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Interstellar Electron Density

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Abstract. We impose the requirement that the spatial distribution of pulsars deduced from their dispersion measures using a model of the galactic electron density (n_e) should be consistent with cylindrical symmetry around the galactic centre (assumed to be 10 kpc from the Sun). Using a carefully selected sub-sample of the pulsars detected by the II Molonglo Survey (II MS), we test a number of simple models and conclude that (i) the effective mean $\langle n_e \rangle$ for the whole galaxy is $0{\cdot}037^{+0.020}_{-0.012}$ cm^{-3}, (ii) the scale height of electrons is greater than 300 pc and probably about 1 kpc or more, and (iii) there is little evidence for variation of n_e with galactic radius R_{GC} for $R_{GC} \gtrsim 5$ kpc. Further, we make a detailed analysis of the contribution to n_e from H II regions. Combining the results of a number of relatively independent calculations, we propose a model for the galactic electron density of the form

$$n_e(z) = 0{\cdot}030 + 0{\cdot}020 \exp(-|z|/70) \text{ cm}^{-3}$$

where z(pc) is the height above the galactic plane and the second term describes the contribution from H II regions. We believe the statistical uncertainties in the parameters of this model are quite small.

Key words: pulsars, dispersion measure—interstellar electron density—H II regions

1. Introduction

The interstellar electron density n_e (cm^{-3}) is an important parameter in pulsar studies since it is used to determine pulsar distances d(pc) from their observed dispersion measures D(pc cm^{-3}). Hall (1980) has summarized in detail the various previous attempts to estimate n_e. Although there have been discussions based on free-free absorption, hydrogen radio recombination lines, Faraday rotation and interstellar scattering, these usually require some assumptions regarding temperature, magnetic field or degree of clumping of the electrons. The most reliable studies of the galactic electron distribution have used the dispersion measures of the few pulsars for which independent distances have been obtained through 21 cm H I absorption measurements. However, mean $\bar{n}_e$ values for these pulsars obtained using the relation

$$\bar{n}_e = D/d \tag{1}$$

range all the way from 0·01 cm^{-3} to 0·2 cm^{-3}, with an average value $\langle n_e \rangle$ of 0·03 cm^{-3}. Obviously one needs other independent studies to fix confidence limits on $\langle n_e \rangle$ more accurately. To our knowledge, the only pulsar dispersion study not depending upon estimates of distances to individual pulsars is that by del Romero and Gomez-Gonzalez (1981), who estimated $\langle n_e \rangle$ to be 0·03 cm^{-3} on the *a priori* assumption that pulsars are predominantly a spiral-arm population. In this paper, we make a further independent study of n_e by assuming that the galactic pulsar population is cylindrically symmetric about the galactic centre. We estimate the mean value of electron density $\langle n_e \rangle$ to be $0{\cdot}037^{+0{\cdot}022}_{-0{\cdot}010}$ cm^{-3}.

The galactic electron density has often been modelled in the exponential form

$$n_e(z) = n_e(0) \exp(-|z|/z_0) \tag{2}$$

where z is the height above the galactic plane. The scale height z_0 has been estimated to be 264 pc by Hall (1980) and 1000 pc by Taylor and Manchester (1977), while Lyne (1980) concluded that it is essentially infinite, barring a component due to H II regions with $z_0 = 70$ pc. One reason for the disparity in these estimates is that $\langle |z| \rangle$ for the pulsars with reliable independent distances (as against distance limits) is only of the order of 100 pc (Table 1); these pulsars are therefore not sensitive probes of large z. The pulsars we use in this paper have a mean $|z|$ of about 350 pc and hence our test has more sensitivity in estimating z_0. On the basis of our results, we rule out low z_0, say below 300 pc, and favour z_0=1000 pc or more.

We have also tested the suggestion that the mean electron density is enhanced in the inner regions of the Galaxy (Ables and Manchester 1976; del Romero and Gomez-Gonzalez 1981, Harding and Harding 1982). We find that such an enhancement is not as large as indicated by earlier studies. We favour a model with $\langle n_e \rangle = 0{\cdot}04$ cm^{-3} within a cylindrical region of galactic radius $R_{GC} \sim 7$ kpc around the galactic centre, and $\langle n_e \rangle = 0{\cdot}03$ cm^{-3} for $R_{GC} > 7$ kpc, though a constant electron density independent of R_{GC} would be nearly as good.

Finally, we have studied the contribution to $\langle n_e \rangle$ from H II regions in the Galaxy. Prentice and ter Haar (1969) have given a procedure to estimate this contribution for known H II regions within 1 kpc of the Sun. At larger distances one can only estimate a statistical contribution. We separate n_e into a uniform component plus a contribution from H II regions of scale height 70 pc as done by Lyne (1980), and estimate the magnitudes of the two components by means of a number of different approaches. Combining all the evidence, we propose the following galactic electron density model (for all R_{GC} except possibly $R_{GC} < 5$ kpc where our sensitivity is poor)

$$n_e(z) = 0{\cdot}030 + 0{\cdot}020 \exp(-|z|/70). \tag{3}$$

From the close agreement of the various independent calculations that we have made, we believe Equation (3) to be a simple formula which probably models the actual

Table 1. Dispersion measures of pulsars with independently measured distances (taken from Manchester and Taylor 1981). The last column shows if the line of sight to the pulsar intersects any known H II region within 1 kpc from the Sun.

