the velocity field; it is shown in Fig. 1. Note that we use the same symbols for density and velocity fields in both real and Fourier space. The curl part of the velocity field in the Fourier space is projected out by multiplying the Euler equation (equation 1) by $\delta_{ij} - \hat{\mathbf{k}}_i\hat{\mathbf{k}}_j$, δ_{ij} being the Kronecker delta function. The time dependence of the curl mode $g_2(t)$ is given in equation (13) and shown in Fig. 1.

From equation (14) the redshift space power spectrum can be written as:

$$(2\pi)^3 P_s(\mathbf{k}, t)\delta_D^3(\mathbf{k} + \mathbf{k}') = \langle(\delta(\mathbf{k}, t) + ik_z v_z(\mathbf{k}, t))(\delta(\mathbf{k}', t) + ik_z' v_z(\mathbf{k}', t))\rangle. \quad (21)$$

This can be expanded as:

$$\begin{aligned} P_s(\mathbf{k}, t) = {} & P(k) f^2(t) - g_1(t)k_z^2\langle v_{dz}(\mathbf{k})v_{dz}(-\mathbf{k})\rangle + ig_1(t)f(t)k_z\langle\delta(\mathbf{k})v_{dz}(-\mathbf{k})\rangle \\ & + ig_1(t)f(t)k_z\langle\delta(-\mathbf{k})v_{dz}(\mathbf{k})\rangle - ig_2(t)f(t)k_z\langle\delta(\mathbf{k})v_{cz}(-\mathbf{k})\rangle \\ & + ig_2(t)f(t)k_z\langle\delta(-\mathbf{k})v_{cz}(\mathbf{k})\rangle + g_2^2(t)k_z^2\langle v_{cz}(\mathbf{k})v_{cz}(-\mathbf{k})\rangle \\ & + k_z^2 g_1(t)g_2(t)\langle v_{dz}(\mathbf{k})v_{cz}(-\mathbf{k})\rangle - k_z^2 g_1(t)g_2(t)\langle v_{dz}(-\mathbf{k})v_{cz}(\mathbf{k})\rangle. \quad (22) \end{aligned}$$

Here $P(k) = \langle\delta(\mathbf{k})\delta(-\mathbf{k})\rangle$ is the real space power spectrum. It is derived in Appendix A. $f(t)$ gives the evolution of density perturbations (equation (11) and Fig. 1). The correlations involving the divergence part of the velocity fields can be readily written using the continuity equation (equation 2):

$$\begin{aligned} \langle\delta(-\mathbf{k})v_{dz}(\mathbf{k})\rangle &= -i\frac{k_z}{k^2}P(k), \\ \langle v_{dz}(\mathbf{k})v_{dz}(-\mathbf{k})\rangle &= -\frac{k_z^2}{k^4}P(k). \quad (23) \end{aligned}$$

Equation (23) along with the first four terms of equation (22) give the usual formula of redshift-distortion first derived by Kaiser (1987): $P_s(\mathbf{k}) = (1 + \mu^2\beta)^2 P(k)$, where $\beta = g_1(t_0)/f(t_0) \simeq \Omega_m^{0.6}$ (Lahav *et al.* 1991). Analysis of 2dF data suggests that $\beta \simeq 0.4$ (Peacock *et al.* 2001). Tangled magnetic fields also generate curl modes, which give rise to additional terms in the power spectrum in redshift space. We show in Appendix A that:

$$\begin{aligned} \langle\delta(-\mathbf{k})v_{cz}(\mathbf{k})\rangle &= 0, \\ \langle v_{dz}(\mathbf{k})v_{cz}(-\mathbf{k})\rangle &= 0. \quad (24) \end{aligned}$$

The non-trivial contribution comes from the term: $\langle v_{cz}(\mathbf{k})v_{cz}(-\mathbf{k})\rangle$. This can be written as:

$$\langle v_{cz}(\mathbf{k})v_{cz}(-\mathbf{k})\rangle = \langle v_z(-\mathbf{k})v_z(\mathbf{k})\rangle - \frac{k_z^2}{k^4}P(k) + i\frac{k_z}{k^2}\langle v_z(\mathbf{k})\delta(-\mathbf{k})\rangle - i\frac{k_z}{k^2}\langle v_z(-\mathbf{k})\delta(\mathbf{k})\rangle. \quad (25)$$