Pulsar	Distance d (kpc)	$\lvert z \rvert$ (pc)	Dispersion measure D(pc cm^{-3})	H II regions
0318 + 59	3·0	110	34·8	no
0329 + 54	2·3	50	26·776	no
0355 + 54	1·6	20	57·03	no
0525 + 21	2·0	240	50·955	no
0531 + 21	2·0	200	56·791	no
0736 − 40	2·5	400	160·8	yes
0740 − 28	1·5	60	73·77	no
0833 − 45	0·5	20	69·08	yes
0835 − 41	2·4	10	147·6	yes
1054 − 62	6·0	310	323·4	yes
1154 − 62	7·0	20	325·2	yes
1240 − 64	12·0	320	297·4	yes
1323 − 62	7·9	30	318·4	no
1356 − 50	8·8	170	295·0	yes
1557 − 50	7·8	220	270·0	no
1558 − 50	2·5	60	169·5	no
1641 − 45	5·3	20	475·0	yes
1859 + 03	11·0	120	402·9	no
1900 + 01	5·0	170	243·4	no
1929 + 10	0·08	5	3·176	no
2002 + 31	8·0	0	233·0	no
2111 + 46	4·3	100	141·5	yes
2319 + 60	2·8	30	96·0	yes

situation rather closely. We do not agree with Arnett and Lerche (1981) who claim that $\langle n_e \rangle$ cannot be known with an accuracy better than a factor of two.

2. Method

All our calculations are based on the assumption of azimuthal symmetry for the galactic pulsar population. The Sun is taken to be situated 10 kpc from the galactic centre. We describe here the basic method employed to determine a uniform mean electron density $\langle n_e \rangle$ for the whole Galaxy. We then proceed to discuss the modifications made in order to study more complicated models of n_e.

It is clear that the observed pulsar distribution will be consistent with cylindrical symmetry about the galactic centre for only a limited range of values of $\langle n_e \rangle$. Distance estimates of pulsars obtained using Equation (1) with over-large values of $\langle n_e \rangle$ would appear to move the centre of gravity of the pulsar distribution away from the galactic centre towards the Sun (after allowing for selection effects), while the converse would be true for too small values of $\langle n_e \rangle$. In our calculations we assume a value of $\langle n_e \rangle$ and compute the corresponding positions of all pulsars in the galaxy. For each pulsar we consider a circle passing through it, centred on the galactic centre and parallel to the galactic plane (Fig. 1 shows the circle projected on

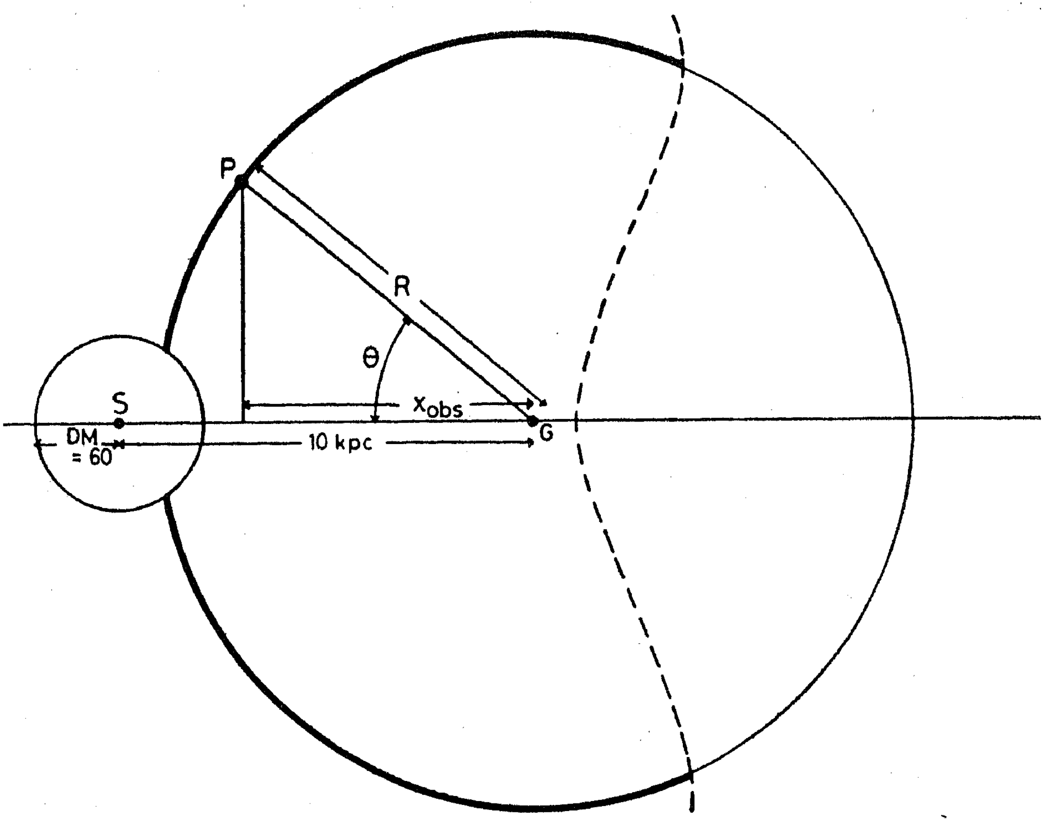

Figure 1. Schematic illustration of a typical pulsar P and its corresponding galactocentric circle, both projected onto the galactic plane. G is the centre of the Galaxy. Around the Sun S an approximately spherical volume of radius corresponding to a dispersion measure of 60 pc cm^{-3} is removed in our calculations for reasons discussed in the text. The dashed curve represents a typical viewing limit for the II Molonglo Survey. For our calculations, we require (i) $x_{obs,\, i}$, the projection of the radius PG onto the line SG, (ii) $x_{exp,\, i}$, the mean value of the projection averaged over the visible portion of the pulsar circle (thick line), and (iii) σ_i^2, the variance of the projection, obtained by averaging the deviation $(x-x_{exp,\, i})^2$ over the visible portion of the circle. These quantities are obtained for each pulsar for a given model of the galactic electron density and used in Equation (4) to compute X. Note that $|\theta|$ could have been used in place of x; however, the sensitivity of the test is then found to decrease.

to the plane of the Galaxy). We then compute x_{obs}, the projection of the derived radius vector from the galactic centre to the pulsar on to the line joining the Sun and the galactic centre. We also compute x_{exp}, the expected value of x for the circle, considering all selection effects and assuming a uniform probability of pulsar occurrence around the circle. Since for a given pulsar period and luminosity only a portion of each circle is visible to the pulsar surveys on Earth due to the various selection effects in pulsar searches (Taylor and Manchester 1977; Vivekanand, Narayan and Radhakrishnan 1982), x_{exp} is generally different from zero. Finally we compute the following mean deviation

$$X(\langle n_e \rangle) = \sum_{i=1}^{N} w_i \{x_{obs,\, i} - x_{exp,\, i}\}/\sigma_i \qquad (4)$$

where σ_i is the calculated variance on $x_{obs,\, i}$. The summation is over all the pulsars included in our calculations and w_i is a weight given to the contribution from the ith pulsar. w_i is estimated on the basis of the effective contribution of the pulsar to our test, which in turn depends upon its radio luminosity. Pulsars with high luminosity can be potentially detected far away from the Sun and are therefore best suited to test for a cylindrical distribution on a galactic scale. The lower luminosity pulsars are closer to the Sun, and so are of lesser importance for our calculations. We have investigated the sensitivity of our estimator $(x_{\rm obs} - x_{\rm exp})/\sigma$ to changes in $\langle n_{\rm e} \rangle$ and have derived a simple weighting scheme in which pulsars with radio luminosity (at 400 MHz and assuming $\langle n_{\rm e} \rangle = 0{\cdot}03$ cm^{-3}) greater than 10 mJy kpc^2 are each given a weight 1·5, those with luminosity less than 10 mJy kpc^2 but greater than 4 mJy kpc^2 are each given a weight 1·0 and pulsars with still lower luminosities are eliminated altogether. These last pulsars are very close to the Sun and only add 'noise' to the estimate of $X(\langle n_{\rm e} \rangle)$ in Equation (4). The particular choice of the projected distance x in Equation (4) was found to be more sensitive than other choices such as $|\theta|$ and was therefore used in all calculations.

Since for the best value of $\langle n_{\rm e} \rangle$, each of the terms $(x_{{\rm obs},\, i} - x_{{\rm exp},\, i})/\sigma_i$ in Equation (4) has an expected mean of 0·0 and a standard deviation of 1·0, the mean value of X is 0·0 while its variance σ_X is given by

$$\sigma_X^2 = \sum_{i=1}^{N} w_i^2. \tag{5}$$

In our calculations, we therefore accept those values of $\langle n_{\rm e} \rangle$ which lead to $(X/\sigma_X)^2 \leqslant 1$ and reject the rest.

The above procedure needs to be modified when testing more complicated electron density models. For example, in testing a model having the form of Equation (2), we need to determine two parameters, $n_{\rm e}$ (0) and z_0. We do this by testing the cylindrical symmetry of pulsars separately in low-z and high-z regions of the galaxy. We choose to divide the pulsars into two classes such that the dividing value of $|z|$ represents the median $|z|$ for the sample. For each choice of $n_{\rm e}$ (0) and z_0, we obtain $X_1, \sigma_{X_1}, X_2, \sigma_{X_2}$ for the two regions separately. Then the criterion for the acceptability of the model is that

$$\Sigma = \left(\frac{X_1^2}{\sigma_{X_1}^2} + \frac{X_2^2}{\sigma_{X_2}^2} \right) \leqslant 1. \tag{6}$$

We restricted our test to the 224 pulsars discovered by the II Molonglo Survey (II MS; Manchester *et al.* 1978) since it is the most extensive survey, and its selection effects are well understood. We have taken the minimum sensitivity S_0 to be 8·0 mJy (Manchester *et. al.* 1978), and used the modified model of the selection effects suggested by Vivekanand, Narayan and Radhakrishnan (1982). We employed three criteria to select a subsample of II MS pulsars. Firstly, all low-luminosity pulsars ($<$ 4 mJy kpc^2) are given weights $w_i = 0$ as discussed earlier. Secondly, nearby pulsars are unreliable for our purposes since the dispersion measure contribution from H II regions can have large fluctuations; this effect is expected to be less significant for more distant pulsars. Consequently, we have removed all pulsars with $D < 60{\cdot}0$ pc cm^{-3}. To be consistent, while computing $x_{\text{exp},\,i}$ and σ_i, we deleted the appropriate segments of those circles which intersect this volume. Thirdly, we have deleted all pulsars whose mean flux densities are below the detection threshold of II MS. This is necessary since we compute $x_{\text{exp},\,i}$ on the basis of the assumed detection threshold. After this selection process, we were finally left with a working sample of 52 pulsars. Fig. 2 shows the distribution of these 52 pulsars projected on the galactic plane. The distances have been computed using the optimized electron density model of Equation (3). It should be noted that very few of the pulsars lie beyond the galactic centre. Therefore our tests may be expected to have rather limited sensitivity.