In Appendix A we show that, $\langle v_z(\mathbf{k})\delta(-\mathbf{k})\rangle = -ik_z/k^2 P(k)$. This simplifies the equation to:

$$\langle v_{cz}(\mathbf{k})v_{cz}(-\mathbf{k})\rangle = \langle v_z(-\mathbf{k})v_z(\mathbf{k})\rangle + 3\frac{k_z^2}{k^4}P(k). \tag{26}$$

The term $\langle v_z(-\mathbf{k})v_z(\mathbf{k})\rangle$ cannot be written in terms of $P(k)$. As shown in Appendix A, it contributes two positive terms proportional to μ^2 (quadrupole) and μ^4 (hexadecapole) with magnitude comparable to $P(k)$ (equation 43). This information along with equation (26) allows us to assess the contribution of the curl component of the velocity field to the redshift space distortion. Its contribution at the present epoch is proportional to $g_2^2(t_0)$. If the magnetic fields originated in the very early universe then the contribution of the curl component of the velocity field is negligible as it doesn't have any growing mode. From Fig. 1, we can see that $g_2(t_0)/g_1(t_0) \ll 1$. However if the magnetic fields have their origin in the recent history of the universe then it is possible to have $g_2(t_0) \simeq g_1(t_0)$. In this case the curl component enhances the contribution in both μ^2 and μ^4 terms. It is interesting to note that unlike the divergence term in which the μ^4 term is smaller than the μ^2 term by a factor of $\beta/2$, the curl contribution is dominated by the μ^4 term. In many models we studied it can be nearly 5 times the μ^2 term. The presence of the curl component leads to the intriguing possibility that the observed redshift space distortion is dominated by the curl mode. In that case it is not possible to infer the value of β from this observation as is usually done (Hamilton *et al.* 2001; Peacock 1998). We illustrate this case in Fig. 2. More realistically however the effect of the curl term might be determined from simultaneously determining the contributions from both the μ^2 and μ^4 terms. It has not so far been possible from observations which have determined only the μ^2 part (Peacock *et al.* 2001). On-going survey SDSS galaxy survey has the potential to test this hypothesis. These redshift space effects are nearly independent of the power spectrum of the tangled magnetic field. We discuss below whether it is possible to construct viable models of density power spectrum from the tangled magnetic field.

For our calculations we take the magnetic field power spectrum to be power law:

$$B^2(k) = Ak^n. \tag{27}$$

We consider the range of k between $k_{\rm min}$, which is taken to be zero unless specified otherwise, and the approximate scale at which the Alfv́en waves damp in the pre-recombination era (Jedamzik, Katalinic & Olinto 1998; Subramanian & Barrow 1998). Following Jedamzik *et al.* (1998), $k_{\rm max} \simeq 60\,{\rm Mpc}^{-1}(B_0/(3\times10^{-9}\,{\rm G})$. B_0, the RMS of magnetic field fluctuations at the present epoch, is defined as:

$$B_0^2 \equiv \langle B_i(\mathbf{x}, t_0)B_i(\mathbf{x}, t_0)\rangle = \frac{1}{\pi^2}\int_0^{k_c} dk k^2 B^2(k). \tag{28}$$

Here $k_c = 1\,h\,{\rm Mpc}^{-1}$ (Subramanian & Barrow 2002). This gives:

$$A = \frac{\pi^2(3+n)}{k_c^{(3+n)}}B_0^2. \tag{29}$$

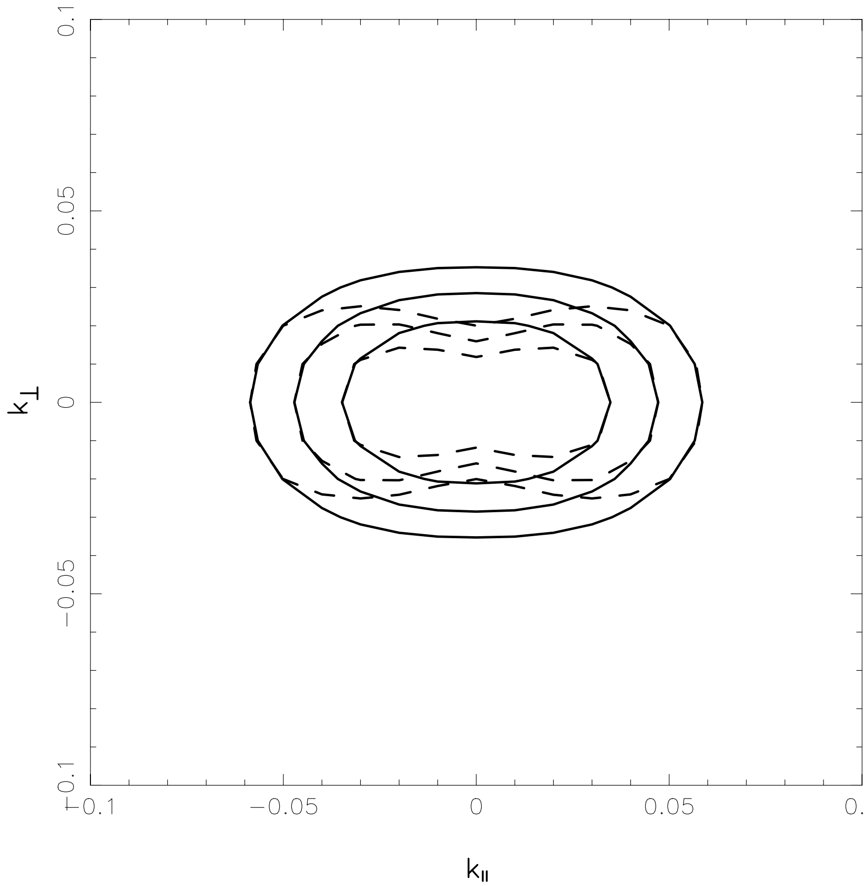

Figure 2. Equal redshift space power spectrum contours are shown. The x- and y-axis correspond to the component of k vector parallel and perpendicular to the line of sight. The solid contours show the contours for $\beta = 0.4$, to match with observations (Peacock *et al.* 2001), with zero curl contribution. The dashed curves correspond to $\beta = 0$ with curl component normalized to give the same quadrupole as in the previous case. Note strong distortions of the curves from the dominant hexadecapole in this case. The contour levels and overall normalization is arbitrary.

3.1 *Power spectrum in real space*

In the previous subsection, we discussed the redshift space effects in the observed power spectrum. Such effects are nearly independent of the power spectrum of the tangled magnetic field. In this section we study the possibility of constructing viable models of density power spectrum from tangled magnetic fields.

It is conceivable that tangled magnetic fields originated in the very early universe during inflationary epoch (Turner & Widrow 1988; Ratra 1992). In this case tangled magnetic fields can have large coherence lengths, or $k_{\min} \simeq 0$ in equation (27). On the other hand magnetic fields could be of more recent origin ($z \lesssim 10$). However, recent astrophysical processes do not generate large scale magnetic fields. Quasar outflows (see e.g., Furlanetto & Loeb 2001) might pollute the intergalactic medium sufficiently for it to have magnetic fields with maximum coherence scales $\simeq 2\,\mathrm{Mpc}$ with magnitudes $\simeq 10^{-9}\,\mathrm{G}$. In both cases magnetic fields can have appreciable effect on the large scale structure in the universe at linear scales. We discuss both these possibilities below.

3.1.1 *Early universe magnetic fields*

Kim, Olinto & Rosner (1996) calculated the density power spectrum in the presence of tangled magnetic fields. They concluded that for magnetic field power spectrum index $4 < n < -1$, the density power spectrum scales as k^4. We confirm their result but also consider smaller values of n. The observed power spectrum (Spergel *et al.* 2003) is consistent with $P(k) \propto k$ at large scales ($k \lesssim 0.002$); at smaller scales $P(k)$ turns around and scales as k^p with p changing from 0 to -3 as the scales become smaller (see e.g., Efstathiou *et al.* 1996; Percival *et al.* 2001; Fig. 3). This clearly means that none of the magnetic field power spectrum index n studied by Kim *et al.* (1996) can explain the data, which they also pointed out. To make atleast the slope of $P(k)$ agree with the large scale structure data, one needs to consider smaller value of n. For $n = -2$, $P(k) \propto k^3$, for $n \lesssim -2.5$, the power spectrum turns even shallower. An analytical understanding of this behaviour is given in Appendix A (equation 37). We consider $n = -2.9$, also studied by Subramanian & Barrow (2002); for this value $P(k)$ scales approximately as k. This suggests that the parameter range of interest lies around this value. As we

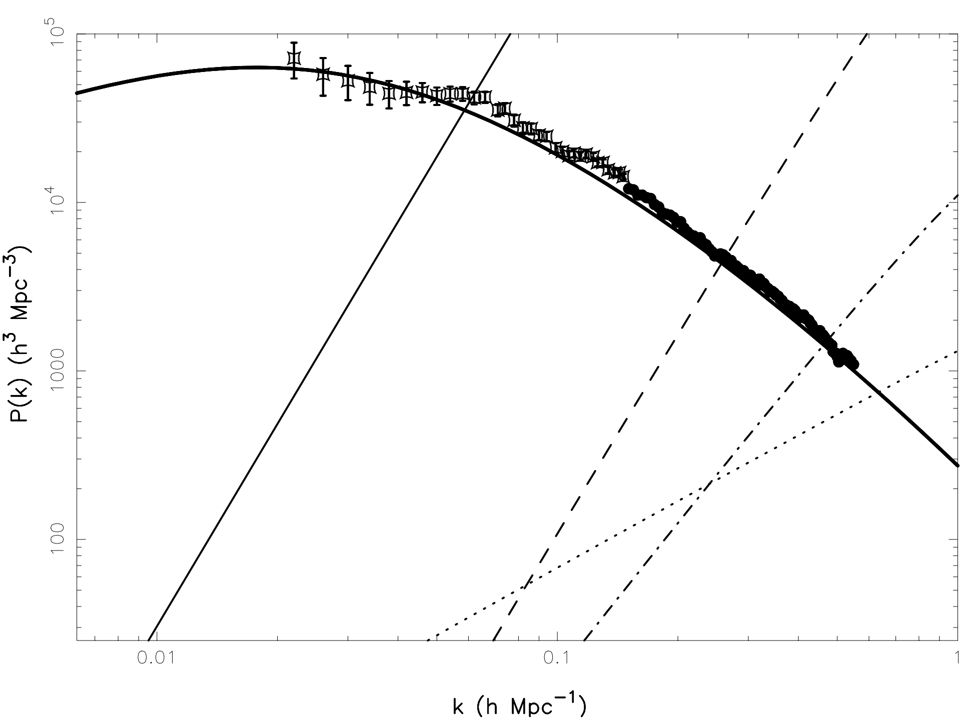

Figure 3. The density power spectrum from tangled magnetic fields is shown along with recent observation of the power spectrum from 2dF galaxy survey (Percival *et al.* 2001) and a variant of CDM model. For all the curves $B_0 = 3 \times 10^{-9}$ G. The curves correspond to different values of n: $n = 0$ (solid line), $n = -1$ (dashed line), $n = -2$ (dot-dashed line), and $n = -2.9$ (dotted line). The thick solid line corresponds to CDM model for a spatially flat universe: $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, $\Omega_b = 0.04$.

discussed above that redshift space effects do not change this conclusion. Subramanian & Barrow (2002) showed that this model can lead to CMBR anisotropies $\simeq 10\,\mu$k for angular scales $1000 < \ell < 2000$ for $B_0 = 3 \times 10^{-9}$, G, which is comparable to the observed anisotropies at these scales (Mason *et al.* 2002). We check if this model, with this normalization, can give reasonable effect on the large scale structure at the present epoch. We plot in Fig. 3 the density power spectra for several values of n. The power spectrum for this model is nearly two orders of magnitude below the observed power spectrum at linear scales. Therefore, even though this model leads to the correct shape of power spectrum at large scales, the normalization needed to give the correct CMBR anisotropy level is too low. We should point out that this result is nearly independent of the upper cut-off $k_{\rm max}$ of the magnetic field power spectrum. Fig. 3 also shows that spectral indices $n \gtrsim -1$ are ruled out by the present data for $B_0 = 3\times10^{-9}$ G. However the results for models with $n \gtrsim -1.5$ are strongly dependent on $k_{\rm max}$ and therefore less reliable and they are also likely to give unacceptably large CMBR anisotropies. Therefore we are led to conclude that if tangled magnetic fields existed at the last scattering surface, they are unlikely to have much impact on the large scale structure in the universe at present at linear scales. It should however be noted from Figure 3 that magnetic fields can have significant effect on the non-linear scales; which in particular will lead to early collapse of structures. This may have important implications for the re-ionization of the universe (Sethi & Subramanian 2003). We do not discuss this scenario in detail in this paper.