3. Some simple models

We have tested a number of simple electron density models that are currently popular. Since ours is an independent test, it gives new bounds on the parameters of these models.

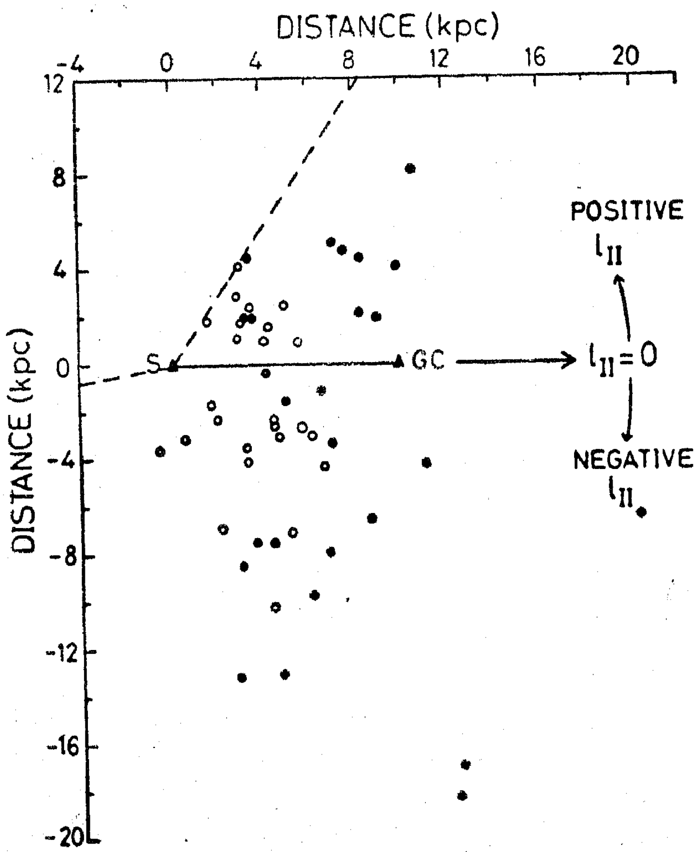

Figure 2. Positions of the 52 pulsars used in our calculations computed using Equation (3) and projected on to the galactic plane. The triangles S and GC mark the positions of the Sun and the galactic centre respectively. The dashed lines represent the longitude limits of the II Molonglo Survey in the galactic plane (corresponding to declination + 20°. Filled circles represent more luminous pulsars which are given a higher weightage (weight = 1·5) in our calculations, as compared to the medium luminosity pulsars which are represented by open circles (weight = 1·0). Note that very few pulsars lie beyond the galactic centre, which might lead to a reduction in our sensitivity

3.1 *Uniform Electron Density Model*

Using the method described in Section 2, we estimate the effective mean electron density in the Galaxy to be $\langle n_e \rangle = 0{\cdot}037^{+0.020}_{-0.012}$ cm^{-3}, where the quoted errors represent satistical fluctuations at the 1σ level. Fig. 3 shows the variation of X/σ_x as a function of the assumed $\langle n_e \rangle$ and illustrates our method of estimating the confidence limits on $\langle n_e \rangle$. Note that the lower bound is rather tight, suggesting that values below 0·025 are unlikely. This is of interest because lower values $\langle n_e \rangle$ have been commonly invoked to resolve the problem of high pulsar birthrates. We now find this improbable.

3.2 *Exponential Model*

We have studied an exponential model of the form of Equation (2) by testing the pulsar distribution separately in high-z and low-z regions (boundary chosen to divide the pulsars equally in the two regions), as described in the previous section. We obtain bounds on n_e (0) at each value of scale height z_0, based on the criterion of Equation (6). The results are shown as the two solid lines in Fig. 4. For very low z_0 values (< 250 pc), the electron density decreases very rapidly with increasing z, and it is impossible to account for the high D of certain pulsars even by placing them at infinity. The dashed line in Fig. 4 is the locus of points at which about 20 per cent of our 52 pulsars run into this problem. In our view, models lying below this line can definitely be rejected. Hall's (1980) model, marked in Fig. 4, is seen