3.1.2 *Low redshift magnetic fields*

We consider a simple model to assess the effect of low redshift magnetic fields on the large scale structure. We assume these fields were created in the post-reionization epoch $z \lesssim 15$ (Spergel *et al.* 2003) and $k_{\rm min} = 6$h Mpc^{-1} and $k_{\rm max} = 30$h Mpc^{-1}, this corresponds roughly to scales between $1\,{\rm h}^{-1}$Mpc and $200\,{\rm h}^{-1}$kpc. The slope of the magnetic field power spectrum and its strength is to be determined by observations. While our choice of $k_{\rm min}$ is motivated by the requirement that astrophysical processes are unlikely to generate larger scale magnetic fields, our choice of $k_{\rm max}$ is largely arbitrary. Our interest is in studying the effect of these fields at scales that are linear at present, i.e., $k \lesssim 0.2$ Mpc. We show in Appendix A (equation (38) that models in which $k \ll k_{\rm min}$ generically give density power spectrum $\propto k^4$, irrespective of the slope of the tangled magnetic field power spectrum n. For $n \gtrsim -1.5$, the density power spectrum is dominated by the upper cut-off $k_{\rm max}$; in the other limit $k_{\rm min}$ determines the amplitude of the power spectrum. In Fig. 4, we show the power spectrum for two values of n for $B_0 = 10^{-9}$ G. For simplicity we take $f(t_0) = 5$. It is seen that if $n \gtrsim 1$ the density power spectrum at linear scales can get appreciable contribution from tangled magnetic fields. However results for these spectral indices depend strongly on the upper cut-off $k_{\rm max}$ and therefore are less reliable. Note that the value of B_0 needed to cause sufficient effect on the large scale structure is quite different from Sethi (2003). This is owing to the fact that $k_{\rm max}$ was taken to be 1 h Mpc^{-1} in that work and fields were assumed to be locally generated, i.e., $f(t_0) = 1$.

A possible criticism of our analysis is the use of linear theory and neglect of the RHS of equation (4). Even though we are interested in density perturbations at linear scales at present, presence of the RHS of equation (4) mixes all modes of tangled magnetic fields and the velocity perturbations and in general cannot be neglected. Our preliminary

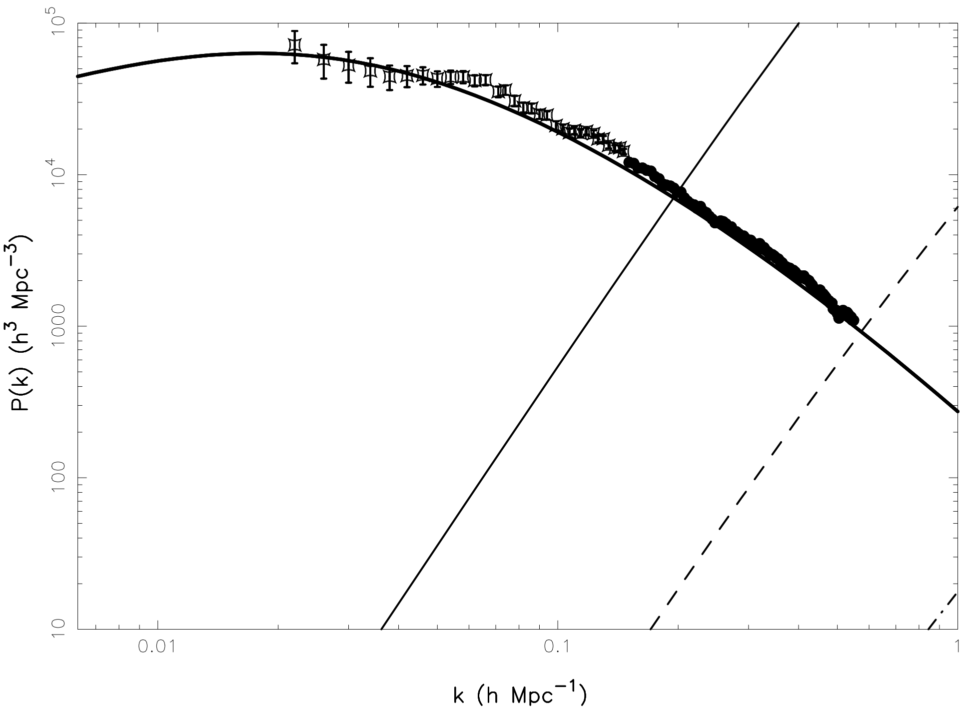

Figure 4. Same as Figure 3 for the model in which the tangled magnetic fields originate at $z \lesssim 10$ (see text for detail). $B_0 = 10^{-9}$ G and the curves correspond to different spectral index values: $n = 2$ (Solid line), $n = 1$ (dashed line).