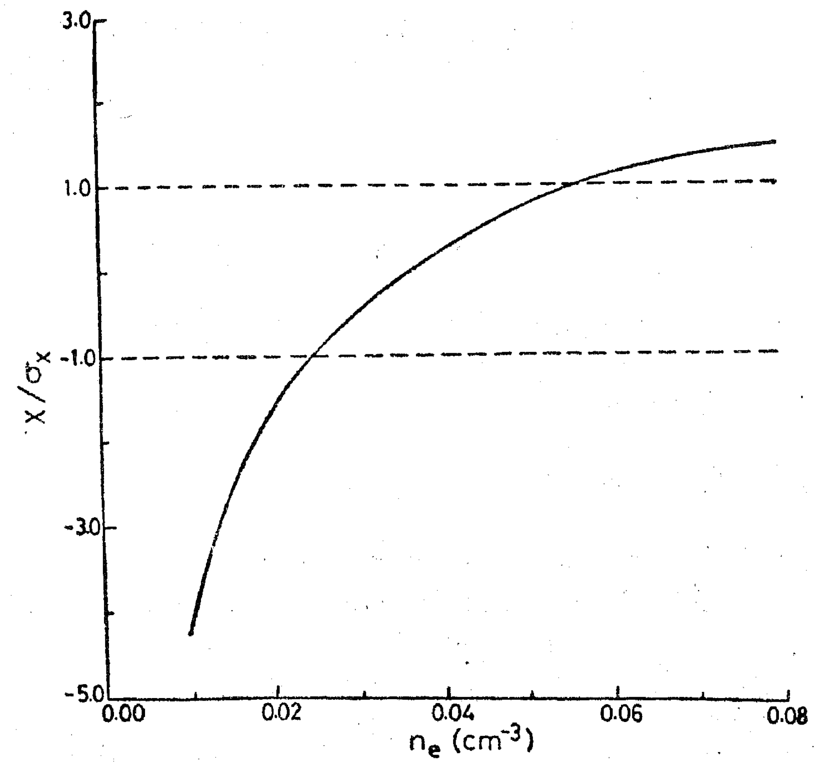

Figure 3. Computed variation of X/σ_x as a function of the assumed $\langle n_e \rangle$. Allowed values of $\langle n_e \rangle$, for which $|X/\sigma_x| \leqslant 1{\cdot}0$, lie within the dashed lines. The curve is very steep at low $\langle n_e \rangle$, allowing us to set confident lower limits on $\langle n_e \rangle$.

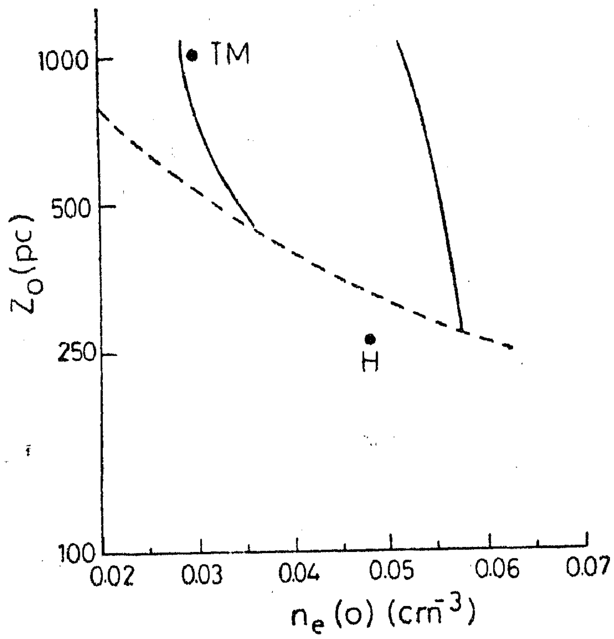

Figure 4. Results for the exponential model of n_e (Equation 2). The solid lines mark the 1σ limits of n_e (0) at each z_0. The dashed line represents points at which the model is unable to explain the observed high dispersion measures of 11 of our 52 pulsars. Models corresponding to points below this line can definitely be rejected. The models proposed by Hall (1980), and Taylor and Manchester (1977) are marked by H and TM.

outside the 'allowed region'. The widely used model proposed by Taylor and Manchester (1977) is acceptable.

Our test rejects low values of z_0. This might have some relevance to the applicability of the McKee and Ostriker (1977) model for the interstellar medium (ISM) where the ionized component (H II) of the ISM is mostly associated with the neutral (H I) clouds. Since H I clouds have a scale height $\simeq$ 170 pc (Crovisier 1978), the same value is implied for H II and hence for n_e. Our test, however, shows that this is unlikely.

3.3 *Variation of Electron Density with Galactic Radius*

We have also studied an electron density model of the form

$$\langle n_e \rangle = n_{e<}, \qquad R_{GC} < R_0,$$

$$= n_{e>}, \qquad R_{GC} \geqslant R_0. \tag{7}$$

As before, we divide the Galaxy into two regions, an inner one ($R_{GC} < R'$) and an outer one ($R_{GC} > R'$), where R' (kpc) is chosen such that each region has approximately the same number of pulsars. We accept only those combinations of $n_{e<}$ and $n_{e>}$ for which Equation (6) is satisfied. Fig. 5 shows the allowed combinations of $n_{e<}$ and $n_{e>}$ for $R_0 = 7$ kpc. There seems to be no reason to suspect significantly different values for $n_{e<}$ and $n_{e>}$, contrary to some recent suggestions. On the basis of Fig. 4 and keeping in mind the evidence of earlier studies (Ables and Manchester 1976; del Romero and Gomez-Gonzalez 1981; Harding and Harding 1982) we sug-

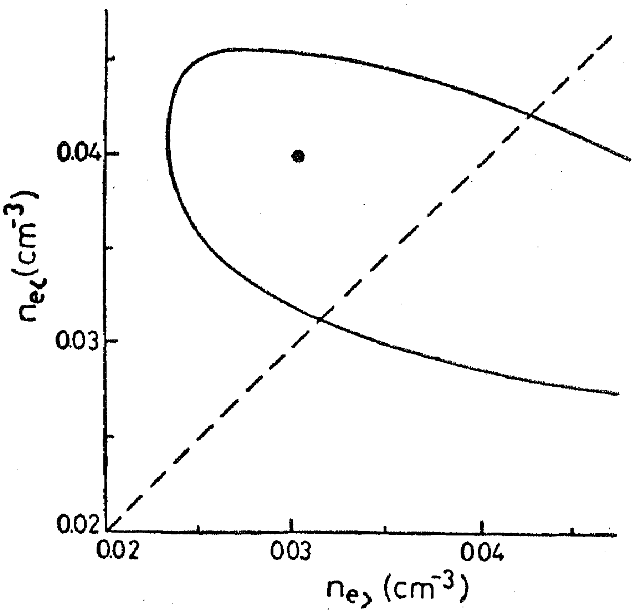

Figure 5. Allowed combinations of $n_{e<}$ (in the inner regions of the Galaxy, $R_{GC} < 7$ kpc) and $n_{e>}$ (in the outer regions, $R_{GC} > 7$ kpc) lie within the solid curve, which represents the 1σ limits on these parameters. The allowed region is nearly equally distributed on either side of the $n_{e<} = n_{e>}$ line (dashed line in the figure). Therefore a uniform electron density model for the whole Galaxy is quite adequate. If at all, $n_{e>}$ appears to be larger than $n_{e<}$. However, since other studies seem to show that $n_{e<} > n_{e>}$, we suggest the model corresponding to the dot may be close to the truth.

gest that $n_{e<} = 0{\cdot}04$ cm^{-3} and $n_{e>} = 0{\cdot}03$ cm^{-3} ($R_0 = 7$ kpc) may be a reasonable model. In fact, for pulsar studies, an $\langle n_e \rangle$ independent of R_{GC} is quite adequate. We note that our test is quite insensitive to the value of n_e in the very inner portion of the galaxy (R_{GC} below say 5 kpc) since very few of our pulsar lines of sight intersect this region. We cannot therefore rule out significantly higher n_e in this region.

4. HII regions

The results of Section 3 show that

(a) the scale height of thermal electrons is most probably quite large;

(b) there is negligible variation of electron density with galactic radius (barring the region $R_{GC} < 5$ kpc which is not very important for pulsar studies).

A constant electron density would therefore appear to be a good model for many purposes. However, we have so far neglected the effect of H II regions. If we include this contribution, a reasonable model for the electron density in the Galaxy would be (Lyne 1980)

$$n_e(z) = n_{e1} + n_{e2} \exp(-|z|/70) \qquad (8)$$

where the second term is due to HII regions which are known to have a scaleheight of about 70 pc. In this section we combine a number of different techniques in order to estimate optimum values of n_{e1} and n_{e2}.

(i) The methods of Sections 2 and 3 can be applied to a model of the type of Equation (8) by dividing pulsars into high and low z categories as before and requiring that Equation (6) be satisfied. The curve labelled A in Fig. 6 shows our results. All points within this curve in the n_{e1}–n_{e2} space are 'allowed' and those outside are unlikely.

(ii) Table 1 shows 23 pulsars for which reliable independent distances are available (Manchester and Taylor 1981). 13 other pulsars for which only distance limits are known have been omitted. For a pulsar at distance d and galactic latitude b (hence $z = d \sin b$), Equation (8) leads to the following expression for the dispersion measure