calculations show that these terms are of order k times the velocity and magnetic field, which means that back reaction of velocity perturbations on the magnetic fields is of higher order than the density power spectrum and could be dropped for studying linear scales. We plan to study this issue in more detail in future. In particular these terms can be neglected if the density perturbation $\delta(\mathbf{k})$ is negligible for all scales in question. The smallest scale at which perturbations can collapse is the magnetic Jeans length $\simeq 100\,\text{kpc}(B_0/(10^{-9}\,\text{G}))$ (Subramanian & Barrow 1998). In practice it is however seen that neglect of non-linear terms to study perturbations at linear scales holds for a wide range of linear scales. For example the use of linear theory in the usual CDM model gives reasonable results for studying perturbations for $k \lesssim 0.2\,\text{h}^{-1}$Mpc which are quasi-linear at present, even though smaller structures could have collapsed at much higher redshifts. One case in which we are justified in neglecting the non-linear terms is when the final result can be shown to be nearly independent of the contribution of large k modes of the magnetic field. This as we discussed above is valid for $n \lesssim -1.5$ if the magnetic fields are generated in the early universe. In other cases neglect of this term should depend on both B_0 and n. Therefore our results on the effect of magnetic fields generated at low redshifts should be considered preliminary.

4. Conclusions

In this paper we studied the effect of tangled magnetic fields on the large scale structure in the universe. We calculated the power spectrum of the tangled magnetic fields and compared it with the observations at the present epoch. Our results can be summarized as:

- If the magnetic field originated in the very early universe. It is possible to construct models in which the shape of density power spectrum $\propto k$, i.e., it agrees with the observed power spectrum shape for $k \lesssim 0.02\,\mathrm{h}^{-1}\,\mathrm{Mpc}^{-1}$. However compatibility with observed CMBR anisotropies suggests that the density power spectrum from tangled magnetic field is smaller than the observed power spectrum by atleast two orders of magnitudes at linear scales ($k \lesssim 0.2\ \mathrm{h}^{-1}\,\mathrm{Mpc}^{-1}$) at present. Therefore very early universe tangled magnetic fields are unlikely to have important impact on the structures in the present universe.
- We consider a simple model in which the magnetic fields were generated with coherence scales $k \gtrsim 2\,\mathrm{h}^{-1}\,\mathrm{Mpc}^{-1}$ in the post-reionization epoch $z \lesssim 10$. In all such models the density power spectrum $\propto k^4$, i.e., the shape of the power spectrum is incompatible with the shape of the observed shape. It is possible to construct models in which the magnetic field can have important contribution to the density power spectrum for $B_0 \simeq 10^{-9}\,\mathrm{G}$. (It should be noted that the density power spectrum from initial conditions which could have originated during inflation adds to the magnetic field-induced density power spectrum as the density fields generated by these two processes are uncorrelated; see equation (11) and the discussion following it.) However these results are quite sensitive to the shape and the upper k cut-off of the tangled magnetic fields power spectrum, which are difficult to fix from either observations or theory.
- The redshift space effects from tangled magnetic fields have additional features owing to curl component of velocities generated by these fields. The curl component increases both the quadrupole (μ^2 term), hexadecapole (μ^4 term) of the redshift space power spectrum. For very early universe magnetic fields the curl component decays so it cannot have important contribution to the redshift space effects. For magnetic fields generated in the more recent epoch, the curl component of the velocity field can be comparable to the divergence component. In this case both quadropole and hexadecapole can be dominated by the curl component as opposed to the usual case of divergence collapse. This leads to the interesting possibility that most of the redshift space effects come from the curl component, and the usual way of determining Ω_m from the redshift space distortion is not entirely valid (Peacock *et al.* 2001). As noted above the density power spectrum from tangled magnetic fields can dominate the observed power spectrum for $B_0 \simeq 10^{-9}\,\mathrm{G}$, and hence can be used to probe tangled fields which are too small to be detected by other methods (see e.g., Sethi 2003)

In summary: Tangled magnetic fields are unlikely to have provided the initial conditions for the formation of presently-observed structure in the universe. In this paper we showed that this conclusion seems inevitable for magnetic fields generated in the very early universe. However we could only study a simple model of tangled magnetic fields which were generated at $z \lesssim 10$. It would be interesting to do a more detailed analysis of this scenario taking into account the non-linear effects.