$$D = n_{e1}\, d + n_{e2}\, d', \tag{9}$$

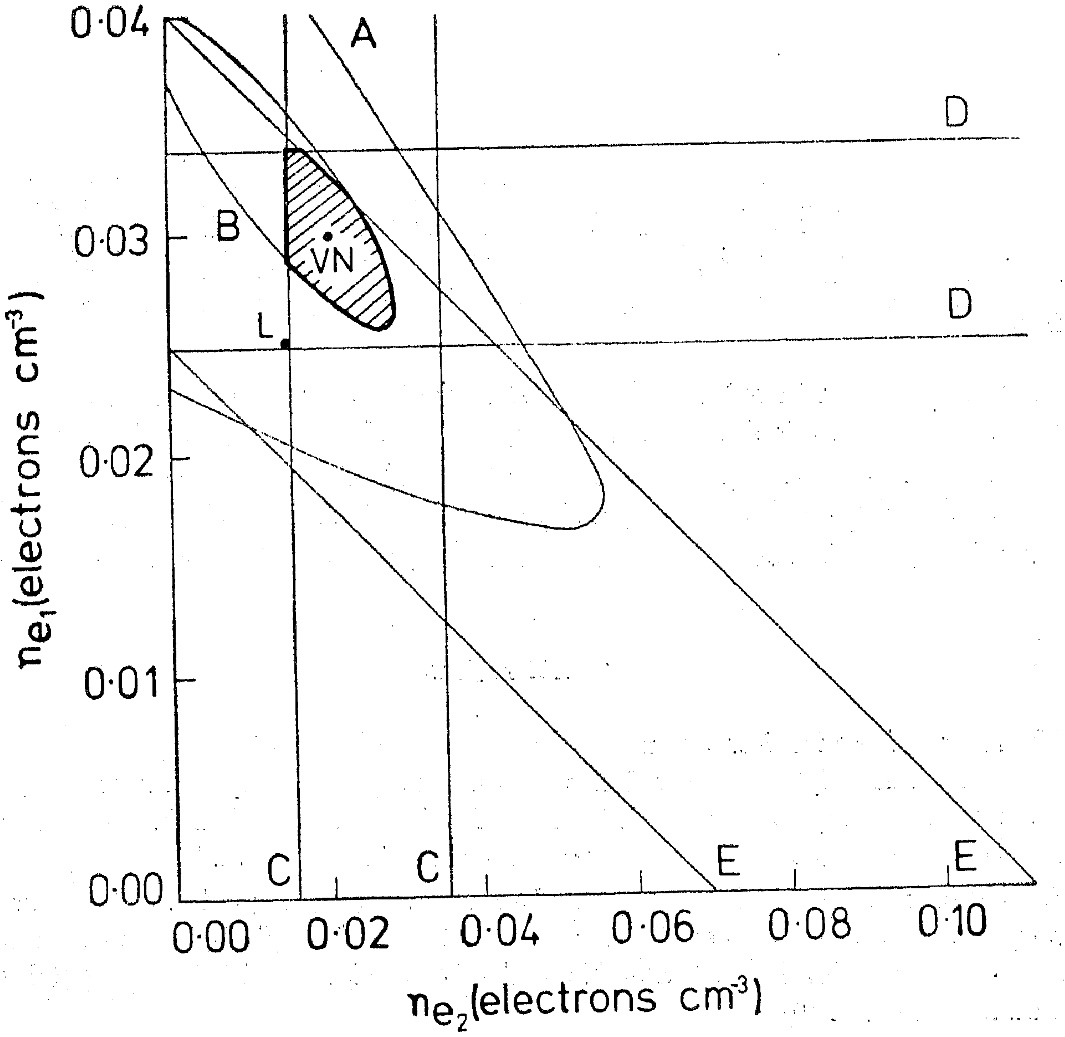

Figure 6. Optimization of the parameters n_{e1} and n_{e2} in an electron density model of the form of Equation (8). Curves labelled from A through E show the respective allowed regions in the n_{e1}–n_{e2} space based on five relatively independent arguments: (a) cylindrical symmetry of the pulsar distribution in the Galaxy, (b) independent pulsar distances of Table 1, (c) calculation of H II region contribution to the dispersion measures as evaluated by Prentice and ter Haar (1969), (D) independent distances of pulsars whose lines of sight do not intersect a known H II region, (e) results of del Romero and Gomez-Gonzalez (1981). The allowed region common to all the five arguments is shown hatched in the figure. The dot in the centre of this region represents our model (Equation 3). Lyne's (1980) model is marked L.

where

$$d' = \frac{70}{|\sin b|}[1 - \exp(-|z|/70)]. \tag{10}$$

Here d' is an effective path length through the H II regions zone of the Galaxy (of electron density n_{e2}). Using the data in Table 1 one can determine n_{e1} and n_{e2} by minimising

$$R = \sum_{i=1}^{18} (n_{e1}\, d_i + n_{e2}\, d_i' - D_i)^2/d_i'. \tag{11}$$

This leads to $n_{e1} = 0{\cdot}0327$ cm^{-3}, $n_{e2} = 0{\cdot}0138$ cm^{-3}. The 1σ permitted region is marked by the curve B in Fig. 6. It is gratifying that curves A and B, obtained by quite independent means, are consistent with each other. Substituting the above values of n_{e1} and n_{e2} into Equation (11) one obtains a value of R which corresponds to a dispersion measure fluctuation of 54·7 pc cm^{-3} per kpc path length. Since the mean D per kpc is itself only of the order of 35 pc cm^{-3}, this shows that the H II regions, if not treated properly, can completely mask the proportionality between D and d at small distances.

(iii) For distances within 1 kpc from the Sun, Prentice and ter Haar (1969) have developed a scheme to treat the known H II regions individually. We have used their scheme to analyse 217 pulsars with computed distances greater than 1 kpc [out of 302 pulsars listed by Manchester and Taylor (1977) and Manchester *et al.* (1978)]. Considering only the lines of sight of these 217 pulsars within 1 kpc of the Sun, we find they have a cumulative d' (Equation 10) of 136·9 kpc and a cumulative D from H II regions of 3225·4 pc cm^{-3}. This corresponds to

$$n_{e2} = 0{\cdot}0236 \text{ cm}^{-3}. \tag{12}$$

Making liberal allowance for errors, we can safely expect

$$n_{e2} > 0{\cdot}0236/1{\cdot}5 = 0{\cdot}0157 \text{ cm}^{-3}; \qquad n_{e2} < 1{\cdot}5 \times 0{\cdot}0236 = 0{\cdot}0353 \text{ cm}^{-3}. \tag{13}$$

These limits have been plotted as the vertical lines marked C in Fig. 6. It is significant that the range of n_{e2} in Equation (13) is in reasonable agreement with that obtained by the method in (ii). Also, the fluctuation in D calculated by the Prentice ter-Haar formula is 43·3 pc cm^{-3} per kpc path length which agrees well with 54·7 pc cm^{-3} per kpc estimated in (ii). All these suggest that the Prentice ter-Haar correction is quite reliable in an average sense, though, in individual cases, it might be significantly in error.

(iv) We have tried to approximately estimate n_{e1} as follows. 13 pulsars in Table 1 do not intersect any of the Prentice ter-Haar H II regions within 1 kpc of the Sun. If we leave out PSR 1323 + 62 and PSR 2002 + 31, the cumulative d' of the others, outside the 1 kpc sphere, is only 10·3 kpc while their cumulative d (including the 1 kpc sphere) is 38·8 kpc. These numbers suggest that these 11 pulsars mostly sample n_{e1} and interact very little with n_{e2}. We can therefore estimate n_{e1} by means of

$$n_{e1} = \left(\sum_{i=1}^{11} D_i - n_{e2} \sum_{i=1}^{11} d_i' \right) \Big/ \left(\sum_{i=1}^{11} d_i \right) \tag{14}$$

where any reasonable value of n_{e2} may be used. Using the limits on n_{e2} given in Equation (13) and also allowing for the fluctuations in D due to H II regions, we obtain the following limits on n_{e1}

$$0{\cdot}0248 \text{ cm}^{-3} < n_{e1} < 0{\cdot}0337 \text{ cm}^{-3}. \tag{15}$$

These are plotted as the horizontal lines D in Fig. 6.

(v) Del Romero and Gomez-Gonzalez (1981) have estimated that the effective $\langle n_e \rangle$ for regions out to about 5 kpc from the Sun is about 0·03 electrons cm^{-3}. By Appendix 1, this implies for the model in Equation (8),

$$\langle n_e \rangle = n_{e1} + 0{\cdot}358\, n_{e2} = 0{\cdot}03 \text{ cm}^{-3}. \tag{16}$$

Del Romero and Gomez-Gonzalez (1981) have not given confidence limits for their estimate of $\langle n_e \rangle$. However, a study of their Fig. 2 suggests that the following are very safe bounds

$$0{\cdot}025 \text{ cm}^{-3} < \langle n_e \rangle \; (= n_{e1} + 0{\cdot}358\, n_{e2}) < 0{\cdot}04 \text{ cm}^{-3}. \tag{17}$$

These lines (marked E) have also been drawn in Fig. 6.

Combining all the above results we see in Fig. 6 that the parameters of Equation (8) are rather well determined. The hatched region shows the ($n_{e1} - n_{e2}$) parameter space that is common to all the different approaches. Our choice for a good model is marked VN near the centre of this region and corresponds to Equation (3). This formula should be used only beyond 1 kpc from the Sun. Within the 1 kpc sphere, we suggest using $n_{e_1} = 0{\cdot}030$ along with the Prentice and ter Haar (1969) correction for H II regions.

The model of Lyne (1980) is marked L in Fig. 6. While it is by no means impossible, we believe our choice (Equation 3) is probably a better approximation to reality. In any case, the results of Fig. 6 show that our knowledge of the galactic electron density is by no means as limited as it has been claimed. The model given by Equation (3) can be used in future pulsar studies with good confidence. We do not expect more than about ~ 20 per cent error on the average (Fig. 6) though, in individual cases, the error may be somewhat larger.

5. Discussion

We have ignored some effects which could possibly affect the validity of our results.

(i) Although it is known that pulsars are found preferably along the spiral arms in the Galaxy (del Romero and Gomez-Gonzalez 1981), we have assumed that the pulsar distribution is cylindrically symmetric around the galactic centre. We believe that, in an average sense, the spiral arm structure can be treated as a cylindrically symmetric system. For example, the distribution of pulsar galactocentric longitudes would be essentially uniform, in spite of the spiral structure. Therefore our simplifying assumption is unlikely to introduce any large systematic error in our results.

(ii) In our calculations, we have treated the H II regions in terms of an equivalent uniform electron density medium. However, the calculations in the previous Section 4 (ii and iii) show that for small distances (< 2 kpc) the D contribution from H II regions can fluctuate considerably. Thus, at such small distances, the proportionality between D and d (Equation 1) which is fundamental to all our calculations may not be valid. We have been cautious in this matter by deleting from our calculations a volume around the Sun of radius approximately 2 kpc ($D \leqslant 60$ pc cm^{-3}). However, even at large distances, some fluctuations in $\langle n_e \rangle$ will be present, which we have ignored. Therefore the statistical errors we have quoted may be underestimated.

(iii) We have not incorporated any selection effects due to interstellar scattering (ISS) of pulsar radiation. ISS increases with increasing D; hence we might miss high D pulsars. This is believed to be strongest in the inner regions of the Galaxy (say, $|l^{\rm II}| < 30°$; Rao 1982, personal communication). However, since the number of pulsars involved in this effect is small, we believe our results will not be significantly affected.

(iv) We have assumed the distance to the galactic centre to be 10 kpc. If the true distance is d, say 8·7 kpc (Oort 1977), then our electron density estimates will need to be multiplied by a factor $(10/d) = 1{\cdot}15$.

None of the above effects is very serious. We therefore believe Equation (3) can be used with confidence in pulsar studies as a reasonable approximation to the galactic electron distribution.

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Appendix 1

Let the scale height of electrons be z_e

$$i.e.\quad n_e(z) = n_e(0) \exp(-|z|/z_e). \tag{A1}$$

Consider a pulsar at height z and galactic latitude b (hence distance $d = |z|/\sin b$). Its dispersion measure is given by

$$D = \frac{n_e(0)\, z_e}{|\sin b|}[1 - \exp(-|z|/z_e)]. \tag{A2}$$

The ' mean ' electron density for this pulsar is

$$\langle n_{\mathrm{eff}}(z)\rangle = \frac{n_{\mathrm{e}}(0)\, z_{\mathrm{e}}}{|z|}[1 - \exp(-|z|/z_{\mathrm{e}})]. \quad \text{(A3)}$$

Let pulsars also be distributed exponentially with scale height z_{p}. Then the effective electron density for the whole pulsar population is

$$\langle n_{\mathrm{e}}\rangle = \frac{n_{\mathrm{e}}(0)\, z_{\mathrm{e}}}{z_{\mathrm{p}}} \int_0^\infty \frac{1}{z}\{1 - \exp(-z/z_{\mathrm{e}})\} \exp(-z/z_{\mathrm{p}})\, dz$$

$$= \frac{n_{\mathrm{e}}(0)\, z_{\mathrm{e}}}{z_{\mathrm{p}}} \ln(1 + z_{\mathrm{p}}/z_{\mathrm{e}}). \quad \text{(A4)}$$

Taking $z_{\mathrm{e}} = 70$ pc (as for H II regions) and $z_{\mathrm{p}} = 350$ pc, we obtain

$$\langle n_{\mathrm{e}}\rangle = 0{\cdot}358\, n_{\mathrm{e}}(0). \quad \text{(A5)}$$

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