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APPENDIX A

In this Appendix, we derive expressions for $P(k)$, $\langle\delta(\mathbf{k})v_z(\mathbf{k})\rangle$, $\langle(v_z(\mathbf{k}))^2\rangle$ and also make an approximate analytical estimate of the small k-dependence of $P(k)$. The real space spatial density contrast and peculiar velocity component along the line of sight are given as:

$$\delta(\mathbf{x}) = \nabla\cdot[\mathbf{B}\times(\nabla\times\mathbf{B})],$$
$$\mathbf{v}(\mathbf{x})\cdot\hat{\mathbf{z}} = [\mathbf{B}\times(\nabla\times\mathbf{B})]\cdot\hat{\mathbf{z}}. \tag{30}$$

Here $B \equiv B(\mathbf{x}, t_0)$, i.e., the value of magnetic field at the present epoch. The Fourier space expressions for the above fields are:

$$\delta(\mathbf{k}) = \int d^3k_1[(\mathbf{k_1}\cdot\mathbf{B}(\mathbf{k}-\mathbf{k}_1))\ (\mathbf{k}\cdot\mathbf{B}(\mathbf{k}_1)) - (\mathbf{k_1}\cdot\mathbf{k})\ (\mathbf{B}(\mathbf{k_1})\cdot\mathbf{B}(\mathbf{k}-\mathbf{k_1}))], \tag{31}$$

$$\mathbf{v}(\mathbf{k}) = -i\int d^3k_1[(\mathbf{B}(\mathbf{k_1})\cdot\mathbf{B}(\mathbf{k}-\mathbf{k_1}))\mathbf{k_1} - (\mathbf{k_1}\cdot\mathbf{B}(\mathbf{k}-\mathbf{k_1}))\mathbf{B}(\mathbf{k_1})]. \tag{32}$$

The volume element in the integrals can be simplified by choosing $\mathbf{k}$ to lie along the z-axis and $\hat{n}$ to lie in the x-z plane. We thus have,

$$\int d^3k_1 = \int dk_1k_1^2\int d\mu\int d\phi. \tag{33}$$

Here, $\mu \equiv \cos\,\theta$ (θ is the angle between k_1 and the z-axis) while ϕ is the azimuthal angle. In the integral, k_1 ranges from $k_{\min}$ to $k_{\max}$, μ from -1 to $+1$ and ϕ from 0 to 2π. Care has to be taken while evaluating multiple integrals formed from above (for e.g., $\langle\delta^2\rangle$) since the presence of terms like $\delta(k_2+k-k_1)$ after integrating over k_2 puts a constraint on the integration range of θ as well. Taking all this into account we can split the integration ranges for the cases of interest in this paper as follows:

For $k_{\min} = 0$ and $0 < k_1 < k_{\max}$,

$$\int d^3k_1 = \int_0^k dk_1\int_{-1}^{+1} d\mu + \int_k^{k_{max}-k} dk_1\int_{-1}^{+1} d\mu + \int_{k_{max}-k}^{k_{max}} dk_1\int_{\mu_{max}}^{1} d\mu. \tag{34}$$

For $k_{\min} \neq 0$ and $0 < k_1 < k_{\min}$,

$$\int d^3k_1 = \int_{k_{\min}}^{k+k_{\min}} dk_1\int_{-1}^{\mu_{\min}} d\mu + \int_{k+k_{\min}}^{k_{\max}-k} dk_1\int_{-1}^{+1} d\mu + \int_{k_{\max}-k}^{k_{\max}} dk_1\int_{\mu_{\max}}^{1} d\mu, \tag{35}$$

where $\mu_{\max} = (k^2+k_1^2-k_{\max}^2)/(2kk_1)$ and $\mu_{\min} = (k^2+k_1^2-k_{\min}^2)/(2kk_1)$.

To calculate $P(k)$ we take the ensemble average of $[\delta(\mathbf{k})]^2$. This product contains terms involving four point functions of $\mathbf{B}$. By assuming that $\mathbf{B}$ is Gaussian distributed in the ensembles such terms can be written as sums of products of two-point functions of **B**. Finally using equation (8) and simplifying, we arrive at the following expression for $P(k)$:

$$P(k) = \int_{k_{\min}}^{k_{\max}} dk_1 \int_{-1}^{+1} d\mu \frac{B^2(k_1)B^2(|\mathbf{k}-\mathbf{k_1}|)}{|\mathbf{k}-\mathbf{k_1}|^2} \; [2k^5k_1^3\mu + k^4k_1^4(1-5\mu^2) + 2k^3k_1^5\mu^3]. \tag{36}$$

We evaluate this double integral numerically. However we can analytically see the form for $P(k)$ when $k \ll k_{\max}$ both for $k_{\min} = 0$ as well as $k_{\min} \neq 0$ as follows:

For $k_{\min} = 0$ and $k \ll k_{\max}$, the relevant case when the magnetic fields originate in the early universe, the only major contribution to the $P(k)$ comes from the second integral in equation (34). We thus have to lowest order in $k/k_{\max}$,

$$P(k) \sim Ak^{2n+7} + Bk_{\max}^{2n+3}k^4 + Ck_{\max}^{2n+1}k^6 + \text{...(higher powers of } k) \tag{37}$$

where, A, B and C are coefficients depending only on n. We thus see that for $n > -1.5$ the leading order term is proportional to k^4 whereas for $n < -1.5$ it is proportional to k^{2n+7}. In particular for $n = -2$, the dependence goes as k^3. Also, $P(k) \to k^1$ as $n \to -3$.

For $k_{\min} \neq 0$ and $k \ll k_{\min}$, the case if the magnetic fields are of more recent origin, the leading contribution to $P(k)$ comes from the third integral in equation (35). Thus, to lowest order in k we get,

$$P(k) \sim Ak^4(k_{\max}^{2n+3} - k_{\min}^{2n+3}) + \text{...(higher powers of } k). \tag{38}$$

Thus, we see that with an infrared cutoff which is much larger than the wavenumber of interest, the dependence of $P(k)$ is generically k^4. The dependence on $k_{\max}$ and $k_{\min}$ is such that for $n > -1.5$, the value of $P(k)$ is determined by and increases with $k_{\max}$. In the other limit $P(k)$ is determined by $k_{\min}$. We now evaluate the correlation $\langle\delta(k)\ \mathbf{v}\cdot\hat{\mathbf{z}}\rangle$ We can show that it is simply proportional to $P(k)$ in the following way: From the assumptions of homogeneity and isotropy, we can write

$$\langle v_i(\mathbf{k}) v_j(\mathbf{q})\rangle = \left(A(k^2)\ \delta_{ij} + B(k^2)\ k_i k_j\right)\ \delta^3(\mathbf{q}-\mathbf{k}) \tag{39}$$

where $A(k^2)$ and $B(k^2)$ are some as yet undetermined coefficients. Thus, using the continuity equation (equation 2):

$$\langle\delta(\mathbf{k})\ \mathbf{v}(\mathbf{k})\cdot\hat{\mathbf{n}}\rangle = -i\,\langle k_i v_i v_j n_j\rangle = -i\mathbf{k}\cdot\hat{\mathbf{n}}[A(k^2) + k^2 B(k^2)]. \tag{40}$$

Similarly we get,

$$P(k) \equiv -\langle k_i v_i k_j v_j\rangle = -k^2[A(k^2) + k^2 B(k^2)]. \tag{41}$$

Thus from these equations we get the following relation:

$$\langle \delta(\mathbf{k})\, \mathbf{v}(\mathbf{k}) \cdot \hat{\mathbf{n}} \rangle = i\,\frac{\mathbf{k} \cdot \hat{\mathbf{n}}}{k^2}\, P(k). \tag{42}$$

From this derivation equation (24) follows. Finally, this allows us to write $\langle(\mathbf{v}(\mathbf{k}) \cdot \hat{\mathbf{n}})^2\rangle$ correlation:

$$\langle(\mathbf{v}(\mathbf{k}) \cdot \hat{\mathbf{n}})^{\mathbf{2}}\rangle = \int_{k_{\min}}^{k_{\max}} dq \int_{-1}^{+1} d\mu \frac{B^2(q) B^2(|\mathbf{k}-\mathbf{q}|)}{|\mathbf{k}-\mathbf{q}|^2} \;\big[\cos^2\alpha\, \big(2k^3q^3\mu - 5k^2q^4\mu^2 + k^2q^4 - q^5k(\mu - 3\mu^3)\big) + q^5k(\mu-\mu^3)\big]. \tag{43}$$

Here, α is the angle between $\mathbf{k}$ and $\hat{n}$.

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